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Equity Pricing: Perfect Foresight versus Rational Expectations

Larry Fogelberg, Chung Baek

Department of Economics and Finance, Sorrell College of Business, Troy University, Troy, AL, USA
Email: cbaek@troy.edu

Abstract

This study demonstrates that when the length of the excess earnings period is not known with certainty, all rational expectations pricing models result in some degree of overpricing when compared ex post facto to perfect foresight models. This study examines the time paths of price under existing valuation models such as Baek *et al.* [1] and Ohlson and Jeuttner-Nauroth [2] under the following stylized facts: we assume that we are dealing with an all-equity firm with opportunity cost of equity of r, and with a proprietary technology which enables it to achieve a marginal return on equity of $R > r$ for approximately N periods, after which a Schumpeterian event S is predicted to occurs and the marginal return on equity is expected to revert to the opportunity cost of equity r. Our study demonstrates a deviation of the predicted rational expectations price from the perfect foresight price and demonstrates that such deviation may become extreme near the end of the excess earnings period, resulting in a catastrophic price adjustment when that period comes to an end.

1. Introduction

In his original formulation of the rational expectations (RE) hypothesis, Muth [3] confined his examination to the special case where: 1) The random disturbances are normally distributed; 2) Certainty equivalents exist for the variables to be predicted; and 3) The equations of the system, including the expectations formulas, are linear. Based upon those assumptions, Muth [3] and others were able to make a number of predictions regarding price fluctuations over time. In particular, RE predicts that, at any one point in time, asset prices should be the

best unbiased estimate of future prices, given the existing information available to market participants. Further, asset prices fluctuations over time should either converge to the perfect foresight price where shocks decay or follow a random walk pattern where shocks have permanent impact. Sargent [4] follows Muth [3] and applies similar assumptions. Feltham and Ohlson [5] apply classical RE methodology to equity pricing, and produce a model of equity pricing which, based upon their assumptions, is unbiased. That study, as well as many others before it, assumed that all shocks to the system would be *iid N(0, σ)*. Then, came the stock market "bubble" of 1999-2000, when catastrophic price adjustments occurred. Many analysts attributed these catastrophic price adjustments to the preexistence of a "bubble," which supposedly had been brought on by "irrational exuberance" on the part of investors. Whether that was an accurate assessment is not debated here; but rather we challenge the assumption that any such catastrophic price decline must necessarily be taken as ex post facto evidence of a preexisting "bubble;" and we challenge the assumption that any such catastrophic price decline must necessarily be taken as ex post facto evidence of a preexisting deviation from rational expectations pricing. Our study challenges those assumptions by providing a simple counterexample.

Recent valuation studies have addressed the problem that all firms will eventually exhaust their opportunities for positive NPV projects. One such study, Feltham and Ohlson [5], addressed the problem where marginal return on equity will eventually converge to the opportunity cost of equity capital. Subsequent to the publication of the original Feltham-Ohlson study, a number of studies have supported the assumption there exists some finite excess earnings period during which the firm will enjoy positive NPV opportunities, after which the marginal return on equity will return to the opportunity cost of equity. Ohlson [6] and Penman [7] acknowledge a finite period during which positive NPV opportunities exists and during which marginal ROE is expected to exceed the opportunity cost; and Soffer [8] provides an important clarification in pointing out that the exhaustion of positive NPV projects does not necessarily result in a return of average, or overall ROE to its opportunity cost, merely that marginal ROE would return to the opportunity cost. While Damodaran [9] and Koller *et al.* [10] have yet to incorporate this assumption into their pricing models, it is now a generally accepted assumption that valuation models should presume that positive NPV opportunities would be exhausted at some point. Baek *et al.* [1] demonstrate that the projected excess earnings period is relatively short, even among the Dow Thirty stocks. While Baek *et al.* [1] claim that ROE would return to the opportunity cost, it is clear from their model that they meant that the marginal ROE would return to the opportunity cost (Causing overall ROE to asymptotically approach the opportunity cost of equity).

Baek *et al.* [1] propose a model where the firm enjoys a marginal return on equity R which is higher than the opportunity cost of equity r, but which is only expected to persist for a finite number of periods N. In this study, we examine the implications of rational expectations pricing where the length of the excess

earnings period is not known with certainty. We propose a model of equity pricing in which we only know the probability that the excess earnings period will come to an end in any given period. We propose that such a model may be a better representation of reality in a high technology economy. No one knows when the next major innovation will occur which will extinguish all future positive NPV opportunities from our present business model; and we have no absolute knowledge of how long it will be before that event occurs. Further, we do not expect to have any update of how much time is remaining until the event occurs and our excess earnings period has come to an end. Our challenge goes to the heart of the question of what we know and when we know it. Thus, the theoretical contribution of our study is to show how the predicted rational expectations price deviates from the perfect foresight price with the occurrence of a Schumpeterian event by examining the time paths of price under existing valuation models and also, demonstrate that such deviation may become extreme near the end of the excess earnings period, resulting in a catastrophic price adjustment when that period comes to an end.

Following Baek *et al.* [1], we adopt the following stylized facts: we assume that we are dealing with an all-equity firm with opportunity cost of equity of r, and with a proprietary technology which enables it to achieve a marginal return on equity of $R > r$ for precisely N periods beyond the end of the current period, after which a Schumpeterian event, S is predicted to occur, making that proprietary technology obsolete and ending that excess earning period. After event S, the marginal return on equity reverts to the opportunity cost of equity r.

2. Perfect Foresight versus Rational Expectations

Under such conditions of perfect foresight, where the length of the excess earning period is known with perfect foresight, Baek *et al.* [1] predict that the firm will follow the optimal retention policy of retaining all earnings during the excess earnings period. Since they use discrete-time valuation, we convert it into continuous-time valuation. Then, under their optimal retention policy, the following price is obtained with an initial earnings per share of α and a retention ratio of λ.

$$P_0 = \frac{\alpha e^{(R-r)N}}{r}$$

Our study does not challenge the substance of Baek *et al.* [1], other than to challenge its assumption of perfect foresight. By definition, a Schumpeterian event is one which makes existing technologies obsolete, and by extension, we would expect most Schumpeterian events to occur unexpectedly. Thus, we propose that a more realistic pricing model would be one where the length of the excess earnings period is not known with perfect foresight, but only estimated, based upon some marginal distribution function $f(x)$.

$$P_0^* = \frac{\alpha}{r} \int_0^\infty e^{(R-r)x} f(x) \, dx$$

For simplicity, we begin with the stylized facts where we do not know the exact length of the excess earnings period, but only the constant probability $m = 1/N$ that it will end in any one year.

$$\frac{f(x)}{1-F(x)} = m$$

We assume that our estimate of the length of the excess earnings period is unbiased.

$$\int_0^\infty x f(x) \, dx = N$$

This results in the constant stopping rate or $m = 1/N$ and the following marginal distribution functions, $f(x)$, is used.

$$f(x) = m e^{-mx}$$

Thus, our rational expectations price can be summarized by the following formula.

$$P_0^* = \frac{\alpha}{r} \int_0^\infty m e^{-mx} e^{(R-r)x} \, dx = \frac{\alpha}{r}\left(\frac{m}{r-R+m}\right)$$

where the sufficient condition of convergence is that $R - r - m < 0$. This of course assumes that state $S_t \in \{0,1\}$ is uncorrelated with the rate of returns on the market M_t. In other words, it assumes that

$$Cov(dS_t, M_t) = 0$$

where $S_t \in \{0,1\}$. The first observation which we make is that rational expectations pricing is biased v is a v is perfect foresight pricing. To illustrate, we assume an initial earnings per share of 2, a marginal return on equity of .10, and an opportunity cost of equity of 0.06, with an expected excess earnings period of 10 years. Under those assumptions, we have a perfect foresight share price $P_0 = 49.73$ versus a rational expectation share price $P_0^* = 55.56$. Further, we can generalize these results to include all distributions $f(x)$ with mean N. We begin with the insight that

$$P^*(N) = \int_0^\infty f(x) P(x) \, dx$$

where $f(x)$ has a mean of N. Further, we can simplify $P(x)$ to read

$$P(x) = ka^x$$

where $k = \dfrac{\alpha e^{Rt}}{r}$, and $a = e^{(R-r)}$. This greatly simplifies the extraction of first and second derivatives. Notably,

$$\frac{\partial P}{\partial x} = ka^x \ln a$$

$$\frac{\partial^2 P}{\partial x^2} = ka^x (\ln a)^2 > 0$$

Thus, we demonstrate that $P(x)$ is convex in x. Further, we demonstrate by

Jensen's inequality that

$$P_t^*(N) > P_t(N)$$

for all $t \geq 0$, all $N > 0$ and for all distributions $f(x)$ with mean of N.

Thus, we are able to demonstrate that there will always be some "overpricing" under rational expectations and under these stylized facts, so long as there exists any uncertainty regarding the time remaining in the excess earnings period; and this will continue to hold even when there is some partial updating of our estimate of the time remaining.

3. Time Path of Equity Price

3.1. Time Path of Price under Perfect Foresight

Having established the fact of rational expectations "overpricing", we now provide evidence of the degree of such "overpricing". To do so, we examine the further divergence of the rational expectations price from the perfect foresight price as we move forward through the excess earnings period.

Under our perfect foresight model, infinitesimal price increase is proportional to the opportunity cost of equity r.

$$P_t = \frac{\alpha e^{Rt}}{r} e^{(R-r)(N-t)}$$

$$\frac{\partial(\ln P_t)}{\partial t} = r$$

3.2. Time Path of Price under Rational Expectations

By contrast, our rational expectations model produces infinitesimal price increase is proportional to the marginal return on equity R.

$$P_t^* = \frac{\alpha e^{Rt}}{r}\left(\frac{m}{r - R + m}\right)$$

$$\frac{\partial(\ln P_t^*)}{\partial t} = R$$

Of course, this cannot persist forever. Eventually, Schumpeterian event S will occur which would bring the excess earnings period to a close, leading to a post-event price of P^{**}, where

$$P_t^{**} = \frac{\alpha e^{Rt}}{r}$$

and this would represent a potentially catastrophic price decline, where

$$\frac{P_t^{**} - P_t^*}{P_t^*} = -(R - r)N$$

3.3. Numerical Example

To illustrate, we return to our numerical assumptions where we assumed initial earnings per share of 2, a marginal return on equity of 0.10, an opportunity cost

of equity of 0.06, an expected excess earnings period of 10 years. As we demonstrated above, the initial price under perfect foresight was 49.73; and the initial price under rational expectations was 55.56.

Moving forward, the expected price increases under perfect foresight would be precisely proportional to 0.06, or the opportunity cost of equity. By contrast, the expected price increases under the rational expectations model would be proportional to 0.10, causing the two price estimates to diverge even further as we move forward through the excess earnings period.

Further, the PE ratios under the two models will continue to diverge. Under perfect foresight, the PE ratio will gradually decline from 24.87 to 16.67 over the 10-year excess earnings period, while the PE ratio will remain fixed at 27.78 under rational expectations until S occurs. At that time, P_{t+1}^{**}/P_t^* will be a catastrophic 0.6, reflecting a price decline of almost 40 percent, as the PE ratio declines from 27.78 to 16.67 in a single period.

4. Conclusions

Clearly, we have departed from the special case of the rational expectation (RE) hypothesis, as originally formulated by Muth [3]. We have presented as set of stylized facts where disturbances are not normally distributed; and we have presented a set of stylized facts where relationships among variables are clearly non-linear. In spite of the restricted assumptions of the original RE hypothesis, it continued to generate valid predictions so long as we were dealing with disturbances which become vanishingly small over vanishingly small intervals of time. Over vanishingly small intervals of time, all relationships among continuously derivable variables become linear. Further, given such assumptions as incorporated into the standard diffusion process, the problems of bias become manageable.

But such is not the case in the example we have provided. We have demonstrated that none of the proposed rational expectations models will help us to anticipate catastrophic price adjustments, given the stylized facts of this study.

Therefore, let us examine our stylized facts to see if they represent the true state of the economy. We assumed that the length of the excess earnings could only be estimated. In the first model we presented, we assumed that the only information available to the investor was the probability that S would occur at the end of any given period. Is that a realistic picture of our economy?

Actually, in most cases we cannot even determine the direction from which an S event will break upon us, much less calculate the time of its arrival. So that aspect of our stylized facts appears to be realistic.

With increasing interrelationships among technologies, where each new technology is expected to have an impact—either negative or positive—upon a host of related technologies, we could expect Schumpeterian events to impact entire sectors of the economy, and not just one or two firms. And because of this possibility, we might no longer be able to claim that S is uncorrelated with the market as a whole; and that leads to problems of establishing a rational expectations price which is beyond the scope of this study.

Further, our stylized facts are consistent with the observation that PE ratios remain relatively fixed for long periods of time, until catastrophic declines occur.

Fama and French [11] provided evidence of long term negative autocorrelation of stock returns, which would be consistent with our stylized facts; and the evidence for "excess volatility", provided by Shiller [12] [13], may possibly be explained by Bayesian adjustments to the estimated length of the excess earnings period. Of course, this is speculation, but what we have clearly demonstrated is that catastrophic price declines can occur at the end of the excess earnings period, even when we know in advance that such events are inevitable, and even when we have an approximate idea of how long the excess earnings period is expected to last. What is clear from this study is that such catastrophic price declines are not necessarily evidence of "irrational exuberance", or of preexisting pricing error.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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