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Pricing Currency Call Options

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Abstract

This paper presents a theoretical model to price foreign currency call options. Currency options are employed in international trade to reduce the risk of loss due to the reduction of revenues obtained in depreciating foreign currency for an exporter, or the escalation of expense from appreciating foreign currency for an importer. Other users include banks and hedge funds who engage in currency speculation. Given the fluctuation of option prices over time, the model describes the distribution of foreign currency as a Weiner process for macroeconomically constrained foreign currencies followed by a Laplace distribution for unconstrained currencies. In a departure from existing currency option models, this model expresses foreign currencies as dependent upon the change in macroeconomic variables, such as inflation, interest rates, and government deficits. The distribution of currency calls is described as a Levy process in the context of an option trader's risk preferences to account for the multiple discontinuities of a jump process. The paper concludes with three models of price functions of the Weiner process for Euro-related currency options, a Weiner process for stable currency options, and a Levy-Khintchine process for volatile currency calls.

Keywords

Laplace, Jump Process, Levy Process, Currency Calls, Weiner Process

1. Introduction

A foreign currency call option permits a buyer to purchase foreign currency at the exercise price for a specific time period for an option premium paid to a call writer. To an exporter facing appreciating foreign currency, the exercise of an option limits foreign currency losses given the purchase of the currency at a cheaper rate. The benefit over purchasing a futures or forward contract is that an option is a nonbinding obligation [1]. A futures or forward contract would require conversion at a binding rate; If exchange rates were to fall, the buyer would

be constrained to convert at the forward rate, sustaining losses. The call option buyer would permit the option to expire, so that the loss would be limited to the negligible amount of the premium, not the entire contract. Foreign currency options further protect against contingency risk exposure. [1] cited the case of an exporter who placed a bid to hedge a receivable. Uncertain that the bid would be accepted, the exporter purchased a put option, which would expire if the bid was rejected, or be exercised to yield the receivable at the highest value of the foreign currency if the bid was accepted. [2] recognized the value of options in reducing exporter risk in their study of Korean firms which found that firms with higher levels of export revenue and foreign currency debt employed options for hedging purposes as did small Indian firms in financial distress and those with less liquid assets [3]. Speculation in foreign currencies abounds with the top 150 US banks being found to reduce portfolio risk with currency derivatives along with their non-bank counterparts ([4] [5]). Risk reduction strategies are being pursued at multiple locations. For example, in India, trading volume in currency options has grown 300% from $940,300 in November 2010 to $2,858,209 in March, 2011 ([6]). In direct competition with banks, firms such as Citadel ([7]) and Gain Capital offer currency options in up to 20 currencies with order entry, tracking reports, and profit-and-loss statements ([8]) both in the United States and in the UK. In Germany, the derivatives exchange, Eurex, has been offering foreign currency options on the Euro with the US Dollar, Euro with the British pound, and Euro with the Swiss Franc, along with the British pound with the US dollar and British pound with the Swiss Franc [9].

This paper proposes a theoretical model to price currency call options. The model contributes to the literature in three ways. It recognizes that an option is a derived instrument. In itself, the option has minimal value. Its true value is based on the value of the underlying asset, the foreign currency. As the value of an option is partly due to the change in price of the option over its life until expiration, we first present the time distribution of the foreign currency. Countries within the European Union are constrained to maintain stability in foreign exchange rates, whereas those in other countries are not held to any restrictions. This paper presents Weiner processes as two constrained foreign currency distributions, and the Laplace distribution as the trajectory of unconstrained exchange rates. This is in contrast to option-only models ([10] [11] [12]) which do not include the time distribution of the foreign currency. Secondly, the paper views foreign currency rates as being dependent on macroeconomic variables. Many existing models overlook macroeconomic variables preferring to be market-based ([12] [13]), or jump diffusion-based ([14] [15] [16]). Thirdly, this paper recognizes that options have higher volatility than the foreign currency. A market model based on the low volatility of the stock market or a jump diffusion model with sporadic currency peaks and troughs may be less effective in describing foreign currency movements than this paper's proposed Levy process with jump discontinuities. The model accounts for varying skewness and kurtosis by incorporating actual measures of each condition into the model, instead of

merely employing high volatility as a variable in a low-volatility equity option model.

The remainder of this paper is organized as follows: Section 2 is a Review of Literature with a description of Macroeconomic Determinants of Exchange Rates and literature on the Distributions of Foreign Currencies and Foreign Currency Options, Section 3 is a Quantitative Model of Foreign Currency Distributions, Section 4 is a Quantitative Model of Foreign Currency Option Distributions, Section 5 provides a Quantitative Model of Pricing Foreign Currency Options, and Section 6 describes Conclusions and Recommendations for Future Research.

2. Review of Literature

2.1. Macroeconomic Determinants of the Value of Foreign Currency

Inflation Rates, Interest Rates, Current Account, and Government Debt. The relative Purchasing Power Parity theory maintains that countries with higher inflation rates experience currency depreciation. As domestic prices rise, each unit of currency diminishes in value, necessitating the use of more units of domestic currency to purchase foreign goods. Consequently, the domestic currency will depreciate ([17]). [18] conclude from an extensive review of tests of the Purchasing Power Parity theory that it explains long-term volatility in the exchange rates of several Western European countries. Higher interest rates attract investors to a country causing a rise in currency exchange rates. A current account deficit usually stems from a negative trade balance. The country spends more of its currency on imports, rather than earning through the sale of exports causing depreciation of the domestic currency. In India, the high inflation and reduction in interest rates from 1990-2016 led to depreciation of the Indian rupee ([19] [20]). While a positive trade balance and increasing foreign reserves ([21]) assisted in increasing currency values, the persistence of a current account deficit depreciated the rupee's value ([20]). Likewise, [22] observed that an unfavorable external trade balance and decline in foreign reserves depleted the current account to the point of continuously declining currency values for a Romanian sample from 2007-2011.

A country with high government debt will be challenged to retain foreign capital. Foreign investors will sell their government bonds depreciating the exchange rate. [23] empirically proved that government debt was a determinant of the declining value of the Polish Zloty. Additionally, fiscal deficits may have a deleterious impact on the volatility of exchange rates as observed in a study of 85 developing and transition economies ([24]). [25] set forth that in the U.K., Austria, Switzerland, Finland, France, and Ireland, the exchange rate is a function of the short-term interest rate, inflation, and the external trade balance, finding that these variables explained exchange rate movements over a 36 month - 48 month period in the pre-1991-1999 period before the formation of the European

Union. [26] described the macroeconomic parameters to qualify for the European Union's exchange rate target of 0% - 1% of normal rates. They include 0% - 0.5% fluctuation in the inflation rate, 0% - 0.7% change in the long-term interest rate, and 0% - 0.25% change in government debt.

Terms of Trade, Political Stability and Speculation. The Harrod-Balassa-Samuelson model posits that the rise in productivity of manufactured exports leads to higher wages. To compete for labor, businesses producing nontradable goods raise wages. The rise in prices of both exports and nontradables results in an increase in currency values, ceteris paribus. [27] empirically justified this model for the United Kingdom, Austria, Switzerland, Denmark, and Italy from 1999-2007. [28] attribute the appreciation of the yen to the dollar largely to Japan's trade surplus with the United States.

Additionally, a country with less political conflict attracts foreign investors, increasing its exchange rate. Speculation exists in that [29] and [30] observed that currency movements in the deutsche mark-U.S. dollar and yen-U.S. dollar pairs were explained by macroeconomic news, including changes in inflation, the money supply, interest rate differentials, and real incomes. Informed traders purchase currency call options in the days before a macroeconomic announcement such as announcement of a change in the money supply or interest rates. This action bids up the prices of the call options, leading to an appreciation of currency values.

2.2. Research on the Distributions of Foreign Currencies

Stochastic foreign currency distributions were historically considered to be similar to distributions describing the movements of stocks. Consequently, the first class of distributions was the stationary Paretian stable or Student t distribution, as in [31] who found evidence of a Paretian stable for 1970s data and [32] who supported the Student t distribution in the 1980s. [33] observed, with the approach of the 1990s, that nonstationarities in single distribution data could be overcome with mixed (Paretian and Student t distributions. All of these distributions were leptokurtic, with thick-tails. [34] identified such distributions as normal distributions with time-varying parameters. He observed that the deviations from predicted foreign currency paths were related to previous deviations, but failed to follow a normal distribution. He termed these periods as following an autoregressive continuous heteroscedasticity process (ARCH) and solved a log likelihood estimator that maximized the product of the conditional densities. The problem of leptokurtosis was solved for the sporadic periods of discontinuities. However, leptokurtosis persisted for other periods, so that [35] and [36] proposed mixed jump-diffusion models in which a lognormal distribution with small variations from predicted currency paths had jumps imposed upon it to account for sharp discontinuities. [35] found that although it improved explanatory power of foreign currency movements for the British pound and Japanese yen from 1973-1985, it failed to account for discontinuities in weekly or daily

data. [37] concurred with the jump diffusion process providing some explanatory power over the aforementioned ARCH process and Paretian stable for 1974-1985 data, though it failed to account for all discontinuities in all periods and subperiods studied. [36] investigated the discontinuities finding that they were time-varying ARCH processes with variances due to specific country factors.

We conclude that a distribution that accounts for leptokurtosis with frequent jumps regularly rather than during a few time periods may be the optimal choice. The [36] finding of the relationship of foreign currency with country-specific factors supports our position that models should include the variation of currency values with macroeconomic variables.

2.3. Research on the Distributions of Foreign Currency Options

How do currency option distributions differ from currency distributions? A call option on foreign currency, for a small investment, promises substantial upside potential, beyond the profits from the increases in the foreign currency. The higher profit potential suggests higher risk for the option. The arrival rate of information is lumpy and discontinuous. Option volumes contain forecasts of future events that affect currency values including tariffs, capital flight, or tax cuts which increase government debt. As option trading firms use superior forecasting tools and have access to the expertise of seasoned traders, they obtain accurate forecasts of macroeconomic variables. Positive news suggests that the exchange rate will rise above the forecasted forward rate, while negative news indicates an actual exchange rate below the forward rate. This may explain the prevalence of small jumps as in the 1980s with U.S. dollar depreciation. In anticipation of dollar depreciation, the distribution of currency exchange rates became skewed to the left [37].

Certain exchange rates had a unique reaction to macroeconomic news, such as inflation announcements in a low inflationary environment, creating a small downward jump in dollar values, while in a high-inflationary environment, such an announcement would cause a large jump in currency values. Such disparate reactions are captured in fat-tailed leptokurtic distributions. In the 1980s, skewness and kurtosis were captured by a drift term in a Brownian motion along with jumps ([38]). Various jump-diffusion models with lognormal distributions upon which jumps were superimposed. They were effective in explaining skewness and kurtosis for small jumps ([39] [40] [36]). In response to [41]'s finding that the diffusion component of jump-diffusion models reflects a noise component, [42] and [11] replaced jump-diffusion models with a variance-gamma process which had a variance rate that assigned higher probabilities to small jumps which were related to prior jumps. However, the variance-gamma process was less effective in accounting for large jumps.

We surmise that option distributions are dependent upon foreign currency distributions, yet differ from currency distributions in that the jumps are stee-

per—a fact that has been empirically observed, but not accounted for theoretically in the literature. Further, the literature does not distinguish between the more stable and volatile foreign currencies. We posit that stable and volatile currencies will have different distributions, and create separate models for them.

$$\Sigma)\sigma^2$$

3. Models of Foreign Currency Distributions σ

3.1. A Continuous Normal Distribution with Constrained Parameters

A continuous normal distribution function may be described as,

$$f(x \mid \mu, \sigma) = 1/\sqrt{2\Pi Var} \exp-\left[(x-\mu)/2Var^2\right] \tag{1}$$

Applying the European Union's requirements, x transforms to x_1, x_2, and x_3, where, x_1 = inflation rate, x_2 = long-term interest rate, x_3 = government debt.

$$f(\mu\sigma) = \left(\sqrt{2\Pi\sigma_1}\right)\exp-1\exp-(x_1-\mu_1)/2\sigma_1\exp^2]$$
$$\exp^2 + 1/\sqrt{2\Pi\sigma_2}\exp^2-(x_2-\mu_2)/2\sigma_2\exp^2] \tag{2}$$
$$\exp^2 + 1/\sqrt{2\Pi\sigma_3\exp^2}]\exp^2$$

Taking the first derivative of Equation (1),

$$f'(\mu\sigma) = -(x-\mu)^2/2\sigma^2\left(1/\sqrt{2\Pi\sigma^2}\right)$$
$$= (x-\mu)^2/2\sqrt{2\Pi\sigma^3} \tag{3}$$

where, σ^3 = skewness,

Expanding to the first derivative of equation (2) yields,

$$f'(x \mid \mu\sigma) = -(x_{1t}-\mu_{1t})^2/2\sqrt{2\Pi\sigma_{1t}^3} - (x_{2t}-\mu_{2t})^2/2\sqrt{2\Pi\sigma_{2t}^3}$$
$$- (x_{3t}-\mu_{3t})^2/2\sqrt{2\sigma_{3t}^3} \tag{4}$$

where, σ_{1t}^3 = skewness due to the Euro being affected by an unexpected increase or decrease in the inflation rate, σ_{2t}^3 = skewness due to the Euro being affected by an unexpected increase or decrease in the long-term interest rate, σ_{3t}^3 = skewness due to the Euro being affected by an unexpected increase or decrease in government debt.

Given the narrow band within which macroeconomic variables in European Union member states must vary, any unexpected changes in inflation rates, interest rates, or government debt that move these values outside of the band will be forced into the band, so that the three forms of skewness and kurtosis will theoretically tend to 0. If the function at time $t = 1$, is extended to all time periods, from $t = 1$, through $t = n$.

Distribution of the Euro for all time periods,

$$\int\left[-(x_{1t}-\mu_{1t})^2/2\sqrt{2\Pi\sigma_{1t}^3} - (x_{2t}-\mu_{2t})^2/2\sqrt{2\Pi\sigma_{2t}^3} - (x_{3t}-\mu_{3t})^2/2\sqrt{2\sigma_{3t}^3}\right] \tag{5}$$

with inflation rates, long-term interest rates, and government debt approaching

their means, the variation of these variables about their means may be described by the variation of a series about a point (*a, b*), which is the following Taylor series expansion generalized to three variables for a single period *t*,

$$
\begin{aligned}
& f(a,b)+(x_{1t}-a)fx_{1t}(a,b)+(x_{2t}-b)fx_{2t}(a,b)+(x_{3t}-b)fx_{3t}(a,b) \\
& +1/2!\Big[(x_{1t}-a)^2 fx_{1t}x_{2t}(a,b)+2(x_{1t}-a)(x_{2t}-b)fx_1x_2(a,b) \\
& +(x_{2t}-b)^2 fx_2x_2(a,b)\Big]+1/2!\Big[(x_{2t}-a)^2 fx_2x_2(a,b) \\
& +2(x_{2t}-a)(x_{3t}-b)fx_2x_3(a,b)+(x_{3t}-b)^2 fx_2x_3(a,b)\Big] \\
& +1/2!\Big[(x_{1t}-a)^2 fx_1x_1(a,b)+2(x_{1t}-a)(x_{3t}-b)fx_1x_3(a,b) \\
& +(x_{3t}-b)^2 fx_3x_3(a,b)\Big]
\end{aligned} \tag{6}
$$

with the subscripts representing partial derivatives ([43]).

$$
T(x)=f(a)+(x-a)TDf(a)+1/2!(x-a)T\left\{D^2 f(a)\right\}(x-a) \tag{7}
$$

To minimize the deviation of x_1, x_2, and x_3 from their means, the Hessian matrix $D^2 f(a) = 0$, is obtained through an iterative process in which the gradient of *f* at point a $D(f)a = 10^{-9}$ in the final iteration of a linear programming model presented in Equation (8), If (5) = (6) or, we may solve the Taylor series expansion (6) by maximizing the value of the Euro as a product of its value and the distribution presented in (5), Maximize,

$$
C*\left[-(x_{1t}-\mu_{1t})^2\Big/2\sqrt{2\Pi\sigma_{1t}^3}-(x_{2t}-\mu_{2t})^2\Big/2\sqrt{2\Pi\sigma_{2t}^3}-(x_{3t}-\mu_{3t})^2\Big/2\sqrt{2\sigma_{3t}^3}\right] \tag{8}
$$

s. t.

$$
x_{1t}-\mu_{1t}\le 0.5
$$
$$
x_{2t}-\mu_{2t}\le 0.7
$$
$$
x_{3t}-\mu_{3t}< 0.25
$$
$$
x_1,x_2,x_3 > 0
$$

3.2. A Weiner Process with Unconstrained Parameters and Small Jumps

For the Japanese yen, Australian dollar, or Canadian dollar, there is no band limiting the movement of exchange rates from the mean. Therefore, foreign currency values will vary with respect to changes in up to 7 macroeconomic variables. However, given the central banks' adherence to a policy of price stability, inflation rates, and long-term riskless rates may remain within 1 standard deviation of the mean. Accordingly, the movement of exchange rates may be described by a Weiner process with a drift term, ([44]). This paper assumes that the government policy of price stability minimizes deviations from the Weiner process, including skewness and kurtosis, which may be captured in the drift term, so that the Weiner process without additional expressions for skewness and kurtosis will suffice. Assuming that foreign currency values depend upon

the following macroeconomic variables, x_{1t} = change in the inflation rate, x_{2t} = change in the short-term interest rate; Rate on a riskless discount bond of the same maturity as the holding period of foreign currency of <1 year, x_{3t} = change in the long-term interest rate; Rate of a riskless discount bond of the same maturity as the holding period of foreign currency of >1 year, x_{4t} = change in government debt, Pt + Δt(Pt + ΔtPPp), x_{5t} = change in export prices, x_{6t} =change in import prices, x_{7t} =varying levels of political stability. Beginning at any point in time t, the foreign currency value X_t, varies in an infinitesimal time interval, Δt. The expected probability of movement of the foreign currency value from X_t to $X_{t+\Delta t}$, is in direct proportion to the change in each macroeconomic variable, x_t to x_t', in an adaptation of the literature [45],

$$E\left(p\left(X_t + 3_t\right)\right) = \int p\left(x_{1t}P_t + \Delta t\left(x_{1t} \mid x_t\right)\right)dx_1 \tag{9}$$

Multiply by $\mathbb{P}_{t,t'}\left(x_t \mid x_t'\right)$, and find the sum over all changes in currency values over a time period for all macroeconomic variables,

$$\int p\left(x_{1t}\right)\int P_t + \Delta_t\left(x_{1t} \mid x_t\right)P_{tt'}\left(x_t \mid x_t'\right)dxdx_1 - p\left(x_t\right)P_{tt'}\left(x_t \mid x_t'\right)dx_1$$
$$+ \int p\left(x_{2t}\right)\int P_t + \Delta_t\left(x_{2t} \mid x_t\right)P_{tt'}\left(x_t \mid x_t'\right)dxdx_2 - \int p\left(x_t\right)P_{tt'}\left(x_t \mid x_t'\right)dx_t$$
$$+ p\left(x_{3t}\right)\int P_t + \Delta_t\left(x_3 \mid x_t\right)P_{tt'}\left(x_t \mid x_t'\right)dxdx_3 - \int p\left(x_t\right)P_{tt'}\int t + \Delta_t\left(x_{5t} \mid x_t\right)$$
$$+ \cdots + \int p\left(x_{7t}\right)\int P_t + \Delta_t\left(x_{7t} \mid x_t\right)P_{tt'}\left(x_t \mid x_t'\right)dxdx_7 - \int p\left(x_t\right)/P_{tt'}\left(x_t \mid x_t'\right)dx_t \tag{10}$$

Assuming that as currency values change from x_t to x_t, inflation adjusts, so that the inflation rate is described by the higher currency value, x_t. The product, $\int P_t + \Delta_t\left(x_{1t} \mid x_t\right)P_{tt'}\left(x_t \mid x_t'\right)dx = P_t + \Delta_t'\left(x_1 \mid x_t\right)$ from Equation (10) transforms to:

$$\int p\left(x_t\right)\int P_t + \Delta_t\left(x_t \mid x_t'\right) - \int p\left(x_t\right)P_{tt'}\left(x_t \mid x_t'\right)dx_t + \int p\left(x_t\right)\int P_t + \Delta_t\left(x_t \mid x_t'\right)x_t$$
$$- \int p\left(x_t\right)P_{tt'}\left(x_t \mid x_t'\right)dx_t + \int p\left(x_t\right)\int P_t + \Delta_t\left(x_t \mid x_t'\right)$$
$$- p\left(x_t\right)/P_{tt'}\left(x_t \mid x_t'\right)dx_t + p(x_t)\int P_t + \Delta_t\left(x_t \mid x_t'\right) - \int p\left(x_t\right)/P_{tt'}\left(x_t \mid x_t'\right)dx_t$$
$$+ p\left(x_t\right)\int P_t + \Delta_t\left(x_t \mid x_t'\right) - p\left(x_t\right)/P_{tt'}\left(x_t \mid x_t'\right) \tag{11}$$

Taking partial derivatives yields the Fokker-Planck equation from [45],

$$\partial t p\left(x,t\right) = -\partial t p\left(x,t\right) + \partial x^2\left(\sigma\left(x,t\right)^2/2 \cdot p\left(x,t\right)\right) \tag{12}$$

where x_t is the currency value based on changes in all 7 macroeconomic variables, so that the following maximum likelihood estimator is to be maximized assuming there are n observations of foreign currency values,

Maximize

$$1/n\log\left[-\partial x\mu\left(x_t\right)\cdot p\left(x_t\right) + \partial x^2\left(\sigma x_t\right)^2/2 \cdot p\left(x,t\right) + \partial x^2\left(\sigma,x_t\right)^2/2p\left(xt\right)\right] \tag{13}$$

Subject to

$$x_t = \partial x/\partial y\left[x_{1t} + x_{2t} + x_{3t} + x_{4t} + x_{5t} + x_{6t} + x_{7t}\right] \tag{14}$$

$$x_{1t} \le \left(x_{1t} - \mu_{1t}\right)^2/\sigma_{1t} \tag{15}$$

$$x_{2t} \le \left(x_{2t} - \mu_{2t}\right)^2/\sigma_{2t} \tag{16}$$

$$x_{3t} \leq \left(x_{3t} - \mu_{3t}\right)^2 \Big/ \sigma_{3t} \tag{17}$$

$$x_{4t} \leq \left(x_{4t} - \mu_{4t}\right)^2 \Big/ \sigma_{4t} \tag{18}$$

$$x_{5t} \leq \left(x_{5t} - \mu_{5t}\right)^2 \Big/ \sigma_{5t} \tag{19}$$

$$x_{6t} \leq \left(x_{6t} - \mu_{6t}\right)^2 \Big/ \sigma_{6t} \tag{20}$$

$$x_{7t} \leq \left(x_{7t} - \mu_{7t}\right) \Big/ \sigma_{7t} \tag{21}$$

$$\partial x^2 \left[\sigma\left(x_t\right)^2 \Big/ 2 \cdot p\left(x_t\right) \right] = 0 \tag{22}$$

The constraint, Equation (14) presents the first-order Kuhn-Tucker condition that the exchange rate at time t is the partial derivative of the composite of inflation rates, the short-term interest rate, the long-term interest rate, government debt, export prices, import prices, and political stability. Equation (15) to Equation (21) present each macroeconomic variable as varying within 1 standard deviation of its mean. Equation (22) sets the second derivative Hessian matrix = 0.

3.3. A Laplace Distribution with Unconstrained Parameters and Large Jumps

For currencies such as the Mexican peso, Turkish lira, and Russian ruble, where there are no restrictions on macroeconomic variables, currency movements may be described by a Laplace distribution. A Laplace distribution is the composite of two exponential distributions, with kurtosis of 3 and 0 skewness. Accordingly, our model will include quantities for excess kurtosis and skewness by including 2 groups of gradient vectors that describe skewness and kurtosis due to the impact on currency values of each macroeconomic variable. Excess kurtosis above 3 is defined in a separate term. The Laplace distribution's double exponents account for the large jumps not captured in other distributions described in Section 3.1 or Section 3.2. The cumulative distribution function of a Laplace distribution may be described as follows from [46]:

$$\int_{-\infty}^{x} F\left(u\right) \mathrm{d}u = \frac{1}{2} + 1/2 \operatorname{sgn}\left(x_t - \mu_t\right)\left(1 - \exp\left(-\frac{\left|x_t - \mu_t\right|}{b}\right)\right) \tag{23}$$

Adding 2 gradient vectors, ∇g_i, to account for skewness, and ∇h_i, to account for kurtosis,

$$\begin{aligned} &= \frac{1}{2} + 1/2 \operatorname{sgn}\left(x_t - \mu_t\right)\left(1 - \exp\left(-\frac{\left|x_t - \mu_t\right|}{b}\right)\right) \\ &+ \sum \mu_t \nabla g_i\left(x_t\right) + \sum \nabla h_i\left(x_t\right) + 1/\sum \sigma_{it}^2) \sum \sigma_{it}^4 \cdot Kurt\left[x_{it}\right] - 3) \end{aligned} \tag{24}$$

Skewness, as the third moment about the mean is defined in Pearson's Moment Coefficient of Skewness. A kurtosis of 3 is captured in the gradient vector, with the excess of kurtosis beyond 3 contained in the last term in Equation (24), $\nabla g_1 x_1$ = skewness in the inflation rate; $\nabla g_2 x_2$ = skewness in the short-term rate;

$\nabla g_3 x_3$ = skewness in the long-term rate; $\nabla g_4 x_4$ = skewness in government debt; $\nabla g_5 x_5$ = skewness in export prices; $\nabla g_6 x_6$ = skewness in import prices; $\nabla g_7 x_7$ = skewness in political stability; $\nabla h_1 x_1$ = kurtosis < 3 in the inflation rate; $\nabla h_2 x_2$ = kurtosis < 3 in the short-term rate_t; $\nabla h_3 x_3$ = kurtosis < 3 in the long-term rate; $\nabla h_4 x_4$ = kurtosis < 3 in government debt; $\nabla h_5 h_5$ = kurtosis < 3 in export prices; $\nabla h_6 x_6$ = kurtosis < 3 in import prices; $\nabla h_7 x_7$ = kurtosis < 3 in political stability.

Taking partial derivatives of each side,

$$
\begin{aligned}
&\partial x_t / \partial x\, F(x_t) + \nabla^2 f(x_t) \\
&= 1/2 \operatorname{sgn}(x_t - \mu_t)^2 / b(1) + \sum \mu_t \nabla^2 g t^2(x_t) + 0.5\sigma_{it} \\
&\quad + \sum \left[4\sigma_{it}^2 + \partial / \partial x\, Kurt(x_{it}) \right] + \sum \lambda_i \nabla^2 h_{12}(x_t)
\end{aligned}
\tag{25}
$$

Since excess kurtosis =

$$
\partial / [Kurt[x_{it}] - 3 = \sum 1/2\,\sigma_i + 4\sigma_{it}^2,
\tag{26}
$$

We can substitute Equation (26) in Equation (25),

$$
\begin{aligned}
&\partial x_t / \partial x\, F(x_t) + \nabla^2 f(x_t) \\
&= 1/2 \operatorname{sgn}(x_t - \mu_t)^2 / b(1) + \sum \mu \nabla^2 g_i^2(x_t) + \sum \lambda_i \nabla^2 h_{12}(x_t) \\
&\quad + \sum / 2\sigma_{it} + \sum 4\sigma_{it}^2 + \sum / 2\sigma_{it} + \sum 4\sigma_{it}^2
\end{aligned}
\tag{27}
$$

Collecting like terms,

$$
\begin{aligned}
&\partial x_t \,|\, \partial x F(x_t) + \nabla^2 f(x_t) \\
&= 1/2 \operatorname{sgn}(x_t - \mu_t)^2 / b(1) + \sum \mu_t \nabla^2 g_i^2(x_t) + \sum \lambda_t \nabla^2 h_i^2(x_t) \\
&\quad + \sum / \sigma_{it} + \sum 8\sigma_{it}^2
\end{aligned}
\tag{28}
$$

This is a necessary condition for solution. The sufficient condition for solution is that the second derivative of Equation (25) = 0. Given that the gradients are in the form of the second derivative, equation may be differentiated thus,

$$
\begin{aligned}
&\partial^2 x / \partial x^2 + F(x_t) + \nabla^2 f(x_t) \\
&= \operatorname{sgn}(x_t - \mu_t) / b + \mu_t g_i^2(xt) + \lambda_j \nabla^2 h_i^2(x_t) + \sum / \sigma_{jt} + \sum 8\sigma_{it}^2
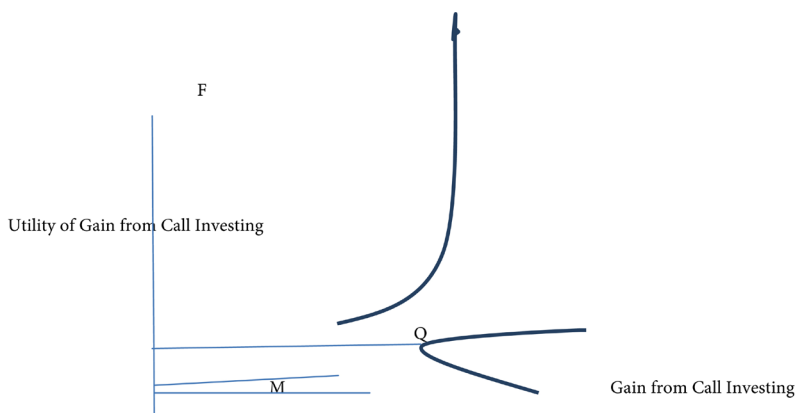\end{aligned}
\tag{29}
$$

At the local minimum, x_t^*, the gradients g_i and h_j are reduced to 0, so that Equation (29) may be solved for x_t,

$$
|x_t - \mu_t| / b + \sum / \sigma_{it} + \sum 8\sigma_{it}^2
\tag{30}
$$

4. The Proposed Model of Currency Call Option Distribution

4.1. A Call Option Distribution on Currencies Meeting Euro Standards

Given the strict band within which macroeconomic variables may vary, call options on the Euro are likely to follow a distribution of small jumps. **Figure 1** shows the 2 possibilities for call option distributions. A single informed trader approaches an options exchange. He or she is risk-averse. He or she will only trade at the level of risk at which the options distribution satisfies personal risk

F

Utility of Gain from Call Investing

Q

M

Gain from Call Investing

Figure 1. Optimal trading levels for currency call options with put sales or small Jumps Equation (32) and Equation (33) expressions presented graphically.

preferences. At point *M*, in **Figure 1**, a highly risk-averse the trader is expected to always opt for a sure amount over a risky bet. The only sure call option strategy is to sell put options to a put buyer, at a premium $= -P_t{}^*n$, where P_t = price of a single put option, and n = number of puts. There is no benefit to waiting for jumps to materialize, so the trader exits the market. In other words, the Coefficient of Absolute Risk Aversion does not increase if there are additional risky opportunities for gain, as the rate of change in risk aversion remains at $u'(c)$.

The Arrow-Pratt Coefficient of Risk Aversion [46],

$$A(c) = -u''(c)/u'(c) = 1 \tag{31}$$

where, $A(c)$ = an individual's propensity to avoid risk, $u(c)$ = the utility function of payoffs to an individual for accepting risk, with $u'(c)$ and $u''(c)$, as first and second derivatives. Small jumps in call option prices ensue with variations of the Euro within its narrow band. A risk-seeking trader has the utility function described in curve *OF* of **Figure 1**.

An increase in wealth (as promised by the gain from trading calls), results in the desire to increase wealth from further investment in call options with future jumps in euro values. An increase in the utility of wealth is a decrease in absolute risk aversion or $A(c) < 0$. In expanded form,

$$\partial A(c)/dc = \left\{-u'(c)u(c) - \left[u(c)\right]^2\right\} \Big/ \left[u'(c)^2\right] \tag{32}$$

c = gain from an investment in call options, = [(Euro Value After the Jump − Exercise Price of the Call Option)] − Call Premium. This model posits that a trader will exercise the option and take the highest gain on a jump at the point at which the rate of change in the utility of wealth (defined in Equation (32)) equals the change in price of a call option along a Levy-Khintchine process (defined in Equation (33) below). This is the point of intersection of the utility function *OF* with the peak of the first jump, *Q*. The Levy process approximates the random walk of a call option with successive discontinuous displacements. Traders make requests for call purchases in a Poisson process, with a multitude of discrete requests in a single interval of time. The Levy-Khintchine expression is as follows,

$$\pmb{\varepsilon}\big[ei\theta x(t)\big] = \exp\Big(t\Big(ai\theta - 0.5\sigma_t^2\theta^2 + \int\big(ei\theta x_t - 1 - i\theta x_t I\,|x_t<1|\big)\Pi\mathrm{d}x\Big)\Big) \quad (33)$$

As the $ai\theta$ quantity is a linear drift, and the $1/2\sigma_t^2\theta^2$ term is a Brownian motion, both of which are independent of jumps, they will be omitted from further consideration.

The first jump in **Figure 1** will be represented by

$$\int\big(ei\theta x_t - 1 - i\theta x_t I\,|x_t<1|\big)\Pi\mathrm{d}x$$

The peak of this jump, at which the trader will realize the maximum gain from investing in the call option is the second derivative of the first jump.

The second derivative is $\big(ei\theta x - i\theta x I\,|x|<t\big)\Pi$.

Differentiating Equation (32) below,

$$\partial/\partial x\big[\partial A(c)/\partial c\big] = \big[-u'(c)u(c) - [u(c)]\big]^2\Big/\big(u'(c)\big)^2 \quad (34)$$

c = gain from an investment in call options, [(Euro Value After the Jump − Exercise Price of the Call Option)] − Call Premium. At the minimum, the second derivative of the utility function = 0,

$$2u''(c) = 0, \text{or Euro value after the jump} - \text{Exercise price of the call} \\ = \text{Call premium} \quad (35)$$

when the condition in Equation (35) is satisfied, a trader will make a trade, or exercise the call option, purchase the euro, and sell it for gain. The trader will continue to make similar trades until the gain < call premium, or the cost of purchasing the option is higher than the profit from the trade.

4.2. A Currency Call Option Distribution for Currencies with Unconstrained Parameters and Small Jumps

Given that the macroeconomic variables may vary no more than 1 standard deviation from the mean, currency calls on currencies such as the Japanese yen, Australian dollar and Canadian dollar, may experience small jumps (See Section 3.2, for a description of the currency distribution). It is unlikely that very risk-averse traders who sell put options would trade options in these currencies, as even a 1 standard deviation change of macroeconomic variables would be considered to be excessively risky.

In Section 3.2, the optimal foreign currency value, x_ρ was presented as the solution to Equation (13).

Where, x_ρ the optimal foreign currency value, is the spot rate, or currency value at a point in time can also be the exercise price on a foreign currency call option as shown in the modification of Equation (35) below.

$$\big[\text{Currency Value After the Jump} - x_t\big] = \text{Gain} > \text{Call Premium} \quad (36)$$

when the condition in Equation (36) is satisfied, a trader will make a trade, or exercise the call option, purchase the currency, and sell it for gain. The trader will continue to make similar trades until the gain < call premium, or the cost of purchasing the option is higher than the profit from the trade.

4.3. A Currency Call Option Distribution with Large Jumps

In **Figure 2**, a call option trader with a propensity for risk-seeking selects calls on highly volatile Mexican pesos or Turkish liras. Both the currency and calls have large jumps, but the calls having the lesser initial investment are riskier, and subject to larger jumps. In **Figure 2**, the jumps represented by *OP* are currency (peso, lira, etc.) 2g2 (jumps, while those in *SR* are call option jumps. The trader's risk preferences are presented in curve *VR*. The trader will purchase the option at the option premium. As call prices escalate beyond the currency price to a peak at, say, point *T*, the trader is satisfied that the option has achieved maximum gain, beyond any gain offered by the foreign currency.

$$\text{Gain on the option} = \text{Gain on the currency} \tag{37}$$

Or,

$$\begin{array}{l} \left[\text{Final currency price} - \text{Exercise price}\right] - \text{Call price} \\ > \text{Final currency price} - \text{Currency purchase price} \end{array} \tag{38}$$

$$\text{Currency function} + \text{Gain on the option} = \text{Levy} - \text{Khintchine option function} \tag{39}$$

If x_t = value of foreign currency

$$\begin{array}{l} 0.5\,\text{sgn}\left(x_t - \mu_t\right)^2 \cdot b^{-1}\sum \mu_t \nabla^2 g^2\left(xt\right) + \sum \hat{\lambda}_t \nabla^2 h^2\left(xt\right) + \sum \left(\sigma_{it}\right)^{-1} + \sum 8\sigma_t^2 \\ + CV - \sum X - CP = ei\theta \boxed{?} \delta - 1 - i\theta\delta I \left|x_t > 1\right| \end{array} \tag{40}$$
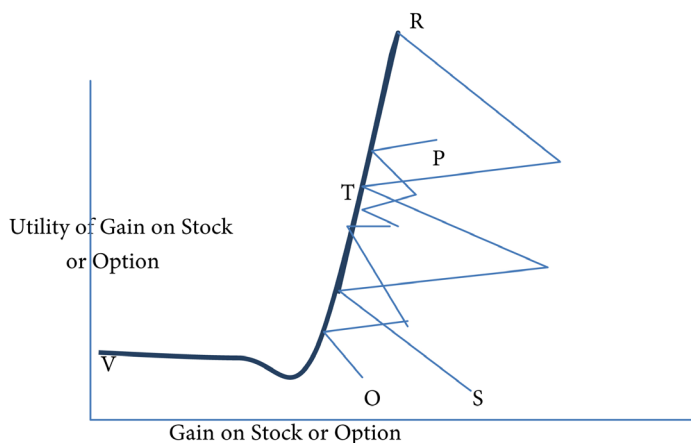
where, δ = jump size, CV = currency value, Ex = exercise price of the currency call option, CP = call premium.

At the point of earning the gain in the call option, *T*, in **Figure 2**, the Levy-Khintchine function is maximized, or, for a jump size, $\delta > 1$, the maximum point is defined by the Right side of Equation (40).

5. Pricing Call Options

5.1. Pricing Currency Calls Meeting Euro Standards

We will draw upon the currency distributions described in Section 3, as well as

Figure 2. Currency call option trading with large jumps Equation (37) presented graphically.

the call option distributions presented in Section 4, to develop expressions for pricing currency calls in this section. This includes combining the currency distribution for euros in Section 3.1, with call option distributions on euros in Section 4.1 to develop the currency call pricing formula in Section 5.1. Likewise, the currency distribution for stable currencies in Section 3.2 will be combined with call option distributions on stable currencies in Section 4.2, and in turn, currency call pricing formulations in Section 5.3. The pattern continues for volatile currencies in Sections 3.3, 4.3, and 5.3. This paper employs the underlying asset pricing concept contained in [47] to formulate the price of call options t- on currencies meeting the restrictions of the European Union, based on the distribution xt presented in Section 3.1. [47]'s Assumption A 6 states that:

The Value of A Contingent Claim such as a call option = (Gain From The Claim − Price Change Due To State Variables) + (Value of the Contingent Claim*. Distribution of the Contingent Claim),

Price of a call

= Present value of currency − Present value of exercise price

  − Price change due to change in macroeconomy

  + Call premium ∗ Distribution of call

$$
= \left(C * e^{-rdt} - \text{Forward rate } e^{-rdt} \right) - \left(C * \left[-\left(x_{1t} + \Delta_t - \mu_{1t} + \Delta_t \right)^2 \Big/ 2\sqrt{2\Pi\sigma_{1t}} + \Delta_t^3 \right. \right.
$$
$$
- \left(x_{2t} + \Delta_t - \mu_{2t} + \Delta_t \right)^2 \Big/ 2\sqrt{2\Pi\sigma_{2t}} + \Delta_t^3
$$
$$
\left. - \left(x_{3t} + \Delta_t - \mu_{3t} + \Delta_t \right)^2 \Big/ 2\sqrt{2\Pi\sigma_{3t}} + \Delta_t^3 \right]
$$
$$
- C * \left[-\left(x_{1t} - \mu_{1t} \right)^2 \Big/ 2\sqrt{2\Pi\sigma_{1t}^3} - \left(x_{2t} - \mu_{2t} \right)^2 \Big/ 2\sqrt{2\Pi\sigma_{2t}^3} \right.
$$
$$
\left. - \left(x_{3t} - \mu_{3t} \right)^2 \Big/ 2\sqrt{2\Pi\sigma_{3t}^3} \right] - \left[L_1 \left(x_{1t} + \Delta_t - \mu_{1t} + \Delta_t - 0.5 \right) \right.
$$
$$
\left. - L_1 \left(x_{1t} - \mu_{1t} - 0.5 \right) \right] - \left[L_2 \left(x_{2t} + \Delta_t - \mu_{2t} + \Delta_t - 0.7 \right) \right.
$$
$$
\left. - L_3 \left(x_{2t} - \mu_{2t} - 0.7 \right) \right] - L \left(x_{3t} + \Delta_t - \mu_{3t} + \Delta_t - 0.25 \right) \tag{41}
$$
$$
- L_3 \left(x_{3t} - \mu_3 - 0.25 \right) + \text{Call premium} * i\theta x_t, -1 - i\theta x I \left| x < 1 \right| \Pi \mathrm{d}x
$$

where, First term = Gain on exercise of the call option at the forward rate, or the present value of the gain (C^*, currency value-forward rate) earned in the future at the foreign interest rate, r_d, over the period, t.

Second and third terms = Objective function of Equation (8) that describes the price change in a currency call option due to change in macroeconomic variables from t to t, Δt. All L terms = Price change due to macroeconomic variables at t and t, Δt. L_1, L_2, L_3 = Lagrange multipliers of constraints in Equation (8). Last term = Call premium*Levi-Khintchine call option distribution listed in Equation (33).

5.2. Pricing Currency Calls with Unconstrained Parameters and Small Jumps

According to [47]'s Assumption A 6, The Value of A Contingent Claim such as a call option = (Gain From The Claim − Price Change Due To State Variables) +

(Value of the Contingent Claim*Distribution of the Contingent Claim),

Price of a call option

$= \big($Present value of the currency

$\quad -$ Present value of the exercise price of the option$\big)$

$\quad -\big($Price change due to change inmacroeconomy$\big)$

$\quad +\big($Call premium $*$ Distribution of the call option$\big)$

while the present value of the currency − present value of exercise price is identical to Section 5.1, other remaining terms need to be adjusted by the expressions in Section 3.2 and Section 4.2, for models with price stability and small jumps,

$$
\begin{aligned}
&= [\big(C * e^{-rdt} - \text{Forward rates } e^{-rdt}\big) - (1/n \log[-\partial x[\mu_t \cdot pxt + \partial x_{2t}/(2\,pxt)]] \\
&- L_1[\partial x/\partial y[x_{1t} + x_{2t} + x_{3t} + x_{4t} + x_{5t} + x_{6t} + x_{7t}] \\
&- L_3\Big[\partial x/\partial y\,(x_{1t} - \mu_t)^2\big/\sigma_{1t}\Big] - L_3\Big[\partial x/\partial y\,(x_{2t} - \mu_t)^2\big/\sigma_{2t}\Big] \\
&- L_4\Big[\partial x/\partial y\,(x_{3t} - \mu_t)^2\big/\sigma_{3t}\Big] - L_5\Big[\partial x/\partial y\,(x_{4t} - \mu_t)^2\big/\sigma_{4t}\Big] \\
&- L_6\Big[\partial x/\partial y\,(x_{5t} - \mu_t)^2\big/\sigma_{5t}\Big] - L_7\Big[\partial x/\partial y\,(x_{6t} - \mu_t)^2\big/\sigma_{6t}\Big] - L_8\Big[\partial x^2\big/2p(xt)\Big]
\end{aligned}
\tag{42}
$$

First term Gain on exercise of the call option at the forward rate, or the present value of the gain (currency value, C-forward rate) earned in the future at the foreign interest rate, r_d, over the period, t.

Second term Objective function of Equation (13) that describes the price change in a currency call option due to change in macroeconomic variables, $1/L$ terms Lagrange multipliers of constraints in Equation (15)-Equation (21), Last term Call premium*Levi-Khintchine call option distribution listed in Equation (33).

5.3. Pricing Currency Calls with Unconstrained Parameters and Large Jumps

We adapt Ito's Lemma [48] to specify the Price Change Due to Change in Macroeconomic Variables, omitting the diffusion term which = 0 for a Laplace distribution,

$$
\text{Price change due to change in macroeconomic variables} = k\big(2T^2 + 2T^3\big) \tag{43}
$$

where k = constant, T = ending time period. 2 Equation (30) describes the value of currency in a Laplace distribution with large jumps, Substituting Equation(30) into Ito's Lemma [48], Upon reduction, the value of currency in a Laplace distribution based upon the change in macroeconomic variables from time = 1, to time = T, it follows that,

$$
\text{Price of a call option} = \big(CV * e^{-rdt} - \text{Forward rate } e^{-rdt}\big)
$$

$$
-\Big[2k\sigma_{it}^2\big(1 + \sigma_{it} - 1 + 8\sigma_{it}^4 + 8\sigma_{it}^5\big)\Big] \cdot b\mu_t + \text{Call premium} * 1/2\,\mathrm{sgn}\,(x_t - \mu_t)^2\big/b \tag{44}
$$

$$
+ \sum \mu_t \nabla^2 g^2(xt) + \sum \lambda_t \nabla^2 h^2(xt) + \sum (\sigma_{it})^{-1} + \sum 8\sigma_t^2 + CV - Ex - CP
$$

CV = currency value, Ex = forward rate, CP = call premium.

6. Conclusions

This paper has updated the sparse publications on stochastic processes in option pricing, most of which belong to the era of the 1980s and 1990s. Unlike papers that modify stock option pricing models, we recognize that stock and foreign currencies are fundamentally different. A share of stock represents ownership of a business, while foreign currency values are determined by macroeconomic forces and government policy. It follows that the values of stock and foreign currency vary in their movements over time. Stock distributions are typically lognormal with minimal skewness and kurtosis. Foreign currency distributions are discontinuous with jumps and skewness and significant fat-tails, or kurtosis. We create foreign currency distributions that account for skewness and kurtosis with high jump-based distributions such as the Laplace distribution. We improve on jump-diffusion and variance gamma distributions which account for small jumps, with Levy processes that explain large jumps.

From a practitioner standpoint, call options on each of the foreign currencies studied may be used to predict future currency values, so that importers who pay in foreign currencies, may protect their accounts payable from currency appreciation. In other words, if a currency rises in value, an importer who has to make a payment in that currency will experience higher expenses, unless they purchase a call option with low exercise price, that permits them to purchase the foreign currency at cheaper rates.

Future research should extend theoretical formulation beyond the three groups of currencies examined. Other currencies may include the Swiss franc, Australian dollar, Hong Kong dollar, Renminbi, baht, krona, and Singapore dollar. Our approach of first developing the formulation of the currency distribution, followed by the option preserves the definition of an option as an instrument whose value is derived upon the foreign currency, so that is recommended over approaches that do not consider the distribution of the underlying foreign currency. Future research should also explore the use of risk preferences to identify option distributions. Trader demand for call options is governed by risk-seeking. Risk-averse traders will seek modest gains, exiting when the risk of investment exceeds the coefficient of risk-aversion. Risk-takers with low coefficients of risk-aversion will trade longer, until higher gains are achieved. Call options assume a market of appreciating foreign currencies. How would traders react to a market of depreciating foreign currencies? Would they short sell the currency, exiting short selling due to regulatory restrictions, satisfying their demand for declining currency with put option purchases? How would those puts be priced? Future research must be directed to answering these questions.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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