Retraction Notice

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The paper does not meet the standards of "Theoretical Economics Letters".

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Garbling in the Principal's Monitoring Device

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Abstract

I examine a principal-agent relation where the principal observes the actions of the agent with error. In order to motivate this situation, I imagine that the principal uses a device for monitoring the actions of an agent without his knowledge. The device potentially garbles, without prejudice, its binary-valued feedback to the principal. I show that as the number of possible errors that the device commits in its observation reduces, the maximum feasible outcome for the principal increases; as the number of observation instances increase, however, the agent's set of feasible actions expands, and this reduces the maximum feasible outcome.

Keywords

Monitoring, Garbling, Information, Principal-Agent

1. Introduction

In the canonical principal-agent relationship with moral hazard, a risk-neutral principal is unable to observe the action undertaken by the risk-averse agent, but she is able to observe the outcome that is generated through a combination of Nature and the agent's input. In this note, I am interested in a somewhat different setup: the principal observes action, but the observation is done through a device that returns a binary message and can garble the information it observes. I use the word "device" in this paper to distance the reader from the idea of a technology as used in the complete contracts literature, which assumes unobserved action. A survey of that literature and interesting results can be found in Silvers [1]. The general impetus for this paper is Holmstrom [2], which would suggest that such a device, no matter how imperfectly it functions, serves the principal when it provides any information on the unobserved actions.

I simplify the setup and focus on providing a metric for the cost of this device related to the efficiency of the device and, thus, the number of observations that the principal would have to make when using such a device.

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Being able to derive a cost in this manner is thus a contribution to the literature that suggests that some contractual adjustments might mitigate shirking, though in a context where observing actions trumps observing outcomes. This needs justification. While it is true that a principal may not even wish to acquire information on the actions of the agent[1], it is also entirely conceivable that she might. Indeed, in several settings, this makes patent sense. One can readily imagine an *R & D* unit, for example, where there may not be any noteworthy outcome for long periods of time, but the actions of each individual researcher in the innovation project matter.

The principal's monitoring device is rarely perfect in practice; there is always the potential that the device provides imperfect, garbled feedback. I show that as the principal's device becomes *more* reliable—and there is less misinformation from its use in the observation of the agent—the best outcome feasible *improves*. In a world tending towards fully state-contingent contracts, this much is perhaps fairly intuitive. Interestingly, when the principal's observation mechanism makes more observations of the employee's actions, the latter is permitted to undertake more actions and this can reduce the principal's best possible outcome. This observation is potentially also relevant to the puzzle of the ubiquity of incomplete contracts observed in practice. For example, Hart and Moore [4] propose the idea of contracts serving as reference points in an exchange between a buyer and a seller, settling ambiguity over the type of performance that a principal might expect from an agent; Fehr, Hart and Zehnder [5] experimentally provide support, suggesting that rigidity in the contracts had the effect of emphasizing the reference point more clearly compared to flexible terms, and thereby suffered from much less ex post regret and shading. In the context of this paper, a "rigid" contract comprises one where the principal makes more observations with a more reliable device, and a flexible contract would entail the principal making several observations, each with error, permitting the employee more opportunities to shade within the scope of the contract.

In the next section, I present the model by first characterizing the device that the principal employs in the monitoring process, followed by a derivation of results on the principal's maximum feasible outcome under both scenarios. I then provide a discussion of the model before making some concluding observations.

2. Model

Let us imagine a principal and an agent with objective functions u_1 and u_2, and with action sets *A* and *B* respectively. Assume that the action sets are compact and that the objective functions are continuous from $A \times B \to \mathbb{R}$. We assume that the principal moves first, however prior to her outcome being determined she has a *monitoring device* of fixed effectiveness that she can use to ascertain some information about the agent's action. The principal uses the device *s* times in evaluating the agent's action $b \in B$. Assume that the device delivers a message comprising a binary response on the agent's action for the principal. Thereafter, the principal makes her choice of action $a \in A$ based upon a *state-contingent plan* that acts as an ex ante mapping between the readings of her screening device and her action set.

2.1. The Monitoring Device

Assume that a proportion q/s of the total observations that the principal wishes to make using the device are garbled, with $q < s$, and that the principal is aware of this shortcoming. Thus, each tranche of *s* observations comprises 2*s* subsets in the space of the agent's action set,

$$\left(P_1^0, P_1^1\right), \left(P_2^0, P_2^1\right), \cdots, \left(P_s^0, P_s^1\right), \tag{1}$$

where the set P_g^1 represents those actions for which the agent is observed displaying a positive behavior and, likewise, P_g^0 represents the negative behavior situations, each at observation *g*. Note that

$$P_g^0 \cap P_g^1 = \varnothing; \; P_g^0 \cup P_g^1 = B \tag{2}$$

where $g = 1, \cdots, s$.

Define a vector containing the observation results from the device, κ with *s* elements taken from the set $S = \{0,1\}^s$ each of which corresponds to $P^\kappa = \bigcap_{g=1}^{s} P_g^{\kappa_g}$. Once the principal receives the results she selects an action, $a^\kappa \in A$. Therefore, the principal's strategy can be defined as depending on the 2*s* sets as in Equation (1)

[1]See, for example, Crémer [3].

that satisfy Equation (2) and with $r = 2^s$ actions.

Assumption 1. *We can further assume that we can correspond* $\boldsymbol{\kappa}$ *with a positive integer from* $(0,1,\cdots,r-1)$ *such that*

$$\left(\kappa_1 2^{s-1} + \kappa_2 2^{s-2} + \cdots + \kappa_s\right) \tag{3}$$

thus making $\kappa_1\kappa_2\cdots\kappa_n$ *the device's binary input.*

Definition 1. *A Hamming distance* [6] *describes the garbling errors of the principal's monitoring device over a period of observation. For any two vectors,* $\boldsymbol{\kappa}$ *and* $\boldsymbol{\vartheta}$, *both with s elements, drawn from S, the Hamming distance* $\mathcal{D}$ *is*

$$\mathcal{D}(\kappa,\vartheta) = \sum_{i=1}^{s} \left|\kappa_i - \vartheta_i\right|. \tag{4}$$

Thus, $\mathcal{D}(\kappa,\vartheta)$ *represents the number of errors that the device makes as it measures* ϑ *at the point of observing the agent and emits* κ *for the principal.*

Definition 2.

Let a garble-adjusted $\mathcal{D}$ *be defined as*

$$\Omega_q = \left(\vartheta \in S : q \ge \mathcal{D}(\kappa,\vartheta)\right). \tag{5}$$

Definition 3.

The principal's strategy is the pair

$$(\hat{a},\odot), \tag{6}$$

where $\left(\hat{a}: S \to A \mid \hat{a}(\kappa) = a^{\kappa}\right)$ *and* $\left(\odot : B \to S \Rightarrow \odot(b) = \kappa \text{ if } b \in P^{\kappa}\right)$.

Given this strategy for the principal, the agent selects an action *b* and the observation conveyed to the principal, ϑ, by the device depends on $b \in P^{\vartheta}$. The principal may make some reading on the device, $\kappa \in \Omega_q(\vartheta)$, and going by her state-contingent plan, she selects the action, a^{κ}. The outcomes of this transaction for the principal and the agent are given simply by $u_1(a^{\kappa},b)$ and $u_2(a^{\kappa},b)$. Note that the agent's outcome is determined only by his actions and the error of device. Naturally it matters what the agent knows about the device, considering it either free of errors or has some prior over the extent of garble and is risk-averse in his choice of action. I consider the former case since, for our purposes, the material argument is unaltered and the analysis is simplified.

Let us assume that, for the agent, there is no uncertainty at the time of selecting his action since he aware of the principal's action and assumes that the device has no errors. The principal acts on the assumption that the agent selects an action from a set of responses $\Theta(\hat{a},\odot)$, defined as follows:

$$\left.\begin{matrix} \left(u_2\left(\hat{a}\left(\odot(b)\right),b\right) = \max_{\tilde{b}\in B} u_2\left(\hat{a}\left(\odot(\tilde{b})\right),\tilde{b}\right)\right) & \text{if } \sup_{\tilde{b}\in B}\left(\hat{a}\left(\odot(\tilde{b})\right),\tilde{b}\right) \\ \left(u_2\left(\hat{a}\left(\odot(b)\right),b\right) \ge \sup_{\tilde{b}\in B} u_2\left(\hat{a}\left(\odot(\tilde{b})\right),\tilde{b}\right) - \zeta\right) & \text{otherwise} \end{matrix}\right\}. \tag{7}$$

2.2. Garbled Results

Consider now the errors in the device causing a garbled result as read by the principal. The errors lead the principal to read $\kappa \in \Omega_q(\odot(b))$ instead of $\odot(b)$, causing her to select $\hat{a}(\kappa)$, rather than $\hat{a}(\odot(b))$. This error carries a real cost to the principal's outcome that cannot be lower than

$$\inf_{b\in\Theta(\hat{a},\odot)} \min_{\kappa\in\Omega_q(\odot(b))} u_1\left(\hat{a}(\kappa),b\right). \tag{8}$$

The principal's maximum feasible outcome, Π^1, is thus

$$\Pi^1_{s,q} = \sup_{(\hat{a},\odot)} \inf_{b\in\Theta(\hat{a},\odot)} \min_{\kappa\in\Omega_q(\odot(b))} u_1\left(\hat{a}(\kappa),b\right). \tag{9}$$

Note that the case where the device does not garble the results for any observations the principal's maximum outcome does not depend on the efficiency of the device. This is obviously not the case in the presence of garbling. Nevertheless, we can show the following:

Proposition 1. *If we define a distance function,* υ, *providing a one-to-one mapping over the binary observations set S, then the principal's outcome is invariant to the strategies* $(\hat{a},\odot)$ *or* $(\hat{a}\circ 1/\upsilon, \upsilon\circ\odot)$. *Thus,* $\forall\vartheta\in S, \exists\upsilon:\vartheta\to 0$.

In order to derive $\Pi^1_{s,q}$ using equation (9) requires contending with $\sup_{(\hat{a},\odot)}$, which is complicated by the fact that $\odot$ has a complex composition. To simplify this propose that:

Proposition 2. *For all strategies of the principal* $(\tilde{b},\oslash)$, *there is a* $(\hat{a},\odot)$ *that permits*

$$\inf_{b\in\Theta(\hat{a},\odot)} \min_{\kappa\in\Omega_q(\odot(b))} u_1(\hat{a}(\kappa),b) \geq \inf_{b\in\Theta(\tilde{b},\oslash)} \min_{\kappa\in\Omega_q(\oslash(b))} u_1(\tilde{b}(\kappa),b)$$

and such that $\sup_{b\in B} u_2(\hat{a}(\odot(b)),b)$ *is met.*

In finding the principal's strategy this then permits us to consider a smaller set of strategies for which $\Theta(\hat{a},\odot)$ is met, for which the agent's outcome $u_2(\hat{a}(\odot(b),b))$ is identical to $\overline{u_2}$. If such a strategy permits $\Pi^1>\mu$ then we should have that $\overline{u_2}\leq u_2(a^\kappa,b)$ and $\mu<u_1(a^\vartheta,b); \vartheta\in\Omega_q(\kappa)$. Further, there would be a result for every action by the agent such that either $u_2(a^\kappa,b)=\overline{u_2}$ and $u_1(a^\vartheta,b)>\mu; \vartheta\in\Omega_q(\kappa)$ or $u_2(a^\kappa,b)<\overline{u_2}$. The former condition applies for any agent action in $\Theta(\hat{a},\odot)$ whereas the latter would apply when the agents action set is parsed into $\Theta(\hat{a},\odot)$ and $\Theta^C(\hat{a},\odot)$ so that either the principal's outcome is greater than μ or the agent's outcome is less than $\overline{u_2}$.

Let the recursive sup function, $\sup_{a^1\in A}\sup_{a^2\in A}\cdots\sup_{a^{r-1}\in A}$, be denoted simply by $\sup_{a\cdots}$. We thus have the following proposition:

Proposition 3. $\beta_q(\mu)>0$, where

$$\beta_q(\mu)=\sup_{a\cdots}\sup_{\overline{u_2}\in\mathbb{R}}\min\Bigg[\sup_{b\in B}\max_{\kappa\in S}\min\Big(\big(u_2(a^\kappa,b)-\overline{u_2}\big),\Big(\min_{\vartheta\in\Omega_q(\kappa)}u_1(a^\vartheta,b)-\mu\Big)\Big),$$
$$\inf_{b\in B}\max_{\kappa\in S}\max\Big(\min\Big(\big(u_2(a^\kappa,b)-\overline{u_2}\big),\Big(\min_{\vartheta\in\Omega_q(\kappa)}u_1(a^\kappa,b)-\mu\Big)\Big),\overline{u_2}-u_2(a^\kappa,b)\Big)\Bigg]$$

Theorem. *The principal's maximum feasible outcome using a monitoring device with garbled results is the solution that minimizes* $\beta_q(\mu)=0$.

Proof. Since $\beta_q(\mu)$ is decreasing and continuous, if it is strictly positive then it must be the case that there is an action for the principal $(\hat{a},\odot)$ that ensures that her outcome is larger than μ. Say that we select some $\overline{u_2}$; $a^0\in A, a^1\in A,\cdots,a^{r-1}\in A$; $b\in B$ and $\breve{\kappa}\in S$ such that

$$\left.\begin{array}{l}\min\Big(\big(u_2(a^{\breve{\kappa}},b)-\overline{u_2}\big),\Big(\min_{\vartheta\in\Omega_q(\breve{\kappa})}u_1(a^\vartheta,b)-\mu\Big)\Big)>0 \quad\text{and}\\ \inf_{b\in B}\max_{\kappa\in S}\max\Big(\min\Big(\big(u_2(a^\kappa,b)-\overline{u_2}\big),\Big(\min_{\vartheta\in\Omega_q(\kappa)}u_1(a^\kappa,b)-\mu\Big)\Big),\overline{u_2}-u_2(a^\kappa,b)\Big)>0\end{array}\right\}. \tag{10}$$

In relation to κ construct sets Γ^κ and Δ^κ, for $b\in B$, as follows

$$\Gamma^\kappa=\Big[\Big(\big(u_2(a^\kappa,b)-\overline{u_2}\big)>0\Big),\min_{\vartheta\in\Omega_q(\kappa)}\Big(\big(u_1(a^\vartheta,b)-\mu\big)>0\Big)\Big]$$

and

$$\Delta^\kappa=\Big[\Big(\big(u_2(a^\kappa,b)-\overline{u_2}\big)\geq 0\Big),\min_{\vartheta\in\Omega_q(\kappa)}\Big(\big(u_1(a^\kappa,b)-\mu\big)\geq 0\Big)\Big],$$

so that if Γ^κ is nonempty then Δ^κ is both nonempty as well as compact, and, therefore, $u_2(a^\kappa,b)$ achieves a maximum that is always greater than $\overline{u_2}$ on Δ^κ. The maximum value for

$$F^{\vartheta} = \max_{b \in \Gamma^{\kappa}} u_2\left(a^{\kappa}, b\right) \tag{11}$$

is achieved at b^{κ} when $\Gamma^{\kappa} \neq \varnothing$. Using *Proposition* 1 we can assume that $\vartheta = 0$, implying $\mu < F^0$.

Define $\Upsilon^{\kappa} = \left(b \in B : \mu - u_2\left(a^{\kappa}, b\right) > 0\right)$. For κ we can then define

$$P^0 = \Upsilon^0; P^{\kappa+1} = \left(\Gamma^{\kappa+1} \cup \Upsilon^{\kappa+1}\right) \setminus \left(\bigcup_{g=0}^{\kappa} P^t\right).$$

Equation (10) implies that *B* is partitioned by $P^0, P^1, \cdots, P^{r-1}$ and $\odot(b) = \kappa$ satisfies $\odot : B \to S$. We can establish that $(\hat{a}, \odot)$ will have the required structure by defining $\hat{a} : S \to A$ as $\hat{a}(\kappa) = a^{\kappa}$. For this strategy we can established that the supremum in $(\hat{a}, \odot)$ is achieved. Note that $F^0 = u_2\left(\hat{a}\left(\odot\left(b^0\right)\right), b^0\right) = u_2\left(a^0, b^0\right)$. At each $b \in P^0$, by Equation (11) we have $u_2\left(\hat{a}\left(\odot(b)\right), b\right) = u_2\left(a^0, b\right) \leq F^0$. However, if $b \in P^{\kappa}$, then we either have a) $b \in P^{\kappa} \cap \Gamma^{\kappa}$ or b) $b \in P^{\kappa} \cap \Upsilon^{\kappa}$. Under a), by Equation (11)

$$u_2\left(\hat{a}\left(\odot(b)\right), b\right) = u_2\left(a^{\kappa}, b\right) \leq F^{\kappa} \leq F^0$$

and under b)

$$u_2\left(\hat{a}\left(\odot(b)\right), b\right) = u_2\left(a^{\kappa}, b\right) \leq \overline{u_2} < F^0.$$

This ensures a maximum is reached. However, under b), we have that

$$\Theta(\hat{a}, \odot) \subset B \setminus \bigcup_{\gamma=0}^{r-1} \Upsilon^{\kappa} \subset \bigcup_{\gamma=1}^{r-1} \Gamma^{\kappa},$$

thus, at each $b \in \Theta(\hat{a}, \odot)$, we obtain $\min_{\kappa \in \Omega_q(\odot(b))} u_1\left(\hat{a}(\kappa), b\right) > \mu$. ►

2.3. Discussion

Let's step back to look at the result. Obviously, when there is a given amount of garbling in the device and $\tilde{s} \leq s$ we would have $\Pi^1_{s,q} \geq \Pi^1_{\tilde{s},q}$.

However, start with a situation where the principal can make $\tilde{s}$ observations with the device and there is no garbling and contrast this with a situation where garbling is $\tilde{q}$ and the principal makes $\breve{s} = \tilde{s}(2q+1)$ observations. What is the maximum feasible outcome for the principal now?

Say that with $\tilde{s}$ observations the principal's action was $\left(\tilde{b}, \oslash\right)$ and that there is a strategy in the situations with $\breve{s}$ observations that leads to the same outcome for her, $(\hat{a}, \odot)$. In this situation the observations needed to select $\left(\tilde{b}, \oslash\right)$ are now made $(2q+1)$ times and because of garbling we get true responses to more than half observations. In other words, we define $\odot : B \to (0,1)^{\breve{s}}$ so that $\odot(b) = \left(\vartheta_1, \vartheta_2, \cdots, \vartheta_{\breve{s}}\right)$, when $\oslash(b) = \left(\kappa_1, \kappa_2, \cdots, \kappa_{\tilde{s}}\right)$, is defined as $\vartheta_h = \kappa_i; (i = 0, 1, \cdots, \tilde{s})$ for $(2q+1)(i-1) < h \leq i(2q+1)$. Further, let $\hat{a}(\kappa) = \tilde{b}\left(E(\kappa)\right)$, and define $E : (0,1)^{\breve{s}} \to (0,1)^{\tilde{s}}$ so that when $E\left(\vartheta_1, \vartheta_2, \cdots, \vartheta_{\breve{s}}\right) = \left(\kappa_1, \kappa_2, \cdots, \kappa_{\tilde{s}}\right)$, $\kappa_h = \left\lfloor 2/(2q+1) \sum_{i=2qh+1}^{(2q+1)h} \vartheta_i \right\rfloor$.

We know that for each action undertaken by the agent it would be true that $\hat{a}\left(\odot(\kappa)\right) = \tilde{b}\left(\oslash(\kappa)\right)$, so the set of the agent's strategies with a principal's action of $(\hat{a}, \odot)$ in the situation where $\breve{s}$ observations are being made by the device is identical to the set under an action of $\left(\tilde{b}, \oslash\right)$ with $\tilde{s}$ questions. By definition, $\odot(b)$ is such that $\vartheta_{((2q+1)(i-1)+1)} = \vartheta_{((2q+1)(i-1)+2)} = \cdots = \vartheta_{(2q+1)i}$ for all *i*. Thus, both strategies offer identical outcomes to the principal and $\Pi^1_{\breve{s},q} \geq \Pi^1_{\tilde{s},q}$; that is, as long as $s \geq (2q+1)\tilde{s}$, $\Pi^1_{s,q} \geq \Pi^1_{\tilde{s},0}$.

3. Concluding Remarks

It is worth underscoring the fundamental tension that forms the thrust of the argument in this note: as the number of possible errors in observation reduces, the garble-adjusted information generated by the principal's monitoring device naturally increases too, and, therefore, the maximum feasible outcome for the principal increases. As the number of observation instances increase, the agent's set of feasible actions expands, and this reduces the maximum feasible outcome.

The analysis in this paper suggests a possibility of using a particular class of monitoring technologies in a principal-agent context where the bias arises from a device that can be considered "unbiased", in that garbling is *not* a function of the message. The problem is in some sense simpler than the usual context where strategic behavior by the agent is an important and integral component of the analysis; in a manner of speaking, the agent too may have a monitoring device over the principal's actions simultaneously.

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