

An Extended Model of Currency Options Applicable as Policy Tool for Central Banks with Inflation Targeting and Dollarized Economies

Luis-Felipe Arizmendi^{1,2}

¹GPI Asset Management S.A., Lima, Perú

²Center for International Policy Studies, Department of Economics, Fordham University, New York, USA
Email: farizmendi@gpiam.com

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ABSTRACT

The purpose of this paper is to provide a new set of tools for policy makers at central banks. Based on the Garman-Kohlhagen [1] formula for currency options, this research extends it with the Taylor-rule expression used for inflation targeting, thus obtaining the corresponding Call and Put options and the first and higher-degree partial derivatives known as “Greeks” for key variables such as the policy target domestic interest rate and the output gap.

Keywords: Inflation Targeting; Central Bank Policies; Exchange Rates; Currency Options

1. Introduction

In spite of the very recent debate in Caballero [2], Good friend [3], Taylor [4,5] and Woodford [6], on some limitations on how to apply it, the well-known Taylor-rule for inflation targeting (IT) has to be recognized as one of the most fruitful tools provided by macroeconomists to facilitate predictability and credibility related to changes in future price level. It is also claimed that for IT to be successful is required to include a high degree of exchange rate flexibility as in Ostry, Ghosh and Chamon [7]. In today’s world, emerging markets economists and policy makers are no longer engaged in just choosing an exchange rate regime as a framework for the economic activity in a country, but rather to optimize growth, employment and private investment. In that sense, the adoption of the Taylor-rule for IT has become not only fashionable but also useful to relate the effects in the nominal interest rate with the desired and observed levels of inflation, changes in the Real GDP and the short-term trends in the business cycle, as seen in Apergis, Miller, Panethimitakis and Vamvakidis [8], and Salvatore [9,10].

There are, however, some potential long-term weaknesses and yet unresolved policy issues when applying IT to dollarized economies. Ozsoz, Rengifo and Salvatore [11] show additional challenges for the policy-makers that come from the dollarization phenomena and the potential currency competition. It brings complex situations,

where the effectiveness of the monetary policy may appear partially or temporarily restricted, opening the case for increasing imbalances that may precipitate financial crises in the medium-term, with massive inflows of foreign speculative capital and abrupt local currency appreciation.

The use of market-related tools, such as currency options, is not strange to literature (see Veestraeten [12]), although they have been just partially employed in a few countries, and none has attempted to merge currency option models, such as the Garman-Kohlhagen [1], within an IT framework.

There are some cases that will help us to set the boundaries of this proposed research. As seen in works of Archer [13], Bernedo and Azañero [14], Breuer [15], Gürsu and Öncü [16], Mandeng [17], Orellana and Rodriguez [18], Suk and Malliaris [19], Taylor [20], and Uribe and Toro [21], there is some recent evidence to assume that exchange rate intervention by employing market-friendly tools such as currency options, can provide the additional instruments many central banks in small dollarized open economies, are looking for.

2. The Model

We will employ the model presented by Taylor [22], Molodtsova and Papell [23], where the most widely used expression by central banks is usually written as:

$$i_t = \pi_t + \varphi(\pi_t - \bar{\pi}) + \gamma Y_t + R \quad (1)$$

where i_t is the policy target domestic interest rate, π_t is the actual inflation rate, $\bar{\pi}$ is the target inflation rate, Y_t is the Output gap (potential vs actual real GDP), and R is the equilibrium real interest rate.

Rearranging some terms to obtain a form easier to compute, we get:

$$i_t - \pi_t = R + \varphi\pi_t - \varphi\bar{\pi} + \gamma Y_t; \quad (1.a)$$

by separating in two terms, we get:

$$i_t = k_t + m, \quad (1.b)$$

$$\text{where } k_t = (1 + \varphi)\pi_t + \gamma Y_t, \quad (1.b.1)$$

$$\text{and } m = R - \varphi\bar{\pi} \quad (1.b.2)$$

As a suggestion to avoid multicollinearity, the regression form may be set as:

$$(i_t - \pi_t) = R + \varphi(\pi_t - \bar{\pi}) + \gamma Y_t + \varepsilon_t \quad (1.a.1),$$

After obtaining the fitted values for $i_t - \pi_t$, and then subtracting π_t , we obtain \hat{i}_t as the estimated target nominal interest rate.

Renaming the terms and using T (for Taylor) instead of t for the purpose to identify them as instrumental values that come from the Taylor expression, the estimated regression can be written as:

$$\hat{i}_T = \hat{k}_T + \hat{m}. \quad (2)$$

Whereas, by grouping terms from (1.a.1), we get:

$$\hat{k}_T = (1 + \varphi)\pi_T + \gamma Y_T + \varepsilon_T \quad (2.a)$$

$$\hat{m} = \hat{R} - \varphi\bar{\pi} \quad (2.b)$$

Being the Garman-Kohlhagen (GK) model an European option, with q being the risk-free foreign interest rate, and $\Phi(d)$, the cumulative distribution function for the standard normal distribution, the values d_1 and d_2 of the GK formula with the coefficients from the Taylor-rule, the modified d_1^T and d_2^T are:

$$d_1^T = \frac{\log(S/X) + \left(\hat{k}_T + \hat{m} - q + \left(\frac{\sigma^2}{2} \right) \right) t}{\sigma\sqrt{t}} \quad (3)$$

and,

$$d_2^T = d_1^T - \sigma\sqrt{t} \quad (4)$$

So, the expressions for the Call and the Put options of the GK-Taylor modified will be:

$$C = Se^{-qt}\Phi(d_1^T) - Xe^{-(\hat{k}_T + \hat{m})t}\Phi(d_2^T) \quad (5)$$

$$P = Xe^{-(\hat{k}_T + \hat{m})t}\Phi(-d_2^T) - Se^{-qt}\Phi(-d_1^T) \quad (6)$$

Being log the natural logarithm, and:

S = the current exchange rate (domestic currency per unit of foreign currency);

X = the strike exchange rate;

$\hat{k}_T + \hat{m}$ = the estimated domestic target interest rate (\hat{i}_T) by the Taylor-rule regression;

q = the continuously compounded foreign risk-free interest rate;

t = the time in years until the expiration of the option;

σ = the implied volatility for the underlying exchange rate;

Φ = the standard normal cumulative distribution function.

Assuming option values at expiration time E to be $C_E = \text{Max}[0, S_E - X]$, and $P_E = \text{Max}[0, X - S_E]$, options having non-negative prices such as $C, P \geq 0$, upper boundaries $C \leq S, P \leq X$, and lower boundaries

$C \geq e^{-qt}S - e^{(\hat{i}_T)t}X$, and $P \geq e^{(\hat{i}_T)t}X - e^{-qt}S$, as described by DeRosa [24], as well as Haup [25] and Steland [26], for further details.

3. First and Higher Order Partial Derivatives

In terms of the GK-Taylor modified model (GK-T), the new ‘‘Greeks’’ are defined in the following way:

Delta (Δ): $\partial C/\partial S$ and $\partial P/\partial S$ will show the change in the price of a corresponding Call or Put to the change in the price of the underlying asset. Delta is known as the ‘‘hedge ratio’’, since it indicates how much of the underlying asset needs to be bought or sold to hedge the option.

For a Call GK-T, $\partial C/\partial S \equiv \Delta_c = e^{-qt}\Phi(d_1^T), 0 < \Delta < 1$.

For a Put GK-T,

$$\partial P/\partial S \equiv \Delta_p = e^{-qt}[\Phi(d_1^T) - 1] - 1 < \Delta < 0.$$

Gamma (Γ): It shows the amount that delta (Δ) changes of C and P when the underlying price changes, in this case S , the current spot exchange rate.

For the GK-T Call,

$$\partial^2 \Delta_c / \partial S^2 \equiv \Gamma_c = \frac{\Phi(d_1^T)e^{-qt}}{S\sigma\sqrt{t}}, \Gamma_c = \Gamma_p > 0 \quad \text{For the GK-}$$

T Put, $\partial \Delta_p / \partial S^2 \equiv \Gamma_p$ = as above

Vega (V): Vega is the amount that the theoretical value of an option changes when the volatility σ changes by one point (either money or percentage). So, $V = \partial C/\partial \sigma, \partial P/\partial \sigma$ being C and P the values of a Call option and a Put option.

By deriving, we obtain

$$V_c = V_p = Se^{-qt}\Phi(d_1^T)\sqrt{t}, V_c = V_p > 0$$

Theta (Θ): Theta represents the loss in value of the option in one time period, all other factors held constant. Then, $\Theta = \partial C/\partial t, \partial P/\partial t$.

By deriving, we obtain the following expressions:

$$\Theta_C = -\frac{Se^{-qt}\Phi(d_1)\sigma}{2\sqrt{t}} + qSe^{-qt}\Phi(d_1^T) - \hat{i}_T Xe^{-\hat{i}_T t}\Phi(d_2^T),$$

$$\Theta_P = -\frac{Se^{-qt}\Phi(d_1)\sigma}{2\sqrt{t}} - qSe^{-qt}\Phi(d_1^T) + \hat{i}_T Xe^{-\hat{i}_T t}\Phi(d_2^T),$$

Being usually Θ_C and $\Theta_P < 0$; and $\hat{i}_T = \hat{k}_T + \hat{m}$.

Rho (ρ): We will obtain several “rho” values for the GK-T model, since we have a domestic target interest rate \hat{i}_T that depends on π and Y , and a foreign q interest rate. Rho is the change in the option price when there is a change in the interest rates. They may be critical for this research, since an important outcome of the “blended” IT model will rely on the central bank’s ability to operate either open or “shadow” currency options, to stabilize the trend of the exchange rate by minimizing the loss of the inflation target forecasting. Then, rho (ρ_q) = $\frac{\partial C}{\partial q}$ will be the first derivative of the Call option price when there is a change in the foreign risk-free interest rate.

Deriving accordingly, we obtain:

1) For domestic rho, we will derive $\partial C/\partial i$ and $\partial P/\partial i$, as well as in terms of π and Y , such as $\partial C/\partial \pi, \partial P/\partial \pi$ and $\partial C/\partial Y, \partial P/\partial Y$, so

$$\partial C/\partial i \equiv \rho_C = Xte^{-(\hat{k}_T + \hat{m})t}\Phi(d_2^T), \rho_C > 0$$

$$\partial P/\partial i \equiv \rho_P = -Xte^{-it}\Phi(d_2^T), \rho_P < 0,$$

Since $\hat{i}_T = (1 + \varphi)\pi_T + \gamma Y_T + \hat{m}$, then

$$\partial C/\partial \pi = -S(1 + \varphi)te^{-(\hat{k}_T + \hat{m})t}\Phi(d_1^T), \text{ and,}$$

$$\partial P/\partial \pi = -S(1 + \varphi)te^{-(\hat{k}_T + \hat{m})t} [1 - \Phi(d_1^T)], \text{ while}$$

$$\partial C/\partial Y = -S\gamma te^{-((1+\varphi)\pi_T + \gamma Y_T + \hat{m})t}\Phi(d_1^T), \text{ and,}$$

$$\partial P/\partial Y = -S\gamma te^{-((1+\varphi)\pi_T + \gamma Y_T + \hat{m})t}\Phi(-d_1^T)$$

2) For foreign rho,

$$\partial C/\partial q \equiv \rho_C = -Ste^{-qt}\Phi(d_1^T), \rho_C > 0$$

$$\partial P/\partial q \equiv \rho_P = -Ste^{-qt}\Phi(-d_1^T), \rho_P < 0$$

Useful higher order partial derivatives for C and P , such as Vanna ($\partial^2 V/\partial S \partial \sigma$), Volga ($\partial^2 V/\partial \sigma^2$), Speed ($\partial^2 V/\partial \sigma^2$), DvegaDtime ($\partial^2 V/\partial \sigma \partial t$), and new “Greeks” (as in Arizmendi [27]), such as DrhoDpi ($\partial^2 C/\partial \pi \partial S; \partial^2 P/\partial e \partial S$), and DrhoDoutput ($\partial^2 C/\partial Y \partial S; \partial^2 P/\partial Y \partial S$), are also obtained as:

$$\mathbf{Vanna} = \frac{q}{S} \left[1 - \frac{d_1^T}{\sigma\sqrt{t}} \right],$$

$$\mathbf{Volga} = \frac{q}{\sigma} (d_1^T d_1^T),$$

$$\mathbf{Speed} = -\frac{e^{-(\hat{k}_T + \hat{m})t}\Phi(d_1^T)}{S^2 \sigma \sqrt{t}} \left[\frac{d_1^T}{\sigma\sqrt{t}} + 1 \right],$$

DvegaDtime

$$= Se^{-(\hat{k}_T + \hat{m})t} \sqrt{t} \Phi(d_1^T) \left[\frac{d_1^T d_2^T - 1}{2t} - \frac{d_1^T (q - \hat{k}_T - \hat{m})}{\sigma\sqrt{t}} - (\hat{k}_T - \hat{m}) \right]$$

DrhoDpi (Call)

$$= -(1 + \varphi)te^{-((1+\varphi)\pi_T + \gamma Y_T + \hat{m})t} \left(\Phi(d_1^T) + \frac{\Phi(d_1^T)}{\sigma\sqrt{t}} \right),$$

DrhoDpi (Put)

$$= -(1 + \varphi)te^{-((1+\varphi)\pi_T + \gamma Y_T + \hat{m})t} \left(\Phi(-d_1^T) + \frac{\Phi(d_1^T)}{\sigma\sqrt{t}} \right);$$

DrhoDoutput (Call)

$$= -\gamma te^{-((1+\varphi)\pi_T + \gamma Y_T + \hat{m})t} \left(\Phi(d_1^T) + \frac{\Phi(d_1^T)}{\sigma\sqrt{t}} \right), \text{ and}$$

DrhoDoutput (Put)

$$= \gamma te^{-((1+\varphi)\pi_T + \gamma Y_T + \hat{m})t} \left(\Phi(-d_1^T) - \frac{\Phi(d_1^T)}{\sigma\sqrt{t}} \right)$$

4. Conclusion

This model may be a useful set of tools for central banks and monetary authorities, especially those in countries with a relatively high level of dependency to a foreign currency, mainly through its local financial market and capital inflows. In particular, Call and Put option pricing—even if such derivatives market is not yet developed—may help central banks to intervene in the FX market when a combination of undesirable or untenable interest rate differentials, persistent inflows of capital, or a series of speculative attacks against the foreign currency in the local FX market, may cause the local currency to appreciate beyond certain level, introducing elements of potential instability, precisely opposed to what is sought by using the IT approach. In a free-float to dirty-float spot FX environment, the central bank of any given relatively dollarized country may diminish the cost of some sterilization through open-market-operations by calculating the relative value of its equivalent currency options, corresponding hedging and timely intervention.

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