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# Kalai-Smorodinsky Bargaining Solution and Alternating Offers Game

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## ABSTRACT

This article presents an alternating offers game that supports a Kalai-Smorodinsky bargaining solution (KSS). It is well known that a solution to an alternating offers game has a breakdown point equivalent to a status quo that converges to its Nash bargaining solution because the probability of breakdown becomes negligible, whereas we show that a KSS is obtained if a breakdown gives everything to the player who rejects. The former option, which is adopted by many application papers may be suitable for *ex ante* production. However, the latter option should be more appropriate for *ex post* production, because players do not need to be concerned with cooperation.

**Keywords:** Bargaining Solution; Alternating Offers Game; Breakdown

## 1. Introduction

Kalai and Smorodinsky [1] proposed an axiomatic bargaining solution, known as the Kalai-Smorodinsky bargaining solution (KSS), that differs from the one pioneered by Nash [2], which imposed monotonicity instead of independence to irrelevant alternatives. Shaked and Sutton [3] connected a Nash bargaining solution with an alternating offers game originated by Rubinstein [4], whereas the relationship between a KSS and an alternating offers game has not yet been clarified. Therefore, this study investigates a KSS for this type of game.

In connection with this study, it is interesting to note that monotonicity is substantially incompatible with the irrelevance of independent alternatives [5]. In addition to our consideration of axiomatic approaches and alternating offers games, it may be important to consider other dimensions such as demand games [6,7] and implementations [8,9]. Extensions of KSS for asymmetry [10], endogenous disagreement [11] and non-convex bargaining sets [12] could be examined in each contrasting dimension.

The remainder of this paper is organized as follows: Section 2 constructs an alternating offers game, Section 3 finds an equilibrium equivalent to a KSS, and Section 4 concludes this paper.

## 2. Model

Two players, 1 and 2, alternately offer their partitions on a strictly convex bargaining set where the frontier is strictly decreasing. Without any loss of generality, such a set is characterized by $x_2 = f(x_1)$, where $x_1 \in [0, \bar{x}_1]$, $x_2 \in [0, \bar{x}_2]$, $\bar{x}_2 = f(0)$ and $\bar{x}_1 = f^{-1}(0)$, a continuous function $f$ is assumed. The game proceeds as follows. Player 1 offers $x_1$ and if player 2 accepts, the game ends with the payoff vector $(x_1, f(x_1))$. If player 2 rejects the offer, the bargain breaks off with a probability $p \in (0,1)$. In that case, the game ends with $(0, f(0))$. If it continues, the players' positions are exchanged. Thus, an offer is $x_2$ and the payoff vectors are $(f^{-1}(x_2), x_2)$, respectively, if the offer is accepted and $(f^{-1}(0), 0)$ if the bargain breaks, while the opportunity to offer reverts to player 1 if the game continues.

## 3. Analysis

This section shows that stationary perfect equilibria in the game converge to the KSS where $f$ intersects the straight line from the origin (**Figure 1**), where the slope is $\bar{x}_2/\bar{x}_1$. No equilibrium consists of repetitive refusals, which expects the payoff vector $\left(\frac{1}{2}\bar{x}_1, \frac{1}{2}\bar{x}_2\right)$, because the bargaining set is strictly convex.

First, the existence of stationary equilibria is assured.

**Proposition 1.** *There is a stationary equilibrium.*

***Proof.*** In stationary equilibria, the one shot deviation properties

$$f(x_1) = p\bar{x}_2 + (1-p)x_2 \quad (1)$$

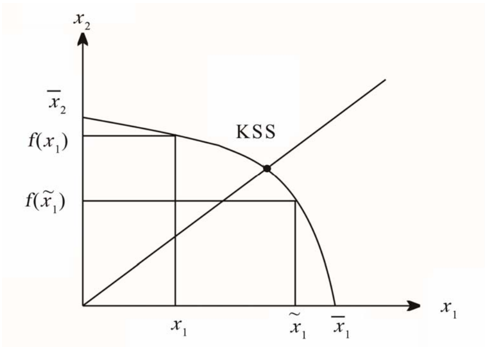


**Figure 1. Allocations on a frontier.**

$$f^{-1}(x_2) = p\bar{x}_1 + (1-p)x_1 \quad (2)$$

must be satisfied. Let $\tilde{x}_1(x_1) = p\bar{x}_1 + (1-p)x_1$ and $\tilde{x}_2(\tilde{x}_1) = p\bar{x}_2 + (1-p)f(\tilde{x}_1)$. Because $f(0) > \tilde{x}_2 \circ \tilde{x}_1(0)$ and $f(\bar{x}_1) < \tilde{x}_2 \circ \tilde{x}_1(\bar{x}_1)$, there is a stationary solution due to continuity. □

Next, the uniqueness of the convergence point is stated. This allocation is the same as that of the KSS.

**Proposition 2.** *Any stationary equilibrium converges on the KSS as* $p \to 0$.

***Proof.*** When $p \to 0$ in Equations (1) and (2), $f(x_1) \to x_2$ and $f^{-1}(x_2) \to x_1$. Thus, it is sufficient to show that

$$\frac{x_2}{f^{-1}(x_2)} \le \frac{\bar{x}_2}{\bar{x}_1} \le \frac{f(x_1)}{x_1},$$

owing to the squeeze theorem.

Suppose that

$$\frac{f(x_1)}{x_1} < \frac{\bar{x}_2}{\bar{x}_1}.$$

then,

$$\tilde{x}_2 > pf(x_1)\frac{\bar{x}_1}{x_1} + (1-p)f(\tilde{x}_1) \to f(x_1) \to x_2$$

as $p \to 0$. This contradicts $\tilde{x}_2 \to x_2$ and it is similar for player 2. □

To eliminate the strictness on the convexity and decrease in $f$, we can impose continuity on a solution with sequences inside and outside the frontiers.

## 4. Conclusion

The above bargain can be broken off with polar allocations whenever a player rejects an offer, such as when an arbiter abandons a wilful player who offers unreasonably and determines that the availability of resources is not settled during a dispute. This implies that each player can only individually use the resources. This type of bargain is concerning during the sharing of *ex post* production. By contrast, a Nash bargaining solution is supported when both parties receive nothing following a breakdown. Cooperation is needed to ensure gain, so this type of bargain is likely to arise during *ex ante* production. Thus, the difference between the two solution concepts may be due to the timing, particularly during competition for resources.

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