

Interaction Dynamics in a Social Network Using Hidden Markov Model

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Abstract

Agents interactions in a social network are dynamic and stochastic. We model the dynamic interactions using the hidden Markov model, a probability model which has a wide array of applications. The transition matrix with three states, forgetting, reinforcement and exploration is estimated using simulation. Singular value decomposition estimates the observation matrix for emission of low, medium and high interaction rates. This is achieved when the rank approximation is applied to the transition matrix. The initial state probabilities are then estimated with rank approximation of the observation matrix. The transition and the observation matrices estimate the state and observed symbols in the model. Agents interactions in a social network account for between 20% and 50% of all the activities in the network. Noise contributes to the other portion due to interaction dynamics and rapid changes observable from the agents transitions in the network. In the model, the interaction proportions are low with 11%, medium with 56% and high with 33%. Hidden Markov model has a strong statistical and mathematical structure to model interactions in a social network.

Keywords

Agents, Interactions, Social Network, Hidden Markov Model, Singular Value Decomposition

1. Introduction

A social network is a set of people, each of whom is acquainted with some subset of the others. This set of people is referred to as agents. The interactions of agents form a social network where they link and stochastically evolve over time [1]. Evolution has history of past and current interactions that contain the information about the agent abilities and dispositions [2]. The social interactions

affect user activity and pairs of individuals with strong ties are likely to exhibit greater similarity than those with weak ties [3]. The social interactions evolve due to agents strategies that are random in order to reinforce the benefits gained. The evolution can lead to decay, forgetting or reinforcement of existing interactions [4] [5].

Networks have vast amount of social networking data necessary to understand the properties and the behavior of social networks [1] [6] [7] [8]. Human interactions study found that the use of networks was important as interactions were dynamic and this had an impact on the properties exhibited by the network evolution [8]. The dynamics, complexities and stochasticity in the network interactions can be captured through Hidden Markov Model (HMM). An analysis of HMM shows that the model is adaptable to a wide array of applications due to its strong statistical foundation; evaluation and training algorithms; ability to handle new data robustly and predict similar patterns efficiently [9]. A number of models of social network interactions have been studied based on different forms and structures [1] [5] [10] [11] [12] [13] [14].

In this paper, we seek to enhance research in this area of interactions in social networks by introducing the use of HMM. We found no similar work that has used HMM and singular value decomposition (SVD) in social network interaction studies. We model social network interactions with a three state transition matrix: forgetting, reinforcement and exploration processes as they drive the dynamics of the network [15]. The observation matrix outputs are low, medium and high interaction levels and the initial state probabilities for the social dynamics are estimated using SVD technique.

2. Dynamic Interactions

Agents in a network make friends on asymmetric weights. The choice is by chance determined by the relative weight each agent has assigned to the other agents. The dynamics of friendships have interactions that are random in which social network structure emerges from the stochastically evolving social networks [5]. A way to improve on the model of social dynamics is through the introduction of the interaction contextual information. The information was incorporated to reflect the agents interaction dynamics better. Information theory is used to select features that generate a compact and effective feature vector for each agent interaction [16]. Social effects in an interaction model using HMM are introduced where the effects influence a user's activity based on the activity of a user's neighbor thus increasing the model explanatory and predictive power [2]. Static interactions of facebook users were modeled and analyzed using graphs. Decay in the rate and amount of interactions between different pairs of users in facebook were observed over time [17]. Activities in a social network are a sequence of events where [11] shows that social relationships changes continuously in a way correlated with the dynamical processes taking place during social interactions. A generative model based on forgetting, reinforcement and

exploration processes in online social network is undertaken where a user has a forgetting behavior that shows decline in pairs of user attraction. A user counteracts this by exhibiting a reinforcing behavior to keep the important relationships alive. Also, each user has an exploring behavior to continuously explore new interaction relationships [15].

Studies involving the use of HMM for varied applications due to its flexible general purpose approach to modeling dynamical systems are presented. HMM do not have a limitation for their applications [18]. HMM modeled trust in a dynamic environment that changes with time [16]. A probabilistic approach used the HMM to model a social network dynamics [2]. The agents' dynamics behavior feedback mechanism was modeled using HMM to enhance the trust and reputation evaluations about the trustee [19]. A HMM approach and SVD estimated trust levels by extracting reputation ratings from the network matrix and a 20% discounting with SVD is found to provide an optimal trust levels in a social network [6].

We model the interaction dynamics of users in a social network using HMM with simulation data. The transition matrix has three processes of forgetting, reinforcement and exploration [15]. The observation matrix is estimated from the transition matrix using the matrix factorization method [20]. The initial state probabilities are estimated from the errors of the matrix factorization method when estimating the observation matrix.

3. Hidden Markov Model

The characterization of HMM is outlined [1] [21].

- The number of states ($M = 3$) in the model with the set of states denoted as $S = \{S_1, S_2, S_3\} = \{F, R, E\} = \{\text{Forgetting, Reinforcement, Exploration}\}$.

- The state transition probability distribution $A = \{a_{ij}\}$, where $a_{ij} = P[q_{t+1} = S_j | q_t = S_i]$, where $1 \leq i, j \leq M, t = 1, T$.

- The number of distinct observation symbols ($K = 3$) per state. We denote the set of observation symbols corresponding to the physical output of the system being modeled as $V = \{v_1, v_2, v_3\} = \{\text{Low, Medium, High}\} = \{L, M, H\}$.

- The observation symbol probability matrix $B = \{b_j(k)\}$, where $b_j(k) = P[v_k | S_j]$, $1 \leq j \leq M, 1 \leq k \leq K$.

- The initial state probability vector $\pi = \{\pi_i\}$, where $\pi = P[q_1 = S_i], 1 \leq i \leq M$.

- Observation sequence $O = O_1, O_2, \dots, O_T$, where each observation O_i is one of the symbols from V and T is the number of observations in the sequence.

A complete set of parameters of the HMM model is represented as $\lambda = (A, B, \pi)$. The social interaction processes of forgetting, reinforcement and exploration that drives dynamics of social interactions over the network forms a Markov process which is the basis for our Hidden Markov transition matrix A . We assume that an agent can transit from one state to another based on transition probabilities. The observation matrix B is estimated using SVD by elimi-

nating errors in matrix A while the initial state probabilities are estimated from the errors eliminated while estimating the observation matrix. The transitions when the noise component was reduced led to the observation of the actual interaction dynamics exhibited by the agents. Noise in the observation matrix represents the component that does not enhance the interactions as it is difficult to decode in the formation of relationships. The decision of who to interact or not interact with increases the noise component in the transition and observation matrix due to the uncertainty of the agent's behavior.

4. User Interaction Analysis

The rate of user interactions across different social networks has been highlighted by different research work. Nearly all interactions can be attributed to only the top 5% of the users in facebook [17]. 30% of facebook user's pairs interact consistently from one month to the next. Interaction edges only accounts for 32% of the total edges, meaning users only interact with a small subset of their friends. Nearly 78% of interactions account for total wall posts which represents the friendship maintenance effort [15].

We conclude that reinforcement is at 32%, exploration at 46% and forgetting is at 22% of all the user activities in a social network. These form the main diagonal of the transition matrix and the other real valued entries of the matrix are derived through simulation. This approach is applied due to lack of real life social networking data. The study varied the entries of the main diagonal of the transition matrix at the analysis level.

5. Singular Value Decomposition

Matrix approximation technique is widely researched and a common tool used heavily in recommendation systems, bioinformatics, computer vision and text processing. SVD is a matrix factorization method that has been used widely in different applications ever since an efficient algorithm for its computation was developed [20]. SVD is a stable and effective method to split the system into a set of linearly independent components, each of them bearing its own energy contribution [22].

The SVD eliminated the noise in the transition matrix A to estimate the observation matrix. The noise in the transition and observation matrices is eliminated by adopting an accuracy threshold level of α . A reduced rank approximation with error of $1-\alpha$ of matrix A to represent matrix B is selected. Let \hat{A} be the matrix for the retained α and \tilde{A} be the matrix for the eliminated $1-\alpha$ of matrix A . The term error in SVD is defined by [23] as:

$$\hat{\alpha} = 1 - \sqrt{\frac{\sum_{i=1}^k \sigma_i^2}{\sum_{i=1}^N \sigma_i^2}}$$

The relative error is estimated for a sum of the first terms of the SVD outer product expansion [20] [23]. SVD is used for optimal low rank approximation and

a partial SVD can be used to construct a rank k approximation. The retained α of the singular values of matrix A are reconstructed to estimate matrix \hat{A} which is normalized to estimate the observation matrix B . The initial state probability vector is then estimated from matrix \tilde{A} of $1-\alpha$ of the singular value of matrix A that was eliminated as noise, due to uncertainty in the agents interactions.

6. Results and Discussions

This section highlights the results and discussions from the agents interactions in a social network.

Table 1 shows the descriptive statistics for two samples generated with two random generators. Each sample has two parts, the first 40% and the second 30% with the error estimation in each case. When compared, the first section of each of the two samples has higher variability as compared to the second part of each sample. This is due to changes observable in the social interactions as agents' interactions in a social network account for around 50% of social dynamics.

In **Table 2**, the observation probabilities increased from low to high when interaction rates ranges between 10% and 50%. The interactions intensity is higher in the high interactions rate for the first partition compared to the second partition. Thus, the most active proportion of agents' is estimated at around 50%.

In **Figure 1**, the determinants of the product between the observation and transition matrices with different rank approximation are plotted. The figure shows the changes observable in a social network in terms of interdependence and independence of the agents interactions. At 10% to 50% interaction rate, the matrix product has a determinant of zero. This high level of dependency amongst the agents shows high levels of interactions. This dependency through the singular matrix as a product of the transition and observation matrices shows that the activities of an agent can be expressed as a linear combination of the other agents in the network. The interactions are intense and agents actively change their positions during these times to achieve their desired relationship levels. This slows down from the error estimation above 50% where independence

Table 1. Descriptive statistics and the interaction rates.

| Statistic | Sample 1 | | Sample 2 | |
|--------------------------|-----------|-----------|-----------|-----------|
| | 10% - 50% | 60% - 90% | 10% - 50% | 60% - 90% |
| Mean | 0.5322 | 0.5040 | 0.5973 | 0.4587 |
| Standard deviation | 0.1158 | 0.0524 | 0.1818 | 0.0807 |
| Coefficient of variation | 21.7587 | 10.3968 | 30.437 | 17.5932 |

Table 2. Observation probabilities and interaction rates.

| Interaction rates | Low | Medium | High |
|-------------------|--------|--------|--------|
| 10% - 50% | 0.2510 | 0.3403 | 0.4087 |
| 60% - 90% | 0.3218 | 0.3231 | 0.3551 |

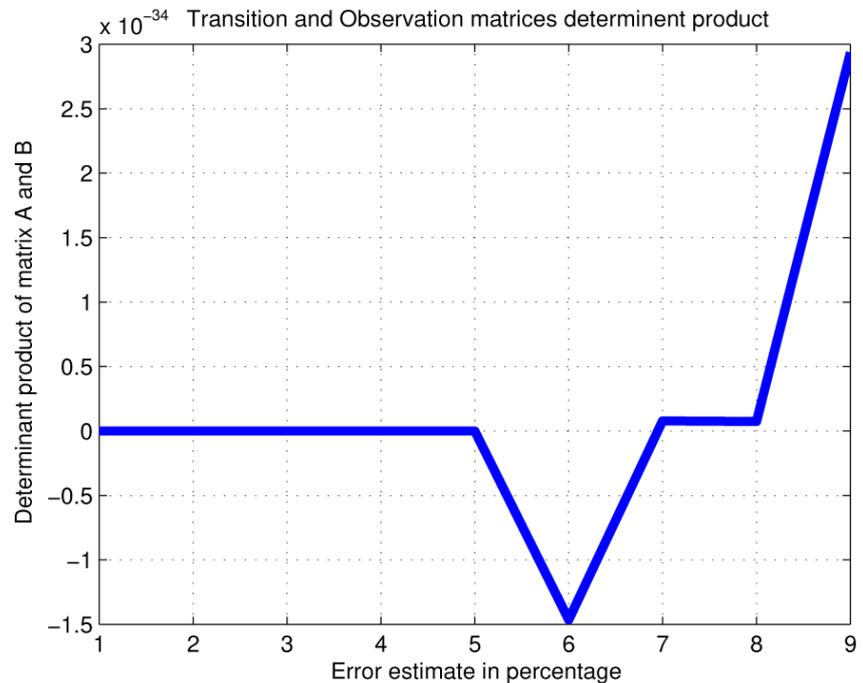


Figure 1. Dynamics of the determinant of the product between the observation and transition matrices.

levels increases.

Figure 1 and **Table 1** highlight that agents are more active in changing positions as they interact with the interaction rates of between 10% and 50%. **Table 1** has high variability estimated by the coefficient of variation. Above 60%, low levels of interdependency are exhibited, an indication of minimum interaction rates in the social network.

In **Figure 2**, the medium and high interaction rates were observable with the estimate error of between 10% and 50%. At 60%, the interaction rates falls to low and then increases again to medium and high as estimation error increases. This compares well with **Figure 1** of the singular matrix emitted by the transitions and observations of the agents. Further observations from **Figure 2** is that low interactions accounts for 11% of all the activities, medium interactions with 56% and high interaction rates accounts for 33%. We used simulation but similar observations were noted [3] [17] [24]. Agents' reinforce and explore new interactions at the start and this dips and increases again with time. A general trend in a social network is for the agents to actively pursue new relationships, some waning with time, others being reinforced and new ones being formed. The other component of the social network is the noise due to these dynamic changes in the network.

7. Conclusion

In our model, agents interactions from the HMM and SVD estimates showed 11% low observations, 56% medium interaction rates and 33% high interactions

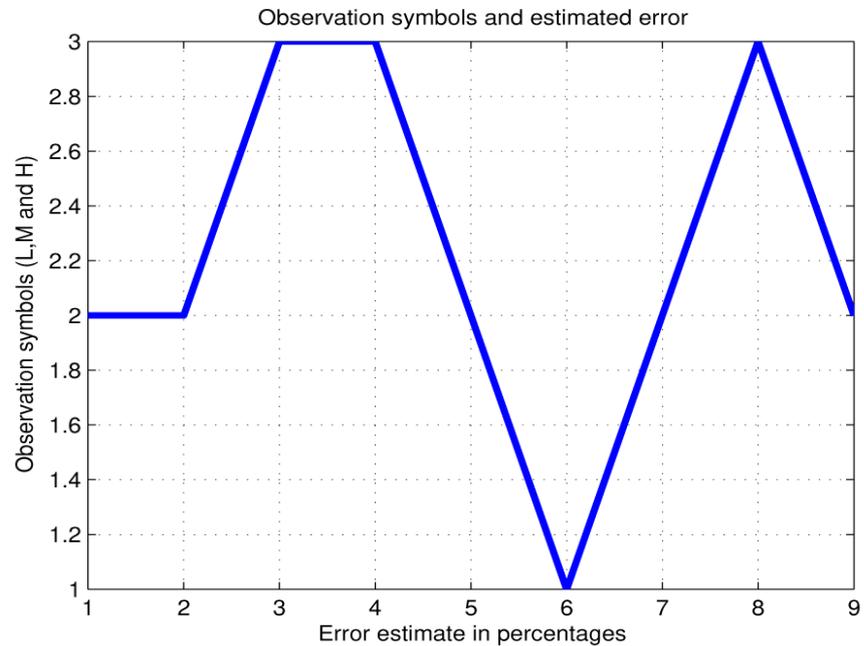


Figure 2. Observation symbols ($L = 1$, $M = 2$ and $H = 3$) versus the estimated errors.

observed. The interaction rate of medium and high is observed within the first 50% of error estimation as the other component is noise in the network due to interaction dynamics and rapid changes from the agents activities. The SVD is an ideal method to estimate the observation matrix from the transition matrix and the initial probabilities from the observation matrix. The HMM is a technique that can model interaction dynamics in a social network due to its adaptability to model stochastic processes.

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