

Dynamic Knowledge—A Century of Evolution

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Received July 25th, 2013; revised August 24th, 2013; accepted September 4th, 2013

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The discovery of non-linear systems dynamics has impacted concepts of knowledge to ascribe to its dynamic properties. It has expanded a development that finds its roots more than hundred years ago. Then, certainty was sought in systems of scientific insight. Such absolute certainty was inevitably static as it would be irrevocable once acquired. Although principal limits to the obtainability of knowledge were defined by scientific and philosophical advances from the 1920s through the mid-twentieth century, the knowledge accessible within those boundaries was considered certain, allowing detailed description and prediction within the recognized limits. The trend shifted away from static theories of knowledge with the discovery of the laws of nature underlying non-linear dynamics. The gnoseology of complex systems has built on insights of non-periodic flow and emergent processes to explain the underpinnings of generation and destruction of information and to unify deterministic and indeterministic descriptions of the world. It has thus opened new opportunities for the discourse of doing research.

Keywords: Theory of Knowledge; Complexity; Information; Chance; Necessity

Introduction

The acquisition of certain, indisputable knowledge has been a fundamental desire throughout the existence of mankind. For this purpose, the basic rules of thought have been established in the subject of logic, going back to Aristotle. The basic rules of inquiry have been developed, mainly since the period of Enlightenment (but rooted as far back as Galileo), in the field of methodology. Advances in both areas have contributed to gains in the content of knowledge through the demarcation of science and the characterization of the scientific approach. In turn, both areas have also shaped our concepts of the nature of knowledge^{1,2}. The interdependence and cross-fertilization between the theory of science and scientific progress has arguably increased in recent history. An investigation into the developments in epistemology over more than a century displays three periods of thought. They evolve from early attempts to define absolute certainty through axiomatization (~1880-1920s) via discoveries of insurmountable limits to the obtainability of knowledge (~1920s-1960s) toward the description of rational inquiry as a dynamic process that has its foundations in insights from non-linear systems research (~1960s onward). This evolution was initially driven by developments in the theory of knowledge, which were then applied to the empirical sciences, but in the second and third phases was increasingly shaped by progres-

sions in the sciences, which required reevaluations in the theory of knowledge. Its outcome is characterized by a redefinition of knowledge from a definitive and cumulative entity to a probabilistic and evolving process.

Evolution of Knowledge

Absolute Certainty

It was the nineteenth century view that the world was a machine, which was fully predictable if all positions and momenta of all its objects could be measured. The predominant scientific philosophical foundation of the time was determinism. In this environment, the mathematical schools around Hilbert and Frege tried to make knowledge definitive through axiomatization. From their basis, Russell developed analytic philosophy, a quasi-reductionist approach that built on logic and mathematics to analyze specific problems. Russell's philosophy was expanded by the Vienna Circle to logical empiricism, which strove to obtain definitive answers in the empirical sciences. The period is characterized by an extension of the formal concepts devised for generating certainty in mathematics via their applications in logic and language to the empirical sciences with the goal of making knowledge in these areas certain as well.

Meta-Mathematics—Hilbert

The axiomatic method in geometry consists of accepting, without proof, certain propositions (axioms), from which all other propositions are derived as theorems. Because mathematics studies strings of signs that have no inherent meaning, a general method for testing internal consistency of the theorems was devised in the conception of models, such that each proposition is converted into a true statement about the model. This

¹Quantum mechanics has precipitated profound gnoseological revisions. As a case in point, the meaning of the term "Verstehen" (German for "understanding") is discussed repeatedly in Heisenberg (1984), Chapters 3,10.

²Non-linear systems research has redefined the nature of chance and determinism (Ruelle, 1991; Favre, et al., 1988). "Computing theory is spawning ways of modeling complexity and disorder by describing information in algorithmic forms. In this way, chaos is revealing fundamental limits to human knowledge in an uncomfortable way." (Hall, 1993: Introduction).

approach has limitations. The interpretation of axioms by models composed of an infinite number of elements makes it impossible to encompass the models in a finite number of observations. Also, the question of consistency of the axiomatic method in geometry may be deferred to a question about the consistency of the model. This was the case for David Hilbert's translation of Euclidean axioms into algebraic truths, which showed that if algebra is consistent so is the Euclidean system of geometry. Therefore, the model method does not provide a final answer to the problem it was designed to solve.

It was Hilbert's declared goal to firmly root arithmetic, and building on it the entirety of mathematics, in an axiomatic system that should be provably free of contradictions (the "Hilbert program" in the context of which he later developed the Hilbert calculus³). He formulated his program with concrete methods for solving the consistency problem. He promoted meta-mathematics as a way of perfecting the axiomatic method via constructing mathematics on a solid and complete logical foundation. Hilbert believed that in principle this could be done, by showing that

- All of mathematics follows from a correctly chosen finite system of axioms;
- There is an axiom system that is consistent, provable through some means such as the epsilon calculus⁴.

Hilbert's approach constituted the shift to the modern axiomatic method, wherein axioms are not taken as self-evident truths but as hypotheses to be tested. Geometry may refer to objects, about which we have strong intuitions, but it is not necessary to assign any explicit meaning to them because only their defined relationships are subject to discussion. Hilbert thus addressed the antinomies of naive set theory and attempted to preserve the entire classical mathematics and logic (without losing Cantor's set theory that had been shaken by the discovery of paradoxes⁵).

Predicate Logic—Frege

In meta-mathematics, a finitistic procedure must show that antinomies cannot be derived by stated rules of inference from the axioms. If the derivation of a single antinomy from the axioms is possible then any formula whatsoever is deducible. Conversely, if there is at least one formula that cannot be derived then the calculus is incomplete. Once consistency is established, it is of interest whether an axiomatized system is complete. To establish consistency and completeness, Gottlob Frege strove to develop a universal language of pure reason, in which "nothing is left to guesswork". He attempted to arithmetize every individual scientific method, so that the truth of every scientific statement can be tested⁶. Universality was a declared goal in the development of the formalism. It was an idea previously conceived of, but not developed by Leibnitz.

Frege's work was intended to fulfill the need of mathematics for exact foundations and stringent axiomatic treatment. He attempted to devise a science of reason, which formalizes content such that it can be logically evaluated. Frege's "Be-

griffsschrift" in 1879 developed this axiomatic form of logic, a second level predicate logic with a concept for identity, which contained the core features of modern formal logic. Central to Frege was the discussion of equivalence in content. He took it that the statements used in mathematics are important only because of the non-linguistic propositions (the "thoughts") they express. Mathematicians working in various languages work on the same subject because their statements express the same thoughts. According to this view, thoughts are the elements that logically imply or contradict one another, that are true or false, and that together constitute mathematical theories. Each thought is about a determinate subject-matter, and makes a true or false statement about that subject-matter. A question about the consistency of a set of geometric axioms is a question about a specific set of thoughts. Because thoughts are determinately true or false, and have a determinate subject-matter, it makes no sense to talk about the "reinterpretation" of thoughts. From Frege's point of view, the kind of reinterpretation Hilbert engaged in (assigning different meanings to specific words) can apply only to statements and never to thoughts. Frege noted a difficulty with Hilbert's approach in the meaning of the term "axioms". If it means the elements for which issues of consistency and independence can arise, then it must refer to thoughts, whereas if it means elements which are susceptible to multiple interpretations, then it must refer to statements. Frege distinguished his work from the theories by Immanuel Kant, who had considered arithmetic statements to be synthetic judgments a priori, and John Stuart Mills, for whom arithmetic statements were general laws of nature confirmed by experience. Bertrand Russell adopted Frege's predicate logic as his primary philosophical method, which he thought could expose the underlying structure of philosophical problems.

Frege's contribution to logic is the development of a formal language, and with it a formalism for proof, which makes him one of the forefathers of analytic philosophy. In contrast to Husserl's 1891 book "Philosophie der Arithmetik", which attempted to show that the concept of the cardinal number is derived from psychological acts of grouping objects and counting them, Frege sought to show that mathematics and logic have their own validity, independent of the judgments or mental states of individual mathematicians and logicians (which were the basis of arithmetic according to the "psychologism" of Husserl's philosophy).

Logicism and Analytic Philosophy—Frege/Russell

Richard Dedekind and Gottlob Frege laid the foundations for the mathematical-philosophical program of logicism. This school of thought in the philosophy of mathematics puts forth the theory that mathematics is an extension of logic and therefore some or all statements of mathematics are reducible to logic. Bertrand Russell and Alfred North Whitehead championed this theory. Like Frege, Russell and Whitehead attempted to show that mathematics is reducible to fundamental logical principles.

³Hilbert strove to carry out final proofs with his formalism. With today's historical perspective, arguably, he succeeded in formalizing computation, not deduction (see also Chaitin, 1999: Chapter I).

⁴The epsilon calculus is an extension of a formal language by the epsilon operator, where the operator substitutes for quantifiers in that language as a method leading to a proof of consistency for the extended formal language; the epsilon operator and epsilon substitution method are typically applied to a first-order predicate calculus, followed by a demonstration of consistency.

⁵Georg Cantor had developed a theory of infinite sets. The ordinal numbers indicated positions on an infinite list, while the cardinal numbers measured the size of infinite sets. He had shown that the set of all subsets of a given set is always bigger than the set itself. Bertrand Russell recognized that the set of all subsets of the universal set cannot be bigger than the universal set itself. He identified a critical paradox in the set of all sets that are not members of themselves—a condition impossible to satisfy.

⁶The Frege program went beyond Hilbert's ambitions by expanding an axiomatic approach beyond mathematics to a language for all sciences.

They collected evidence in support of the assertion of logicism in their *Principia Mathematica*. However, logicism was brought to a deep crisis with the discovery of the classical paradoxes of set theory (by Cantor in 1896, by Zermelo and Russell in 1900-1901). Frege gave up on the project after Russell communicated his exposition of an inconsistency in naïve set theory (the “Russell antinomy”⁷). Nevertheless, Frege’s research had provided the groundwork for others to develop the logicistic program.

Late nineteenth-century English philosophy was dominated by British idealism⁸, as taught by philosophers such as Francis Herbert Bradley and Thomas Hill Green. Against this intellectual background, Bertrand Russell and George Edward Moore, articulated their program of analytic philosophy, a basic principle of which is conceptual clarity. Inspired by the developments in logic, specifically Frege’s predicate logic, Russell claimed that the problems of philosophy can be solved by demonstrating the simple constituents of complex notions. This approach differs from that of Locke, Berkeley, and Hume by its incorporation of mathematics and its development of a logical technique. It is thus able to achieve definite answers to certain problems, which have the quality of a science rather than of a philosophy. Compared with the philosophies of system-builders, the quasi-reductionist approach of analytic philosophy is able to tackle its problems one at a time, instead of having to devise a theory of the whole universe. Its methods, in this respect, resemble those of the applied sciences. Russell had no doubt that, in so far as philosophical knowledge was possible, it had to be sought by such approaches which could make many long-standing problems completely solvable.

Logical Empiricism—The Vienna Circle

The Vienna Circle (“Der Wiener Kreis”) was a group of philosophers, gathered around the University of Vienna in 1922, that developed the formalisms of Bertrand Russell and Ludwig Wittgenstein into the school of logical positivism (neopositivism, which later evolved into logical empiricism). Logical empiricism used formal logic to underpin an empiricist account for our knowledge of the world (Hahn et al., 1929⁹). Similar philosophical concepts were pursued simultaneously by the Berlin Circle (“Berliner Gruppe”, later “Berliner Gesellschaft für empirische Philosophie”).

The Vienna Circle considered logic and mathematics to be analytic in nature. Extending Wittgenstein’s insights about logical truths to mathematical ones, the Vienna Circle viewed both as tautological. Like the true statements of logic, true statements of mathematics did not express factual truths. Being devoid of empirical content, they only concerned ways of representing the world by spelling out implicit relations between statements. The knowledge claims of logic and mathematics gained their justification on purely formal grounds, by proof of their derivability via stated rules from stated axioms and premises. Thus, the contribution of pure reason to knowledge (in the form of logic and mathematics) was thought to be easily integrated into the empiricist framework¹⁰.

The synthetic statements of the empirical sciences were held to be cognitively meaningful if—and only if—they were em-

pirically testable in some sense. These statements derived their justification as knowledge claims from successful tests. For this purpose, the Vienna Circle applied a meaning criterion. While the correct formulation was much debated, it mandated that synthetic statements, which failed testability in principle, were considered to be cognitively meaningless and to give rise only to pseudo-problems¹¹. No third category of significance besides that of a priori analytic and a posteriori synthetic statements was admitted. In particular, Kant’s synthetic a priori was banned as having been refuted by the progress of science. Hence, the Vienna Circle rejected the knowledge claims of metaphysics as being neither analytic and a priori nor empirical and synthetic. Combined with the rejection of rational intuition, the Vienna Circle’s exclusive apportionment of reason into either formal a priori reasoning, issuing in analytic truths or contradictions, or substantive a posteriori reasoning, issuing in synthetic truths or falsehoods, was very characteristic for the philosophy of the time. The logical empiricist principle stated that there are no specifically philosophical truths and that the object of philosophy is the logical clarification of thoughts.

Thus, a theory of scientific knowledge was propagated that sought to renew empiricism by freeing it from the impossible task of justifying the claims of the formal sciences. The Vienna Circle strove to reconceptualize empiricism by means of their interpretation of then recent advances in the physical and formal sciences. Their anti-metaphysical stance¹² was supported by an empiricist criterion of meaning and a broadly logicist conception of mathematics. Moreover, the Circle sought to account for the presuppositions of scientific theories by regimenting such theories within a logical framework so that the important role played by conventions, either in the form of definitions or of other analytical framework principles, became evident. The theories of the Vienna Circle helped to provide the blueprint for an analytical philosophy of science as a meta-theory.

Limits to Absolute Knowledge

The early part of the twentieth century was the time when principal limits to the obtainability of knowledge were identified. For the researchers and philosophers of those days, the boundaries of knowledge were increasingly revealed during efforts to complete the edifice of the preceding period and put the scientific discourse on absolutely certain foundations. Contributions to defining these limits came from the natural sciences, mathematics/computation, and philosophy. Thermodynamics and particle physics began to expose the confines of the nineteenth century mechanistic and deterministic world view. Yet, these were initially observations of specialized sciences that seemed to show practical rather than profound constraints. However, in a largely parallel development, attempts at final proofs in meta-

¹⁰“Wir haben die wissenschaftliche Weltauffassung im wesentlichen durch zwei Bestimmungen charakterisiert. Erstens ist sie empiristisch und positivistisch: Es gibt nur Erfahrungserkenntnis, die auf dem unmittelbar Gegebenen beruht. Hiermit ist die Grenze für den Inhalt legitimer Wissenschaft gezogen. Zweitens ist die wissenschaftliche Weltauffassung gekennzeichnet durch die Anwendung einer bestimmten Methode, nämlich der der logischen Analyse. Das Bestreben der wissenschaftlichen Arbeit geht dahin, das Ziel, die Einheitswissenschaft, durch Anwendung dieser logischen Analyse auf das empirische Material zu erreichen.” (Hahn et al., 1929).

¹¹Note the central role of testability that later also played a fundamental role in Popper’s philosophy, but became limited to being falsifiable.

¹²“Es hat sich immer deutlicher gezeigt, daß die nicht nur metaphysikfreie, sondern antimetaphysische Einstellung das gemeinsame Ziel aller bedeutet.” (Hahn et al., 1929).

⁷See Footnote 5.

⁸British idealism is broadly characterized by a belief in a single all-encompassing reality—an absolute, the assignment of reason as the faculty to grasp the absolute, the rejection of a dichotomy between thought and object.

⁹The exact authorship of the brochure is subject to some debate (see Uebel 2008).

mathematics (a continuation of the programs developed by Hilbert and Frege) revealed incompleteness and uncomputability. Hence, the restrictions to what is knowable transcend the applications of physics. Philosophy revealed the logical shortcomings in the principle of induction for testing hypotheses (the synthetic statements of logical empiricism), a recognition that led to the development of a hypothetical-deductive theory of science. Rather than defining a path to absolute knowledge, this philosophy elucidated a principal limit within the empirical sciences in the impossibility to verify general theories. Despite these fundamental developments showing the unachievability of absolute certainty, it was a characteristic of that time that the knowledge accessible within the confines of the scientific systems of thought was considered stable and definitive¹³.

Uncertainty—Heisenberg

In 1900, Max Planck suggested that waves could not be emitted at an arbitrary rate but only in quanta, each of which had a certain amount of energy that increased with the frequency of the waves. There was intense debate over the formal explanation for this phenomenon. Schrödinger's equation in quantum mechanics, like the canonical equation in classical physics, expresses a reversible and deterministic process. If the wave function at a given instant is known it can be calculated for any previous or subsequent instant. However, the properties of particles are measurable only in terms of probability distributions. The Schrödinger equation predicts what the probability distributions are, but fundamentally cannot predict the exact result of each measurement. In quantum mechanics, classical determinism becomes inapplicable; and statistical considerations, introduced through the wave intensity, play a central role.

In 1926, Werner Heisenberg used Planck's model to describe the uncertainty principle. The precise description of the future of a particle depends on the exact determination of its present position and velocity. Quantum mechanics has shown that no measurement ever leaves the system to be measured undisturbed. So, the position of a particle cannot be measured more precisely than the wavelength of the light used for its measurement. The shorter the wavelength of the light the higher its energy, the more it will perturb the velocity of the particle measured. Hence, the more accurately the position of a particle is determined the less accurately its velocity can be assessed. The analogous relationship exists between energy and time. The uncertainty principle describes a physical limit to the precision of obtainable knowledge¹⁴. Phase space is divisible into blocks of minimum size that represent states. This limits the precision of obtainable knowledge. It also has implications for the information content of an event, and for the analysis of non-linear events in the ensuing phase of inquiry.

Incompleteness—Gödel

Gödel demonstrated the limits of the axiomatic method. He constructed an arithmetical formula that represents the meta-mathematical statement "This formula is not demonstrable" and showed that it is demonstrable only if its negation also is demonstrable. The formula is, therefore, true (by meta-mathematical criteria) and undecidable within the confines of arithmetic, implying that the axioms of arithmetic are incomplete. Even if additional axioms were to be assumed so that the true

formula could be derived from the set of arguments, another true but undecided formula could be constructed in the expanded system. This conclusion holds, no matter how often the original system is enlarged. Next, Gödel described how to construct an arithmetical formula that represents the meta-mathematic statement "arithmetic is consistent" and he proved that the formula "if arithmetic is consistent then this formula is not demonstrable" is formally demonstrable while the statement "arithmetic is consistent" is not. It follows that the consistency of arithmetic cannot be established by an argument that can be represented in the formal arithmetical calculus. Gödel showed that it is impossible to give a meta-mathematical proof of the consistency of a system comprehensive enough to contain the whole of arithmetic unless the proof itself employs rules of inference different from the transformation rules used in deriving theorems within the system. Therefore, the consistency of the assumptions in the reasoning is as subject to doubt as is the consistency of arithmetic. Furthermore, Gödel characterized a fundamental limitation in the power of the axiomatic method by showing that any system within which arithmetic can be developed is essentially incomplete, that is there are true arithmetical statements that cannot be derived from the set of underlying axioms. He demonstrated the untenability of the assumption that the totality of true propositions can be developed systematically from a set of axioms. It is impossible to establish the internal logical consistency of a large class of deductive systems unless one adopts principles so complex that their internal consistency is as open to doubt as that of the systems themselves (Gödel, 1931; Nagel/Newman, 1958).

Uncomputability—Turing/Kolmogorov/Chaitin

With the aim of solving Hilbert's Entscheidungsproblem challenge to automate testing the truth of mathematical statements, Turing introduced a mechanistic approach to a procedure that could decide their validity. The model of computation he proposed, now called the Turing machine (a universal computer programmed to carry out any computation whatsoever), consists of an infinite tape that stores symbols and a finite-state controller that sequentially reads symbols from the tape and writes symbols to it. The Turing machine is deterministic insofar as the tape contents exactly determine the machine's behavior. Given the present state of the controller and the next symbol read off the tape, the controller goes to a unique next state, writing at most one symbol to the tape. The input determines the next step of the machine, and the tape input determines the entire sequence of steps the Turing machine goes through.

According to the Church-Turing thesis, established in 1936 by Alan Turing and Alonzo Church (Emil Post developed similar concepts independently), a universal Turing machine can compute anything at all computable. At the most basic level, the Turing machine uses discrete symbols and advances in discrete time steps. However, not every Turing computation halts when presented with a given input string. With this recognition, Alan Turing expanded Gödel's theorem to state that it may not be possible to predict whether a universal computer will ever halt when started with a given input data string¹⁵. Turing deduced as

¹⁵"Metamathematics was promoted, mostly by Hilbert, as a way of perfecting the axiomatic method, as a way of eliminating all doubts. But this meta-mathematical endeavor exploded into mathematicians' faces, because, to everyone's surprise, it turned out impossible to do. Instead it led to the discovery by Gödel, Turing, and [Chaitin] of metamathematical results, incompleteness theorems, that place severe limits on the power of mathematical reasoning and on the power of the axiomatic method." (Chaitin, 1999: Chapter I).

¹³This is reflected in the literature of that period, which contains ample references to the limits of objective knowledge (Einstein, 1954; Popper, 1972; Barrow 1998).

¹⁴For a philosophical discussion about the inevitably resulting incompleteness of knowledge see Heisenberg (1984) Chapter 10.

a corollary that there is also no axiomatic system to predict whether an arbitrary program will ever halt. While Turing's analysis demonstrated the completeness of computing formalisms¹⁶, it showed the incompleteness of deductive formalisms, which helped start the field of numerical analysis. According to the principle of computational equivalence (Wolfram, 2002¹⁷), all systems that exhibit more than simple behavior have equal computational powers and can serve as universal computers. Since no universal computer can outstrip any other, most processes in the world are inherently computationally irreducible. Even a set of ultimate rules that run the universe would not allow any predictions about its outcome without running it through a computer program. Many simple combinatorial systems have complicated and unpredictable behavior, which means they achieve computational universality.

The paradoxes of meta-mathematics (described above) led to the development of a new formalism in symbolic logic that attempted to avoid them. From it, programming languages were developed¹⁸. Kolmogorov, Chaitin and Solomonoff put forward the idea that the complexity of a string of data can be defined by the shortest binary computer program for computing the string. Thus, the complexity is the minimal description length. Chaitin refined computational complexity and algorithmic information theory (Chaitin, 1975). The halting probability (the Chaitin constant Ω) is a real number that represents the probability that a randomly chosen computer program, having been presented with an input string, will halt. The complexity of a binary string is measured by the size of the smallest program for calculating it. Chaitin defined randomness (lack of structure) via incompressibility. A string is random when it cannot be compressed: a random string is its own minimal program. He reinterpreted the results from the works of Gödel and Turing by demonstrating that any attempt to show the randomness of a sufficiently long binary string is inherently doomed to failure. Hence, there can be no formal proof whether or not a sufficiently long string is random¹⁹. In some areas, mathematical truth is completely unstructured and incomprehensible. This occurs in elementary number theory and in Peano arithmetic. Further, axioms cannot be used to derive results of higher complexity than their own. To derive conclusions of high complexity, a highly complicated axiomatic system is required (Chaitin, 1999).

Unverifiability—Popper

Karl Popper coined the term "critical rationalism" to describe his philosophy (Popper, 1963²⁰). It indicates his rejection of classical empiricism, and the classical observational-inductive method of science that was derived from it. Prior to Popper, induction had been an accepted research approach. In it, conclusions are drawn from specific statements to more general

statements. In the empirical sciences, the erection of hypothesis- and theory-systems by induction from specific observations was considered appropriate. The technique of complete induction had been formalized in mathematics by Blaise Pascal. Although induction in the applied sciences is never complete, Bertrand Russell had acknowledged that extrapolation from scientific observations to general laws of science (which are presumed to hold in the future) is impossible unless the inductive principle is assumed. Scientific progress would grind to a halt if one did not assume the legitimacy of extrapolations from (reproduced) observations to general principles. Investigation would be trapped in a never-ending process of reconfirming experiments of the past. The inductive principle is exemplified in the notion that the sun will rise tomorrow because it has risen every day thus far.

Popper built on the recognition by David Hume that induction has logical shortcomings²¹. He realized that a verification of all-statements was neither logically consistent nor practically feasible. Theories are never empirically verifiable. His account of the logical asymmetry between verifiability and falsifiability lies at the heart of his philosophy of science. In Popper's example, no matter how many white swans are observed it does not allow the conclusion that all swans are white. By contrast, the observation of a single black swan is sufficient to support the conclusion that not all swans are white²². Hence, the falsification of hypotheses and theories is supported by deductive logic and can be accomplished with one counter-example (the falsification) if the hypotheses or theories in question are all-statements. The term "falsifiable" means that if a hypothesis is false this can be shown by observation or experiment. Logically, no number of positive outcomes at the level of experimental testing can confirm a scientific theory, but a single counter-example is logically decisive: it shows the theory, from which the implication is derived, to be false. The shortcomings of induction therefore led Popper to the development of a deductive method of testing. He held that one should rationally prefer the least likely (simplest, most easily falsifiable) theory that explains known facts. It is impossible, Popper argued, to ensure that a theory is true; it is more important that its falsity can be detected as easily as possible. We cannot know with certainty what is always true, only what is not.

The demarcation between scientific and transcendental problems has been an important question in philosophy. In induction logic, the criterion for the demarcation of the empirical sciences from mathematics, logic, and metaphysics is definitive, because the logical form of its statements is such that their verification or falsification is finally decidable. This is not the case in the hypothetical-deductive theory of knowledge. Popper took falsifiability as his criterion of demarcation between what is, and is not, genuinely scientific²³: a theory should be considered scientific if—and only if—it is falsifiable (Popper, 1935). Like the Vienna Circle, Popper investigated the testing of hypotheses

¹⁶The capability of almost any computer programming language to express all possible algorithms is now known as computational universality (Wolfram, 2002).

¹⁷Chapter 12: The principle of computational equivalence.

¹⁸Chaitin points out that his work addresses the Berry paradox ("the first ordinary number that cannot be named in a finite number of words"); he delineates it from Gödel's focus on the liar paradox ("this statement is false" and Turing's work on the Russell paradox (see Footnote 5) (Chaitin, 1999). Here, we support the view that thegnoseological relevance of his work, like Turing's, is the elucidation of principal limitations to computability.

¹⁹"[...] incompleteness is not accidental, but ubiquitous [...]: the probability that a true sentence of length n is provable in the theory tends to zero when n tends to infinity, while the probability that a sentence of length n is true is strictly positive." (Calud, 2005).

²⁰Introduction, Section XV.

²¹Hume had shown in his 1739 Treatise of Human Nature that reliance on experience to draw conclusions to unobserved cases would lead to an infinite regress, as discussed by Popper (1982, Neuer Anhang VII).

²²"Bekanntlich berechtigen uns noch so viele Beobachtungen von weissen Schwänen nicht zu dem Satz, dass alle Schwäne weiss sind." (Popper, 1935, Kapitel I.1.).

²³For Popper, this was the demarcation of the empirical sciences from mathematics, logic and metaphysics (Popper, 1935, Kapitel I.4.). Here, mathematics is considered one of the sciences and the relevant demarcation is the one from metaphysics.

(“synthetic statements”). However, rather than defining a path to absolute knowledge he identified the principal limitation that verification of general hypotheses (“all-statements”) is impossible.

Although associated with some great advances, this period was largely characterized by defining the limitations in the strife for absolute knowledge that had been initiated from around 1880 through the 1920s. To some degree, it generated a crisis in the theory of knowledge because rather than elucidating new possibilities, its most influential works espoused on limits to obtainable knowledge (in essence elucidating impossibilities).

Dynamic Knowledge

The discovery of non-linear systems dynamics, mainly in the 1960s, and its ensuing rapid research progress (aided in part by the increasing availability of computer simulations) has profoundly impacted the natural sciences. The inherent emergent properties rooted in a high sensitivity to the initial conditions of such systems also have required reevaluations of existing theories of knowledge. No longer is certainty attainable within the limits defined from the 1920s through the 1960s. Knowledge is fluid—it can be produced and destroyed, and it is always probabilistic.

The dynamic nature of knowledge has been established in complex systems research of non-periodic flow (by Lorenz) and emergent processes (by Prigogine and Kauffman), in which information is generated and lost (Shaw). Complexity research has also broken the dichotomy between chance and necessity with the definition of degrees of randomness (Crutchfield). The investigations are strongly influenced by two recognitions of the preceding period, uncertainty and computational complexity.

- Non-linear systems research often describes events as trajectories in phase space. The uncertainty principle assures that two trajectories become indistinguishable after they have approached each other below a minimum distance. Further, bifurcation points, where a system can evolve toward one state or another, are at the heart of non-linear systems. The infinite accuracy of measurement at the bifurcation point that would be required to predict which state a system will assume is impossible. Hence, unpredictability and changes in information content by complex systems are rooted in the uncertainty principle.
- Non-linear systems dynamics draws heavily on information theory to establish new concepts of chance and necessity. In the 1940s, Claude Shannon (1948) had developed the modern concept of information theory. Communication occurs between a sender and a receiver via a channel. The channel capacity is a critical determinant, which is calculated from the noise characteristics of the channel. For all communication rates below channel capacity, the probability of error can be made arbitrarily small. However, theoretically optimized communication schemes may be computationally impractical. Random processes have an irreducible complexity

²⁴The lack of closed form solutions for non-linear differential equations has elevated the status of computer modeling in research from its use as a rather preliminary analysis that merely guides the path toward formal proof to a third methodology beside experimentation and logical deduction.

²⁵“For those systems with bounded solutions, (...) non-periodic solutions are ordinarily unstable [under the influence of] small modifications, so that slightly differing initial states can evolve into considerably different [outcome] states” (Lorenz, 1963).

below which the signal cannot be compressed. Shannon named the ultimate data compression the entropy. Entropy and mutual information are functions of the probability distributions that underlie the process of communication. The Kolmogorov-Chaitin complexity (K) is approximately equal to the Shannon entropy (H) if the sequence of the string under study is drawn at random from a distribution that has the entropy H (Kolmogorov, 1968). Specifically, for almost all infinite sequences produced by a stationary process the growth rate of the Kolmogorov-Chaitin complexity is the Shannon entropy rate. Thus, the insights derived from uncomputability contribute to the foundations of non-linear systems research and its epistemological implications.

Non-Periodic Flow—Lorenz

A lack of periodicity is very common in natural systems, and is one of the distinguishing features of turbulent flow. Because instantaneous turbulent flow patterns are so irregular, attention to them was often confined to the statistics of turbulence, which, in contrast to the details of turbulence, often behave in a regular well-organized manner. A closed hydrodynamic system of finite mass may ostensibly be treated mathematically as a (usually very large) finite collection of molecules, in which case the governing laws are expressible as a finite set of ordinary differential equations. These equations are generally highly intractable, and the ensemble of molecules is usually approximated by a continuous distribution of mass. The governing laws are then expressed as a set of partial differential equations, containing such quantities as velocity, density, and pressure as dependent variables. It is sometimes possible to obtain particular solutions of these equations analytically, especially when these solutions are periodic or invariant with time. Ordinarily, however, non-periodic solutions cannot readily be determined, except by numerical procedures²⁴.

A finite system of ordinary differential equations representing forced dissipative flow often has the property that all of its solutions are ultimately confined within the same bounds. A non-periodic solution with no transient component must be unstable in the sense that solutions temporarily approximating it do not continue to do so. A non-periodic solution with a transient component is sometimes stable, but in this case its stability is one of its transient properties, which tends to die out (Lorenz, 1963). Finite systems of deterministic ordinary non-linear differential equations may be designed to represent forced dissipative hydrodynamic flow. Solutions of these equations can be identified with trajectories in phase space. Systems with bounded solutions possess bounded numerical solutions²⁵.

Prediction of the sufficiently distant future is impossible by any method, unless the initial conditions are known exactly (a feat impossible to accomplish according to Heisenberg’s uncertainty relation). The foundation of Lorenz’s principal result is the eventual necessity for any bounded system of finite dimensionality to come arbitrarily close to acquiring a state it has previously assumed. Only if the system is stable, will its future development then remain arbitrarily close to its past history, and it will be quasi-periodic²⁶. The discovery by Lorenz of non-linear dynamic flow, the outcome of which sensitively depends on the initial conditions, confined the dictum of predict-

²⁶Instable systems display the now famous “butterfly effect”: One flap of a butterfly’s wings may change the future course of the weather in a place far away.

ability in science²⁷.

Emergence—Prigogine/Kauffman

The emergence of ordered structures from non-equilibrium conditions was described in chemistry by Ilya Prigogine and in evolution by Stuart Kauffman. Historically, physics has studied reversible processes. Classical physics, including quantum theory and relativity theory, have provided only limited models of temporal development. The past and the future are described as trajectories in phase space, which implies that both are somehow contained in the present. The Schrödinger equation is completely deterministic²⁸, and the macroscopic thermodynamic description typically focuses on mean values for which random fluctuations became negligible, whereas quantum mechanics introduced a probabilistic description on the microscopic level. States in thermodynamic equilibrium (or states that equate to a minimal entropy production in the linear thermodynamics of non-equilibrium) are stable states. Yet, irreversible processes play a fundamental constructive role in the physical world. The laws of irreversible processes (Prigogine, 1980) embed dynamics in a more comprehensive formalism that includes unstable states. Non-equilibrium can lead to dissipative structures, wherein fluctuations introduce a stochastic description into the macroscopic level. Instabilities far from equilibrium are essential elements for emerging systems. In the vicinity of their bifurcations the law of large numbers is not valid anymore. While the molecular interactions in chemical reactions far from equilibrium are the same as in equilibrium, they also become dependent on global conditions. The transition from the dynamic, time-reversible description of mechanics to the description of emerging processes is accomplished through a particular form of a non-local transformation, in which the homogeneity of the space-time structure is destroyed, entropy and time become operators. This transition involves an internal time that is derived from the indeterminism of the trajectories in unstable dynamic systems. The transformation leads to a spacio-temporally non-local description.

The initiating event for every step in evolution is an error in reproductive invariance²⁹ (a mutation). Such chance event is the origin of any innovation and creation in living nature. Once a mutation has taken place, its penetration of the population is subjected to the rule of selection. However, simple and complex systems can exhibit powerful self-organization. The effects of mutation and selection are diminished when operating on

²⁷“The result has far-reaching consequences when the system being considered is an observable nonperiodic system whose future state we may desire to predict. It implies that two states differing by imperceptible amounts may eventually evolve into two considerably different states. If, then, there is any error whatever in observing the present state-and in any real system such errors seem inevitable-an acceptable prediction of an instantaneous state in the distant future may well be impossible.” (Lorenz, 1963).

²⁸Compare the section on uncertainty above.

²⁹The term reproductive invariance was originally used by Monod (1985).

³⁰“[...] to combine the themes of self-organization and selection, we must expand evolutionary theory so that it stands on a broader foundation and then raise the new edifice. That edifice has at least three tiers:

- We must delineate the spontaneous sources of order, the self-organized properties of simple and complex systems which provide the inherent order evolution has to work with *ab initio* and always.
- We must understand how such self-ordered properties permit, enable, and limit the efficacy of natural selection. [...] In short, we must integrate the fact that selection is not the sole source of order in organisms.
- We must understand which properties in complex living systems confer on the systems their capacities to adapt. [...] (Kauffman, 1993).

systems that have their own rich and robust self-ordered properties³⁰. As the complexity of regulatory networks under selection increases (“complexity catastrophe”), selection is ultimately limited by:

- being too weak in the face of mutations to hold a population at small volumes of the ensemble, which exhibit rare properties; hence, typical properties are encountered instead
- or if selection is very strong, the population typically becomes trapped on suboptimal peaks of an adaptive landscape, which do not differ substantially from the average properties of the ensemble.

Evolution can be viewed as occurring in an imaginary space, the shape of which is defined by the distribution of properties across an ensemble (a “fitness landscape”³¹) (Kauffman, 1993). Spontaneous order is maintained despite selection, not because of it. However, selection may be able to change ensembles of self-organized systems by mitigating the tendency for adaptive processes to become trapped on continuously lower local optima of fitness as complexity increases. Below a critical complexity of an organism, the selective force is stronger than the mutational force. Selection can either hold the population at the global optimum or pull it there from a suboptimal genotype. Above the critical complexity, the dispersing mutational pressure increases, and the population falls from the global optimum to a suboptimal stationary steady state.

Generation and Destruction of Information—Shaw

The energy of physical systems can be described on the macro-scale, which in classical mechanics is completely intelligible, and the micro-scale of thermal motion, which to classical mechanics is unintelligible but can be successfully ignored³². Shaw applied information theory to the measurements of dynamical systems. It was his recognition that there are non-conservative systems, where there may be an active flow of information between the macro- and micro-scales. Simple system equations displaying turbulent behavior are capable of acting as an information source. According to Heisenberg’s uncertainty principle, trajectories in phase space are distinguishable only to a lower limit of distance between them³³. In laminar flow, motion is governed by boundary and initial conditions, no new information is generated. In turbulent flow, information is continuously generated by the flow itself. The transition of a system from laminar to turbulent behavior corresponds to a change of the system from an information sink to an information source³⁴. The new information of turbulent systems precludes prediction past a certain time, when new information has accumulated to

³¹Local optima on a rugged fitness landscape and attractors in phase space are alternative metaphors for the same phenomenon. They map a preferred state to be assumed by a dynamic system. The shape of the attractor or the ruggedness of the fitness landscape are reflections of the complexity of the system.

³²These micro-scales constitute a lower limit of explanation. In the eras preceding non-linear systems research, their states were assumed to be uniform or stochastic (McKelvey, 1998).

³³In applying the uncertainty principle that identifies the minimum resolvable product of bandwidth and time in the description of the frequency of a photon, Shaw divides up phase space into minimum resolvable blocks that identify “states”. (Shaw, 1981).

³⁴“The chief qualitative difference between laminar and turbulent flow is the direction of information flow between the macroscopic and microscopic length scales. (...) Entropy increases in both laminar and turbulent systems, that is, energy in both cases moves from macroscopic to microscopic degrees of freedom.” (Shaw, 1981).

displace the initial data³⁵. In non-periodic flow, closed form predictions are impossible because the information they would represent simply does not exist prior to the operation of the mechanism. “New information is continuously being injected into the macroscopic degrees of freedom of the world by every puff of wind and every swirl of water” (Shaw, 1981). This establishes, by law of nature, a transience for the intelligibility of the universe. With the inevitable and always prevalent generation and destruction of information in non-linear systems, knowledge has become fluid and dynamic.

Degrees of Randomness—Crutchfield

One designs clocks to be as regular as physically possible, so much so that they are the very instruments of determinism. The coin flip, by contrast, expresses our ideal of total randomness. Although randomness is as necessary to physics as determinism, the clock and the coin flip are mathematical ideals³⁶. Many domains face the confounding problems of detecting random and patterned components in processes under study. These tasks translate into measuring their intrinsic computation. Like Shaw, Crutchfield applied Shannon’s information theory to the analysis of complex systems, viewing every process as a channel that communicates its past to its future through its present³⁷. Similarly, he viewed model building in terms of a channel through which experimentalists communicate results to one another.

Crutchfield (2012) compared the deterministic and statistical descriptions of complexities, which despite their different teleologies are related and essentially complementary in physical systems.

- One approach that models system behaviors by applying exact deterministic representations leads to the deterministic complexity that allows us to measure degrees of randomness. Kolmogorov-Chaitin complexity is a measure of randomness, not a measure of structure.
- Ensembles of behaviors can be measured with statistical complexity that assesses degrees of structural organization. One solution, familiar in the physical sciences, is to discount for randomness by describing the complexity in ensembles of behaviors. The unpredictability of deterministic chaos forces investigators to use the ensemble approach.

A synthesis of those descriptions is articulated in computational mechanics, an extension of statistical mechanics that describes not only a system’s statistical properties but also how it stores and processes information—how it computes³⁸. At root, extracting the representation of a process is accomplished by grouping histories together that make the same predictions, the groups themselves capture the relevant information for predicting the future. This leads to the definition that the equivalence classes of the relation are the process’s causal states S (its reconstructed state space), and the induced state-to-state transitions are the process’s dynamic T (its equations of motion).

³⁵“The transition of a system from laminar to turbulent behavior is understandable in terms of the change of [the Liapunov characteristic exponent] from negative to positive, corresponding to the change of the system from an information sink to a source. The new information of turbulent systems precludes predictability past a certain time; when information accumulates to displace the initial data, the system is undetermined.” (Shaw, 1981: Abstract)

³⁶“The extreme difficulties of engineering the perfect clock and implementing a source of randomness as pure as the fair coin testify to the fact that determinism and randomness are two inherent aspects of all physical processes” (Crutchfield, 2012).

³⁷We start from the simple principle that model variables should, as much as possible, render the future and past conditionally independent (Still, 2007).

Together, the states S and dynamic T give the process’s so-called ϵ -machine that describes the effective states, that is the property of the statistical complexity as the amount of information the process stores in its causal states. The ϵ -machine (states plus dynamic) forms a semi-group that gives all of a process’s symmetries, including noisy symmetries (Shalizi, 2001). The statistical complexity has an essential kind of representational independence. The causal equivalence relation, in effect, extracts the representation from a process’s behavior. Causal equivalence can be applied to any class of system—continuous, quantum, stochastic or discrete.

The statistical complexity defined in terms of the ϵ -machine solves the main problems of the Kolmogorov-Chaitin complexity by being representation independent, constructive, the complexity of an ensemble, and a measure of structure. In these ways, the ϵ -machine gives a baseline against which any measures of complexity, or modeling in general can be compared. It is a minimal sufficient statistic that captures a system’s pattern in the algebraic structure of the ϵ -machine. The degree of randomness of a system is defined as a process’s ϵ -machine Shannon entropy rate. Its amount of organization is defined in a process with its ϵ -machine’s statistical complexity. The ϵ -machine approach demonstrates how the framework of deterministic complexity relates to computational mechanics. With it, Crutchfield was able to break down the dichotomy between necessity and chance.

Complexity often arises at the order/disorder border. There is a tendency for natural systems to balance order and chaos, to move to this complex interface between predictability and uncertainty. This often appears as a change in a system’s intrinsic computational capability. Natural systems that evolve by interaction with their immediate environment exhibit both structural order and dynamical chaos. Order is the foundation of communication between elements at any level of organization. Chaos is the dynamical mechanism by which nature develops constrained and useful randomness. From it follows diversity (Crutchfield, 2012).

Conclusion

The evolution in the concepts of knowledge over the past century has important implications. The historical development outlined here reflects the persistence of key questions and key techniques over decades, which are applied to enhance certainty, but result in displaying its limits. In using meta-mathematical formulations, Gödel found limitations in the axiomatic method. In an attempt to automate testing the truth of mathematical statements, Turing and later Kolmogorov and Chaitin discovered uncomputability. Even though these discoveries identified boundaries to what is knowable, they provided techniques for analyzing systems that were previously intractable. The Kolmogorov complexity is approximately equal to the Shannon entropy of information theory. Complexity and entropy are two measures that have been amply applied to describe the dynamic nature of knowledge.

We live in a culture that treats knowledge as cumulative, as persistently increasing³⁹. Yet, non-linear systems dynamics dem-

³⁸“[...] one sees that many domains face the confounding problems of detecting randomness and pattern. I argued that these tasks translate into measuring intrinsic computation in processes and that the answers give us insights into how nature computes.” (Crutchfield, 2012).

³⁹Note the subtitle “The Growth of Scientific Knowledge” in Popper (1963).

onstrates that the loss of information, and with it the loss of knowledge, is as inevitable as the emergence of new information by non-periodic flow. Intuitively, the claim that knowledge—once acquired—is not permanent may seem defective. However, the dynamic nature of knowledge may be well illustrated with the example of archeology. This branch of science tries to recover information that once was obvious but has been lost. A full information content of the past can never be reconstructed (as delineated by Shaw (1981), the old information has been replaced). Conversely, some information (for example the DNA sequence of dinosaurs) was implicit in the system, but can be explicated as knowledge only with today's technology. Ergo, knowledge is in flux, being constantly generated and destroyed.

It could be argued that the concept of knowledge espoused here is flawed as knowledge does not equate to information, so the generation or destruction of information has no bearing on the evolution of knowledge. While we concur in making a differentiation between information and knowledge we nevertheless assert that knowledge, in a scientific sense, requires information as its basis. Without the empirical component of information collected, knowledge turns into a transcendental type of certainty, which is outside the realm of science. Of note, Shannon (1948) used his definition of information as a basis for his analyses of communication—an essential component in the generation of knowledge.

The continuity of experience causes us to perceive the universe as one entity. In contrast, the description of nature typically categorizes observations and creates opposites that are seemingly unrelated, thus generating sub-entities of the world that are mutually disconnected. Among the starkest of these opposites is the separation of causative events from chance events. An entirely deterministic world view makes human decisions futile and leads to fatalism, whereas a stochastic view eliminates the need for decisions due to the random nature of future events, in finality leading to nihilism. Currently, the most prevalent way of dealing with this conflict is the perception that some events are subject to cause-effect relationships, while others are stochastic (random) in nature. This dualism inevitably creates two worlds, which are mutually unconnected. The evolution in the theory of knowledge over the past century has accomplished (in its most recent, third period) a reconnection of the two world views, not as opposites that compete for the control over nature but as alternative descriptions of one unified nature. Yet, this progress has forced us to give up on the idealistic goals of certainty and completeness in knowledge.

Scientific observation always originates in a hypothesis and is deduced therefrom according to set rules of logic and methodology⁴⁰. The axioms in mathematics are, in fact, hypotheses, rendering this field of inquiry a hypothetical-deductive science, like the natural sciences. As hypothesis-free observation does not exist, research moves from hypotheses (which in philosophical terms are the prior, starting points that can be chosen quite freely) to deductions and observations that are consistent with them. It formulates coalescent theories as mutually consistent sets of hypotheses. However, it must also permanently question and reexamine the original hypotheses and their roots.

⁴⁰Remarkably, Heisenberg's discovery of the uncertainty relation was guided by this recognition, which had earlier been communicated to him by Einstein "Erst die Theorie entscheidet darüber, was man beobachten kann". Heisenberg (1984), Chapter 6.

⁴¹This requirement is mandated by the recognition that a single antinomy is sufficient to derive ("prove") any conclusion whatsoever.

Scientific inquiry needs to move from the prior to all possible directions, to more basic as well as more applied questions. The starting hypothesis is strengthened or weakened by the preponderance of evidence, not refuted by final proof (as envisioned by Popper). If hypotheses and theories can neither be proven nor refuted definitively—if the best that can be achieved is preponderance of evidence in favor or against a hypothesis, what constitutes the ultimate goal of the scientific enterprise? We can assume that the number of possible hypotheses about the world is practically limitless. However, the ideal theory of the world, while always incomplete, will be inherently flawlessly consistent⁴¹. We speculate that there is only one such possibility. Implied in this model is the notion that scientific progress is accomplished by addressing and eliminating inconsistencies.

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