

# Optimal Scheduling of Cascaded Hydrothermal Systems Using a New Improved Particle Swarm Optimization Technique

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Received February 18, 2011; revised May 12, 2011; accepted May 19, 2011.

## ABSTRACT

Optimum scheduling of hydrothermal plants generation is of great importance to electric utilities. Many evolutionary techniques such as particle swarm optimization, differential evolution have been applied to solve these problems and found to perform in a better way in comparison with conventional optimization methods. But often these methods converge to a sub-optimal solution prematurely. This paper presents a new improved particle swarm optimization technique called self-organizing hierarchical particle swarm optimization technique with time-varying acceleration coefficients (SOHPSO\_TVAC) for solving short-term economic generation scheduling of hydrothermal systems to avoid premature convergence. A multi-reservoir cascaded hydrothermal system with nonlinear relationship between water discharge rate, power generation and net head is considered here. The performance of the proposed method is demonstrated on two test systems comprising of hydro and thermal units. The results obtained by the proposed methods are compared with other methods. The results show that the proposed technique is capable of producing better results.

**Keywords:** Hydrothermal Systems, Cascaded Reservoirs, Self-Organizing Hierarchical Particle Swarm Optimization with Time-Varying Acceleration Coefficients (SOHPSO\_TVAC)

## 1. Introduction

The optimum scheduling of hydrothermal plants is one of the important planning task in power system operation. The generation scheduling problem consists of determining the optimal operation strategy for the next scheduling period, subjected to a variety of constraints. The main objective is to allocate generations of hydroelectric and thermal plants in such a way so as to minimize the total operation cost of the systems subjected to variety of constraints. With the insignificant operating cost of hydroelectric plants, the problem of minimizing the operational cost of a hydrothermal system essentially reduces to that of minimizing the fuel cost for thermal plants under the various constraints on the hydraulic and power system network. It is basically a nonlinear programming problem involving non-linear objective function and a mixture of linear and non-linear constraints.

The main constraints include the cascaded nature of the hydraulic network, the time coupling effect of the hydro sub problem where the water inflow of an earlier time interval affects the discharge capability at a later

period of time, the varying hourly reservoir inflows, the physical limitations on the reservoir storage and turbine flow rate, the varying system load demand and the loading limits of both thermal and hydro plants.

The hydrothermal scheduling problem has been the subject of investigation for several decades and many methods have been applied to solve this problem. Some of these solution methods include decomposition techniques [1], dynamic programming [2], semi-definite programming [3] and concept of non-linear network flow [4]. In recent times, optimal hydrothermal scheduling problems have been solved by different heuristic techniques such as genetic algorithm [5] simulated annealing technique [6], evolutionary programming technique [7] etc. Yu, *et al.* used particle swarm optimization technique to solve short-term hydrothermal scheduling problem [8] with an equivalent thermal unit having smooth cost functions. A modified hybrid differential evolution technique was applied by Lakshminarasimman, *et al.* [9] to solve short-term hydrothermal generation scheduling problems with promising results. A comparison of particle swarm optimization and dynamic programming for large scale

hydro unit load dispatch was made by Cheng, *et al.* [10]. Recently, Catalao, *et al.* [11] applied mixed-integer quadratic programming method to determine scheduling of head dependent cascaded hydro systems.

Particle swarm optimization (PSO) happens to be a comparatively new combinatorial metaheuristic technique which is based on the social metaphor of bird flocking or fish schooling [12]. This algorithm has come to existence in mid 90's and has gained prominence from late 90's. The PSO technique has been applied to various fields of power system optimization. Gaing used PSO to solve economic dispatch problem considering generator constraints [13]. Abido proposed a revised PSO technique for optimal design of voltage stabilizer [14]. Park, *et al.* presented a method for solving economic dispatch with non-smooth cost functions [15]. A hybrid method for optimal scheduling of short-term electric power generation of cascaded hydroelectric plants based on particle swarm optimization and chance-constrained programming was presented by Jiekang, *et al.* [16].

A novel parameter automation strategy called self-organizing hierarchical particle swarm optimization technique with time-varying acceleration coefficients (SOHPSO\_TVAC) is applied in this paper for the hydrothermal scheduling to address the problem of premature convergence. In this case, the particle velocities are reinitialized whenever the population stagnates at local optima during the search. A relatively high value of the cognitive component results in excessive wandering of particles while a higher value of the social component causes premature convergence of particles. Hence, time-varying acceleration coefficients (TVAC) [17] are employed to strike a proper balance between the cognitive and social component during the search. The proposed approach was first tested on a simple test system comprising of one equivalent thermal unit and four cascaded hydro unit and then the effectiveness of the SOHPSO\_TVAC was demonstrated on a more practical system comprising of six thermal units and four cascaded hydro units. The results have been compared with other evolutionary methods and found to be superior.

## 2. Problem Formulation

Economic generation scheduling of hydrothermal systems involves the optimization of a problem with non-linear objective function subject to a mixture of linear, non-linear constraints. As the fuel cost of hydroelectric plants is insignificant in comparison with that of thermal power plants, the objective is to minimize the fuel cost of thermal power plants, while making use of the availability of hydro-resources as much as possible. The objective function and associated constraints are described as follows:

Minimize

$$F(P_{sit}) = \sum_{t=1}^T \sum_{i=1}^{N_s} [f_{it}(P_{sit})] \quad (1)$$

where,  $F(P_{sit})$  is the total fuel cost,  $T$  is the number of time interval for scheduling horizon,  $N_s$  is the number of thermal plants and  $P_{sit}$  is the power generation by the  $i$ -th thermal plants at time  $t$ .

Conventionally, the fuel cost curve for any unit can be represented by segments of quadratic functions of the active power output of the generator and can be expressed as

$$f_{it}(P_{sit}) = a_{si} + b_{si}P_{sit} + c_{si}P_{sit}^2 \quad (2)$$

where,  $a_{si}, b_{si}, c_{si}$ : fuel cost coefficients of the  $i$ -th thermal unit.

For more practical and accurate modeling of fuel cost function, the above expression needs to be modified suitably. Modern thermal power plants comprise of generating units having multi-valve steam turbines in order to incorporate flexible operational facilities. The generating units with multi-valve turbines have very different cost curve compared with that defined by (2). The effect of valve-point effect loading may be considered by adding a sinusoidal function [9] to the quadratic cost function described above. Hence, the function described by (2) is revised as follows:

$$f_{it}^v(P_{sit}) = a_{si} + b_{si}P_{sit} + c_{si}P_{sit}^2 + \left| e_{si} \times \sin \left\{ f_{si} \times (P_{sit}^{\min} - P_{sit}) \right\} \right| \quad (3)$$

where  $f_{it}^v(P_{sit})$  is the fuel cost function of thermal units including the valve point loading effect and  $e_{si}, f_{si}$  are fuel cost coefficients of the  $i$ -th thermal generating unit reflecting the valve-point effect.

The above objective function described by (3) is to be minimized subject to a variety of constraints as follows:

1) Demand constraints

The total power generated must balance the power demand plus losses, at each time interval over the entire scheduling period

$$\sum_{i=1}^{N_s} P_{sit} + \sum_{j=1}^{N_h} P_{hjt} - P_{Dt} - P_{Lt} = 0 \quad (4)$$

where  $P_{hjt}$  is the power generation of  $j$ th hydro generating unit at time  $t$ ,  $P_{Dt}$  is power demand at time  $t$  and  $P_{Lt}$  is total transmission loss at the corresponding time.

The hydropower generation is a function of water discharge rate and reservoir storage volume, which can be described by (5) as follow:

$$P_{hjt} = C_{1j}V_{hjt}^2 + C_{2j}Q_{hjt}^2 + C_{3j}V_{hjt}Q_{hjt} + C_{4j}V_{hjt} + C_{5j}Q_{hjt} + C_{6j} \quad (5)$$

where  $C_{1j}, C_{2j}, C_{3j}, C_{4j}, C_{5j}, C_{6j}$  are power generation coefficients of  $j$ th hydro generating unit,  $V_{hjt}$  is the storage volume of  $j$ -th reservoir at time  $t$  and  $Q_{hjt}$  is water discharge rate of  $j$ -th reservoir at time  $t$ .

2) Power generation constraints

$$P_{si}^{\min} \leq P_{sit} \leq P_{si}^{\max} \quad (6)$$

$$P_{hj}^{\min} \leq P_{hjt} \leq P_{hj}^{\max} \quad (7)$$

where  $P_{si}^{\min}$  and  $P_{si}^{\max}$  are the minimum and maximum power generation by  $i$ -th thermal generating unit,  $P_{hj}^{\min}$  and  $P_{hj}^{\max}$  are the minimum and maximum power generation by the  $j$ -th hydro generating unit respectively.

3) Water dynamic balance

$$V_{hjt} = V_{hj,t-1} + I_{hjt} - Q_{hjt} - S_{hjt} + \sum_{m=1}^{R_{uj}} (Q_{hm,t-\tau_{mj}} + S_{hm,t-\tau_{mj}}) \quad (8)$$

where  $I_{hjt}$  is natural inflow of  $j$ -th hydro reservoir at time  $t$ ,  $S_{hjt}$  is spillage discharge rate of  $j$ -th hydro generating unit at time  $t$ ,  $\tau_{mj}$  is the water transport delay from reservoir  $m$  to  $j$  and  $R_{uj}$  is the number of upstream hydro generating plants immediately above the  $j$ -th reservoir.

4) Reservoir storage volume constraints

$$V_{hj}^{\min} \leq V_{hjt} \leq V_{hj}^{\max} \quad (9)$$

where  $V_{hj}^{\min}, V_{hj}^{\max}$  are the minimum and maximum storage volume of  $j$ th reservoir.

5) Water discharge rate limit

$$Q_{hj}^{\min} \leq Q_{hjt} \leq Q_{hj}^{\max} \quad (10)$$

where,  $Q_{hj}^{\min}$  and  $Q_{hj}^{\max}$  are the minimum and maximum water discharge rate of the  $j$ -th reservoir respectively.

### 3. Overview of Some PSO Strategies

There are several variants of PSO. Some of the commonly used PSO are discussed in the following sections.

#### 3.1. Classical PSO

The Particle Swarm Optimization (PSO) is one of the recent developments in the category of heuristic optimization technique. Kennedy and Eberhart [12] originally developed the PSO concept based on the behavior of individuals (*i.e.* particles or agents) of a swarm or group. PSO, as an optimization tool, provides a population-based search procedure in which individuals called agents or particles change their position with time. In a PSO algorithm, the particles fly around the multidimensional search space in order to find the optimum solution. Each particle adjusts its position according to its own experience and the experience of neighboring particle.

Let in a physical  $d$ -dimensional search space, the position and velocity of the  $i$ -th particle (*i.e.*  $i$ -th individual in the population of particles) be represented as the vectors  $X_i = (x_{i1}, x_{i2}, \dots, x_{id})$  and  $V_i = (v_{i1}, v_{i2}, \dots, v_{id})$  respectively. The previous best position of the  $i$ -th particle is recorded and represented as

$pbest_i = (pbest_{i1}, pbest_{i2}, \dots, pbest_{id})$ . The index of the best particle among all the particles in the group is represented by the  $gbest_d$ . The modified velocity and position of each particle can be calculated using the current velocity and the distance from  $pbest_{id}$  to  $gbest_d$  as shown below:

$$V_{id}^{k+1} = w \times V_{id}^k + C_1 \times rand(\ ) \times (pbest_{id} - X_{id}^k) + C_2 \times rand(\ ) \times (gbest_d - X_{id}^k) \quad (11)$$

$$i = 1, 2, \dots, N_p \quad d = 1, 2, \dots, N_g$$

where  $N_p$  is the number of particles in a swarm or group,  $N_g$  is the number of members or elements in a particle,  $V_{id}^k$  is the velocity of individual  $i$  at iteration  $k$ ,  $w$  is the weight parameter or swarm inertia,  $C_1$  and  $C_2$  are the acceleration constants and  $rand(\ )$  is uniform random number in the range [0 1].

The updated velocity can be used to change the position of each particle in the swarm as depicted in (12) as:

$$X_{id}^{k+1} = X_{id}^k + V_{id}^{k+1} \quad (12)$$

Suitable selection of inertia weight  $w$  provides a balance between global and local explorations, thus requiring less iteration on average to find a sufficiently optimal solution. In general, the inertia weight  $w$  is set according to the following equation:

$$w = w_{\max} - \frac{w_{\max} - w_{\min}}{iter_{\max}} \times iter \quad (13)$$

where  $iter_{\max}$  is the maximum number of iterations and  $iter$  is the current number of iterations.

The constants  $C_1$  and  $C_2$  represent the weighting of the stochastic acceleration terms that pull each particle toward the  $pbest$  and  $gbest$  positions.

#### 3.2. Concept of Time-Varying Acceleration Coefficients (TVAC)

It is observed from (11) that the search toward the optimum solution is heavily dependent on the two stochastic acceleration components (*i.e.* the cognitive component and the social component). Thus, it is very important to control these two components properly in order to get optimum solution efficiently and accurately. It has been reported [18] that in PSO, problem-based tuning of parameters is a key factor to find the optimum solution accurately and efficiently. Kennedy and Eberhart [12] re-

ported that a relatively higher value of the cognitive component, compared with the social component, results in excessive roaming of individuals through a larger search space. On the other hand, a relatively high value of the social component may lead particles to rush toward a local optimum prematurely.

In general, for any population-based optimization method like PSO, it is always desired to encourage the individuals to wander through the entire search space, during the initial part of the search, without clustering around local optima. In contrast, during the latter stages, it is desirable to enhance convergence towards the global optima so that optimum solution can be achieved efficiently. For this, the concept of parameter automation strategy called Time Varying acceleration Coefficients (TVAC) had been introduced [17]. The main purpose of this concept is to enhance the global search capability during the early part of the optimization process and to promote the particles to converge toward the global optimum at the end of the search. For this, the cognitive component should be reduced while the social component should be increased during search procedure. In TVAC, this can be achieved by changing the acceleration coefficients with time. With a large cognitive component and small social component at the beginning, the particles are encouraged to move around the search space, instead of moving towards the population best prematurely. On the other hand, during the latter stage of optimization, a small cognitive component and a large social component encourage the particles to converge towards the global optimum. The concept of time varying acceleration coefficients (TVAC) can be introduced mathematically as follows [17].

$$C_1 = (C_{1f} - C_{1i}) \frac{iter}{iter_{max}} + C_{1i} \quad (14)$$

$$C_2 = (C_{2f} - C_{2i}) \frac{iter}{iter_{max}} + C_{2i} \quad (15)$$

where  $C_{1i}, C_{1f}, C_{2i}, C_{2f}$  are constants representing initial and final values of cognitive and social acceleration factors respectively.

### 3.3. Self-Organizing Hierarchical PSO with TVAC (SOHPSO\_TVAC)

It is seen that the classical PSO is either based on a constant inertia weight factor or a linearly varying inertia weight factor. It is reported that the particles in classical PSO may converge to a local minimum prematurely due to lack of diversity in the population, particularly for complex problems [17]. In SOHPSO\_TVAC, the previous velocity term in (11) is kept at zero. It is observed that in the absence of previous velocity term the particles

rapidly rush towards a local optimum solution and then stagnate due to the lack of momentum. In fact in the absence of velocity term, the optimum solution depends highly on the initial population. To overcome this difficulty, the modulus of velocity vector of a particle is reinitialized with a random velocity (called reinitialization velocity) whenever it stagnates in the search space. Stagnation of particles highly influences the performance of PSO in searching global optimum. When a particle is stagnated, it's  $pbest$  remains unchanged over a large number of iterations. When more particles are stagnated, the  $gbest$  also remains unchanged and the PSO algorithm converges to a local minimum prematurely. The necessary momentum is imparted to the particles by reinitialization of velocity vector with a random velocity. The above method can be implemented as follows [17]:

$$V_{id}^{k+1} = \left( (C_{1f} - C_{1i}) \frac{iter}{iter_{max}} + C_{1i} \right) \times rand_1 \times (pbest_{id} - X_{id}^k) + \left( (C_{2f} - C_{2i}) \frac{iter}{iter_{max}} \right) \times rand_2 \times (gbest_d - X_{id}^k) \quad (16)$$

If  $V_{id} = 0$  and  $rand_3 < 0.5$  then

$$V_{id} = rand_4 \times V_{dmax} \quad \text{else} \quad V_{id} = -rand_5 \times V_{dmax} \quad (17)$$

Thus a series of particle swarm optimizers are generated automatically inside the main PSO according to the behavior of the particles in the search space until the convergence criteria is satisfied. The variables  $rand_1, rand_2, rand_3, rand_4$  and  $rand_5$  are numbers generated randomly between 0 and 1. A time varying reinitialization strategy is used to overcome the difficulties of selecting appropriate reinitialization velocities.

## 4. Development of the Proposed Algorithm

In this section, an algorithm based on SOHPSO\_TVAC is described to obtain quality solutions for scheduling problems of hydrothermal systems with cascaded reservoirs. For any population based evolutionary algorithm like PSO, the representation of individuals and their elements is very important. For the present problem, the position of each particle (*i.e.* each individual in the population of particles) is composed of a set of elements and for the present problem it is the discharge rate of each hydro plant and the power generated by each thermal unit. The algorithm starts with the initialization process. Let  $P^{(0)} = [X_1^{(0)}, X_2^{(0)}, \dots, X_k^{(0)}, \dots, X_{N_p}^{(0)}]$  be the initial population of  $N_p$  number of particles. For a system with  $N_h$  number of hydro units and  $N_s$  number of thermal units, position of  $k$ -th individual of a population is ini-

tialized randomly satisfying the constraints defined by (6) and (10) and can be represented by

$$X_k^{(0)} = [Q_{h1}^{(0)}, \dots, Q_{hj}^{(0)}, \dots, Q_{hN_h}^{(0)}, P_{s1}^{(0)}, \dots, P_{si}^{(0)}, \dots, P_{sN_s}^{(0)}]^T \quad (18)$$

with  $Q_{hj}^{(0)} = [Q_{hj1}^{(0)}, Q_{hj2}^{(0)}, \dots, Q_{hjt}^{(0)}, \dots, Q_{hjt}^{(0)}]^T$  and

$P_{si}^{(0)} = [P_{si1}^{(0)}, P_{si2}^{(0)}, \dots, P_{sit}^{(0)}, \dots, P_{sit}^{(0)}]^T$ . The elements  $Q_{hjt}^{(0)}$  and  $P_{sit}^{(0)}$  are the discharge rate of the  $j$ -th hydro plant and the power output of the  $i$ -th thermal unit at time  $t$ . The range of the elements  $Q_{hjt}^{(0)}$  and  $P_{sit}^{(0)}$  must satisfy the water discharge rate and the thermal generating capacity constraints as depicted in (6) and (10) respectively. Assuming the spillage in (8) to be zero for simplicity, the water discharge rate of the  $j$ -th hydro plant in the dependent interval is then calculated using (8) to meet exactly the restrictions on the initial and final reservoir storage. The dependent water discharge rate must satisfy the constraints in (10). At the same time, to meet exactly the power balance constraints, the thermal generation of the dependent thermal generating unit is calculated using (4). Thus, the initial generation is checked against all the constraints. If the constraints are satisfied then movement towards the next step is undertaken. Now, the algorithm can be described as follows:

Step 1: Initialize randomly each particle according to the limit of each unit including individual dimensions, searching points and velocities according to (18). These initial particles must be feasible candidates for solutions that satisfy the practical operating constraints.

Step 2: For each particle, calculate fitness value according to (3).

Step3: If the fitness value is better than the best fitness value in history, set current value as the *pbest*.

Step 4: Modify the member velocity of each particle according to (16) and reinitialize it according to (17).

Step 5: Choose the particle with the best fitness value of all the particles as the *gbest*.

Step 6: If the number of iterations reaches the maximum, then go to Step 7 else go to Step 4.

Step 7: The individual that generates the latest *gbest* is the solution of the problem.

### 5. Simulation Results

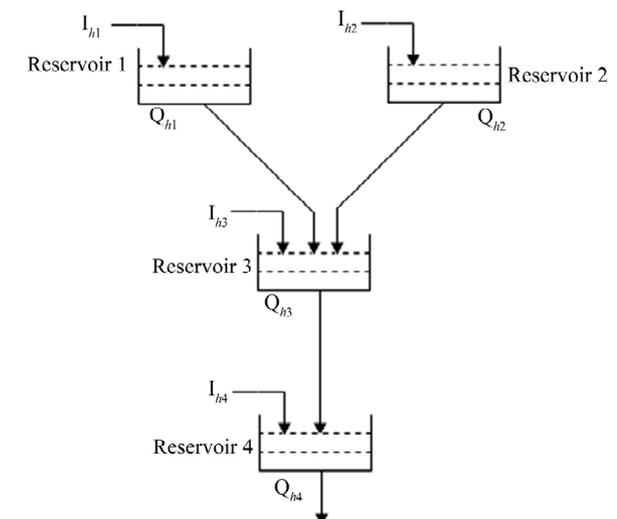
The proposed algorithm was implemented using in house Matlab code on 3.0 MHz, 2.0 GB RAM PC. It was applied on two illustrative test systems to obtain the simulation results.

#### 5.1. Test System-I

The proposed method has been initially applied to a test system consisting of four cascaded hydro units and an equivalent thermal plant.

This has been done to evaluate the performance of the proposed method in comparison to other population based methods [8,9], on the same test system. The scheduling period has been kept to 24 hours with one hour time interval. The water transport delay between connected reservoirs is also considered. For a direct comparison, effect of valve point loading is not considered in this case. The hydraulic network is shown in **Figure 1**. The load demand, hydro unit power generation coefficients, reservoir inflows, reservoir limits, the generation limits, cost coefficients of thermal unit and other data are taken from [8,9] and hence they are not shown in this paper.

The performance of PSO algorithm is quite sensitive to the various parameter settings. Tuning of parameters is essential in all PSO based methods. Based on empirical studies on a number of mathematical benchmark functions [17] it has been reported the best range of variation as 2.5 - 0.5 for  $C_1$  and 0.5 - 2.5 for  $C_2$ . The idea is to use a high initial value of the cognitive coefficient to make use of full range of the search space and to avoid premature convergence with a low social coefficient. We experimented with the same range and several combinations of the values of  $C_1$  and  $C_2$  were tested. The best results were obtained for 2.5 - 1.2 for  $C_1$  and 0.8 - 2.5 for  $C_2$  out of 50 trial runs. The optimization is done with a randomly initialized population of 30 swarms. The maximum iteration was set at 500.



Plant	1	2	3	4
$R_u$	0	0	2	1
$t_d$	2	3	4	0

$R_u$  : no of upstream plants,  $t_d$  : time delay to immediate downstream plant.

**Figure 1. Hydraulic system networks.**

**Table 1** shows the optimal water discharge obtained by the proposed method. The optimal hydrothermal generation schedule along with demand for 24 hours is shown in **Table 2**. From the **Table 2**, it is clearly seen that total demand is met by the total power (both hydro and thermal power) generated in every scheduling interval. The proposed method converges to the optimal solution in 6.65 seconds. The optimal cost is found to be \$ 922018.24. The results of the proposed method are compared with the results obtained by various versions of particle swarm optimization (PSO) techniques and genetic algorithm [8], modified hybrid differential evolution [9], improved PSO technique [19] and are shown in **Table 3**. It is clearly seen that the proposed method based SOHPSO\_TVAC yields comparable results.

## 5.2. Test System-II

To evaluate the performance of the proposed method based on SOHPSO\_TVAC, it was further applied to a test system that consists of a multi-chain cascade of four hydro units and six thermal units. The effect of valve point loading has been taken into account in this case to illustrate the robustness of the proposed algorithm. The scheduling period has been kept to 24 hours with one

hour time interval. The water transport delay between connected reservoirs is also considered. The hydro sub-system configuration, hydro unit power generation coefficients, reservoir inflows, reservoir limits and other data related to hydro sub-system are same as that of test system-I [8,9]. The load demand, generation limits and cost coefficients of six thermal units are given in **Tables 4** and **5** respectively.

In this case also, the parameters were selected separately. Out of 50 trial runs, best results were obtained for 2.5 - 1.0 for  $C_1$  and 1.08 - 2.5 for  $C_2$ . The optimization is done with a randomly initialized population of 50 swarms. The maximum iteration for this case was set at 1000. The iteration number was increased at a step of fifty and beyond 1000 no improvement in results was obtained. Hence, maximum iteration was set at this value.

Optimal hourly water discharge rate obtained by the proposed algorithm for this system is shown in **Table 6**. **Table 7** presents the optimal hydro-generation schedule including total hydropower generation while **Table 8** presents the optimal thermal-generation along with total thermal-power generation. From the **Tables 7** and **8**, it is clearly seen that total demand is exactly met by the total power (both hydro and thermal power) generated in

**Table 1. Hourly discharge ( $\times 10^4 \text{ m}^3$ ) for Test system-I using SOHPSO\_TVAC.**

Hour	$Q_{h1}$	$Q_{h2}$	$Q_{h3}$	$Q_{h4}$
1	13.8283	14.4910	24.2658	15.2399
2	12.6601	9.2900	24.9872	17.2769
3	5.9466	6.6413	22.9332	13.6824
4	13.3810	14.3373	29.3514	18.3014
5	8.2904	6.0950	28.7873	16.5410
6	11.8885	11.9559	28.6013	14.0943
7	12.6919	6.2446	15.9601	18.0801
8	9.3017	8.6030	19.3372	14.6328
9	11.3282	8.9676	14.8883	13.0162
10	10.9708	7.6717	16.7851	16.4236
11	9.4602	13.9516	15.3201	18.9266
12	11.1395	14.0177	14.0084	13.1561
13	14.5317	9.4261	14.0798	17.8119
14	7.9011	8.9554	18.0524	16.6606
15	8.3188	10.2877	24.4559	15.5128
16	9.8519	7.3214	13.7804	13.0263
17	6.6072	12.1301	22.8774	14.1860
18	11.8547	6.4092	10.0867	16.8845
19	12.9146	13.9743	13.8062	15.9441
20	12.5296	12.0523	19.1564	19.4293
21	8.9751	14.7372	17.1883	15.3933
22	8.3536	11.6243	29.4380	17.6711
23	13.9647	8.6715	15.8585	16.2179
24	9.7047	13.2850	14.4794	19.7077

**Table 2. Hydrothermal generation (MW) for Test System-I using SOHPSO\_TVAC.**

Hour	$P_{h1}$	$P_{h2}$	$P_{h3}$	$P_{h4}$	$P_{s1}$
1	97.4547	87.6571	21.8256	230.3377	932.7249
2	93.5193	64.7935	23.6970	229.6978	978.2924
3	58.4330	78.5222	57.6787	210.2464	955.1197
4	94.1196	83.3466	0.0000	198.7914	913.7424
5	92.6744	72.8456	0.0000	220.9078	903.5722
6	86.4760	74.3160	0.0000	191.6303	1057.5777
7	85.4516	41.7571	43.1969	229.6711	1249.9233
8	72.1956	55.8172	34.0058	219.9818	1617.9996
9	79.3600	76.5117	49.5002	232.1563	1802.4718
10	77.5266	48.6986	46.1335	266.1956	1681.4457
11	72.0148	74.7016	50.6196	283.0959	1749.5681
12	79.5559	70.6179	53.8710	237.5837	1868.3715
13	83.9767	50.1964	55.1515	278.0854	1762.5900
14	63.1928	46.7289	51.5653	268.6745	1769.8385
15	67.5760	52.7787	24.7423	258.2166	1726.6864
16	76.1820	36.8627	60.4846	234.3241	1662.1466
17	58.5041	58.7856	45.1654	246.2596	1721.2853
18	84.1886	27.4923	58.0117	272.4697	1697.8377
19	83.9924	57.9884	62.2518	272.4128	1763.3546
20	79.5395	46.5762	55.1625	295.3375	1803.3843
21	64.5494	47.7753	60.4145	268.9274	1798.3334
22	60.4338	35.1203	64.1562	281.8141	1678.4756
23	74.3806	41.0008	63.0802	276.4471	1395.0913
24	62.6654	35.5388	64.6617	294.0484	1133.0857

**Table 3. Comparison of cost and Computation time [8,9,19] for Test System-I using SOHPSO\_TVAC.**

Method	Cost (\$)	CPU Time (sec)
GCPSO [8]	927288.00	182.4
GWPSO [8]	930622.50	129.10
LCPSO [8]	925618.50	103.50
LWPSO [8]	925383.80	82.9
GA [8]	942600.00	-
MHDE [9]	921893.94	8
HDE [9]	922872.68	9
MDE [9]	922555.44	45
DE [9]	923574.31	50
IPSO [19]	922553.49	38.46
Proposed SOHPSO_TVAC	922018.24	6.65

**Table 4. Load demand for Test System-II.**

Hour	$P_D$ (MW)	Hour	$P_D$ (MW)	Hour	$P_D$ (MW)
1	1270	9	1640	17	1330
2	1290	10	1520	18	1540
3	1260	11	1330	19	1340
4	1190	12	1310	20	1280
5	1190	13	1430	21	1540
6	1310	14	1500	22	1120
7	1450	15	1130	23	1450
8	1800	16	1270	24	1590

**Table 5. Cost curve coefficients and operating limits of thermal generators for Test system-II.**

Unit	$P_s^{\min}$	$P_s^{\max}$	$a_s$	$b_s$	$c_s$	$e_s$	$f_s$
1	40	415	0.0050	1.89	150	300	0.035
2	35	350	0.0055	2.00	115	200	0.042
3	35	425	0.0060	3.50	40	200	0.042
4	35	410	0.0050	3.15	122	150	0.063
5	50	450	0.0050	3.05	125	150	0.063
6	75	550	0.0070	2.75	120	150	0.063

**Table 6. Hourly plant discharge ( $\times 10^4 \text{ m}^3$ ) for Test System-II for using SOHPSO\_TVAC.**

Hour	$Q_{h1}$	$Q_{h2}$	$Q_{h3}$	$Q_{h4}$
1	6.972502	11.740150	24.907634	17.407985
2	7.731941	13.124870	16.186119	14.178098
3	7.477768	9.158731	13.950951	18.784392
4	8.871571	14.010161	29.042359	15.487399
5	14.277798	10.796809	21.769518	15.139450
6	12.732249	11.668549	26.451547	17.772207
7	10.138482	9.483738	15.355559	19.677885
8	5.742317	8.004957	14.481912	20.665652
9	6.944741	9.636653	20.646292	15.486904
10	7.408659	7.848469	14.401946	20.656160
11	8.948285	14.384019	10.811545	19.714787
12	7.472149	14.970795	18.763718	14.584901
13	7.505869	13.909323	23.308321	21.484800
14	5.776865	6.598329	10.781980	22.006157
15	5.015201	13.372890	28.618021	13.571865
16	13.114330	14.072622	28.718159	19.642249
17	13.178272	13.790026	15.070220	17.845279
18	8.196122	12.311397	23.422576	23.441177
19	12.524022	8.468255	19.765975	23.113251
20	9.938308	7.244151	18.779339	17.879239
21	8.160802	14.330810	10.896191	20.778777
22	7.448584	11.340049	16.514698	22.410447
23	8.411573	8.439337	22.846284	17.289667
24	8.632123	6.812816	21.503769	18.486091

every scheduling interval. For example, total power (both hydro and thermal power) generated during hour 12 is 1310 MW while demand during the same interval is 1310 MW as seen in **Table 4**. Computation time for optimal solution for this case is found to be 76.25 seconds and optimal fuel cost is found to be \$104232.48. The same problem was also solved by the so called classical PSO described in section 3.1. For this case optimal cost was found to be \$106322.23. **Figure 2** compares the convergence characteristic of fuel cost classical PSO and pro-

posed SOHPSO\_TVAC. It is clearly seen from the convergence characteristics that classical PSO converges to sub-optimal solution prematurely. It also seen that the proposed SOHPSO\_TVAC algorithm successfully addresses the problem of premature convergence and produces better results.

**6. Conclusions**

Optimum scheduling of hydrothermal plants generation is of great importance to electric utilities. In this paper, a

**Table 7. Hydro Generation (MW) for Test System-II using SOHPSO\_TVAC.**

Hour	$P_{h1}$	$P_{h2}$	$P_{h3}$	$P_{h4}$	Total MW (Hydro)
1	68.223902	79.870276	17.415086	245.771899	411.281163
2	74.25182	81.695087	51.847937	204.964697	412.759541
3	72.869711	62.468965	52.829906	219.589260	407.757842
4	81.748249	79.889387	0.000000	176.960986	338.598622
5	99.394337	66.076948	29.128339	187.504885	382.104509
6	92.957195	67.112659	0.000000	205.072090	365.141944
7	82.085906	54.436703	53.410569	208.514325	398.447503
8	55.002407	44.041311	57.276431	225.984299	382.304448
9	64.941358	51.400626	43.193648	200.031335	359.566967
10	69.115574	40.982128	58.410617	242.928173	411.436492
11	79.568629	65.687318	57.545585	231.659008	434.460540
12	71.617824	61.408143	51.601508	194.081949	378.709424
13	72.543089	52.843556	30.587430	239.752425	395.726500
14	60.290326	18.338580	58.858229	231.123599	368.610734
15	54.554216	48.210457	0.000000	170.544137	273.308810
16	103.048297	44.824854	0.000000	214.028806	361.901957
17	102.219552	37.652148	59.021664	210.627995	409.521359
18	79.125646	26.879337	32.984490	220.090011	359.079484
19	99.045047	6.025023	51.820825	227.501503	384.392398
20	87.391146	0.000000	55.708136	215.896854	358.996136
21	76.429248	21.975673	61.093279	225.072949	384.571149
22	71.492977	9.468806	62.964007	234.036600	377.962390
23	77.764954	0.000000	42.199644	208.777956	328.742554
24	79.240273	0.000000	48.618568	217.279495	345.138336

**Table 8. Thermal Generation (MW) for Test System-II using SOHPSO\_TVAC.**

Hour	$P_{s1}$	$P_{s2}$	$P_{s3}$	$P_{s4}$	$P_{s5}$	$P_{s6}$	Total MW (Thermal)
1	209.348051	184.281276	108.431265	84.899313	147.603506	124.155426	858.718837
2	211.054495	189.794321	111.901474	137.639007	101.434318	125.416844	877.240459
3	297.724906	184.630092	110.354280	83.873035	99.625233	76.034612	852.242158
4	209.547896	182.388407	106.315822	83.703034	147.519088	121.927131	851.401378
5	208.651252	183.183484	107.001379	134.774553	99.117685	75.167138	807.895491
6	297.026672	183.828760	107.799894	133.337042	99.471626	123.394062	944.858056
7	210.589658	260.300077	114.324085	136.639403	201.259709	128.439565	1051.552497
8	299.252972	333.112614	183.337976	229.165002	198.923506	173.903482	1417.695552
9	299.503327	260.615281	111.098576	234.256055	199.78993	175.169864	1280.433033
10	299.772920	185.682623	113.141228	134.802676	199.416465	175.747596	1108.563508
11	210.167576	260.689032	111.548850	87.224691	100.723202	125.186109	895.539460
12	208.355239	259.064644	105.717785	85.035917	149.160068	123.956923	931.290576
13	208.945245	258.821928	109.157954	134.641022	198.380268	124.327083	1034.273500
14	299.398231	260.628553	109.993129	135.516992	150.867596	174.984765	1131.389266
15	298.049942	109.739612	109.174060	84.678568	129.990498	125.058510	856.691190
16	299.165804	185.252126	110.279713	134.897376	101.171056	77.331968	908.098043
17	210.044864	188.282655	109.392485	87.773298	150.453875	174.531464	920.478641
18	299.586350	260.219714	110.827288	184.992184	150.009217	175.285763	1180.920516
19	299.559604	184.661993	110.535431	134.980262	100.257089	125.613223	955.607602
20	210.025480	187.944929	111.772368	134.894143	151.137992	125.228952	921.003864
21	297.326699	259.307088	109.783293	185.226451	198.381285	105.404035	1155.428851
22	206.793580	181.526398	97.609726	83.142509	97.924818	75.040579	742.037610
23	298.701053	257.472455	107.142068	184.288955	149.221054	124.431861	1121.257446
24	298.757300	258.115619	184.684892	184.158452	148.407766	170.737635	1244.861664

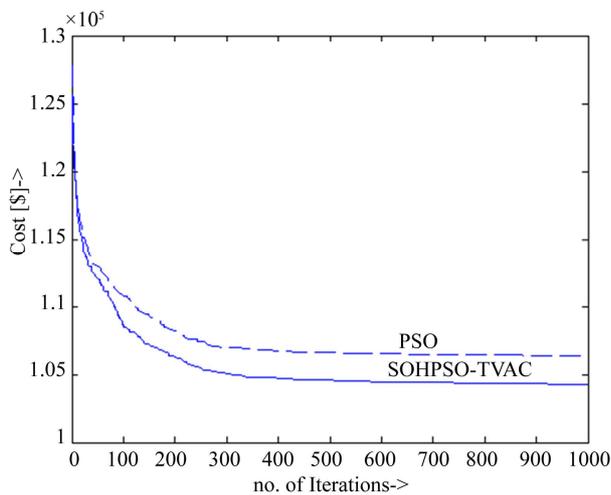


Figure 2. Convergence Characteristics for fuel cost.

novel algorithm called self-organizing hierarchical particle swarm optimization technique with time-varying acceleration coefficients (SOHPSO\_TVAC) for solving short-term economic generation scheduling of hydrothermal systems to avoid premature convergence has been proposed and successfully applied to solve daily hydrothermal scheduling problem. To evaluate the performance of the proposed algorithm, it has been applied on two test systems comprising of a multi-chain cascade of hydro units and several thermal units and results are presented. The effect of valve point loading is also considered. The results obtained by the proposed method have been compared with other evolutionary algorithms like improved PSO, GA and modified hybrid differential evolution (MHDE). It is found that proposed method SOHPSO\_TVAC can address the problem of premature convergence and produce better results.

## 7. Acknowledgments

We would like to acknowledge and thank Jadavpur University, Kolkata, India for providing all the necessary help to carry out this work.

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### Appendix A: List of Symbols

$P_{sit}$  : output power of  $i$  th thermal unit at time  $t$   
 $P_{si}^{\min}$ ,  $P_{si}^{\max}$  : lower and upper generation limits for  $i$  th thermal unit  
 $a_{si}$ ,  $b_{si}$ ,  $c_{si}$ ,  $e_{si}$ ,  $f_{si}$  : cost curve coefficients of  $i$  th thermal unit  
 $P_{Dt}$  : load demand at time  $t$   
 $P_{hjt}$  : output power of  $j$ -th hydro unit at time  $t$   
 $P_{hj}^{\min}$ ,  $P_{hj}^{\max}$  : lower and upper generation limits for  $j$  th hydro unit  
 $Q_{hjt}$  : water discharge rate of  $j$ -th reservoir at time  $t$   
 $V_{hjt}$  : storage volume of  $j$ -th reservoir at time  $t$   
 $Q_{hj}^{\min}$ ,  $Q_{hj}^{\max}$  : minimum and maximum water discharge

rate of  $j$ -th reservoir  
 $V_{hj}^{\min}$ ,  $V_{hj}^{\max}$  : minimum and maximum storage volume of  $j$  th reservoir  
 $C_{1j}$ ,  $C_{2j}$ ,  $C_{3j}$ ,  $C_{4j}$ ,  $C_{5j}$ ,  $C_{6j}$  : power generation coefficients of  $j$ -th hydro unit  
 $I_{hjt}$  : inflow rate of  $j$  th reservoir at time  $t$   
 $R_{uj}$  : number of upstream units directly above  $j$  th hydro plant  
 $S_{hjt}$  : spillage of  $j$  th reservoir at time  $t$   
 $\tau_{mj}$  : water transport delay from reservoir  $m$  to  $j$   
 $N_s$  : number of thermal generating units  
 $N_h$  : number of hydro generating units