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Research of Impact of Geografical Latitute and Residual Ionospheric Noises on Informativeness of Measuring of Zenith Wet Delay of GPS Signals

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Abstract

It is noted that necessity of further increase of accuracy of GPS positioning systems requires development of more perfect methods to compensate information losses occurred due to residual ionospheric delay by using optimization procedures. According to the conditions of formulated optimization task, the signal/noise ratio in measurements of zenith wet delay depends on the second order ionospheric errors, geographic latitude and day of year. At the same time if we assume that the number of measurements at the fixed geographic site is proportional to geographic latitude and if we accept existence of only two antiphase scenarios for variation of residual ionospheric delay on latitude normed by their specific constant, there should be optimum functional dependence of precipitated water on latitude upon which the quantity of measuring information reaches the maximum. The mathematical grounding of solution of formulated optimization task is given.

Keywords

Zenith Wet Delay, Information, Optimization, GPS Measurements, Ionosphere, Geographic Latitude

1. Introduction

As it is noted in the work [1], the microwave signals of satellite navigation systems are subjected to delay upon propagation in atmosphere. The major components of such delay are ionospheric and tropospheric delays. The

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ionospheric delay depends on frequency and almost can be removed.

The tropospheric delay reaches 2.5 m in zenith direction, or 25 m upon 5˚ elevation angle and hardly can be compensated. In its turn, the tropospheric delay contains hydrostatic component (by percentage reaching 90%) and wet delay. The hydrostatic delay can be determined by measuring the atmospheric pressure at the antenn location zone. The wet delay cannot be determined by only using the ground measurements. As it is noted in the work [1], the zenith delay can be recalculated for direction of observation of satellite. Such a recalculation is carried out by using the mapping functions. If the elevation angle decreases, the delay of satellite signals will increase.

But in order to carry out the true analyses of error generated by such delays, one should know the amount of the wet delay in zenith direction. According to the work [1], mostly used models of the wet delay are following:

1. Model of MOPS;
2. Hopfield's model;
3. Mendes's model.

In all abovementioned models the root mean square error decreases by the increase of the geographic latittude, because the tropics are featured by the higher level humidity, therefore, by big amount of wet delay.

As it was noted above, at present time the ionospheric delay of GPS signals can be removed on the whole. According to the work [2], the delay of the signal upon propagation in ionosphere depends on the total amount of electrons along the path of the signal. That delay depends on frequency of signal, geographic location and current time. For the single frequency GPS receivers the model of Klobuchar has been developed. According to this mode, the zenith ionospheric delay at the moment t can be determined as

$$T^{iom} = A_1 + A_2 \cos\frac{2\pi\left(t - A_3\right)}{A_4}, \tag{1}$$

where A_1: night-time value of zenith ionospheric; $A_1, A_2, A_3 = \text{const}$; A_2 and A_4: parameters transferred by each GPS satellite in navigational massage depending on position of user, azimuth of the satellite, satellite's height and local time. According to [3], in single frequency GPS systems the ionospheric delay can cause the positioning error in amount of 5 - 15 m, but during the period of solar activity this error can reach 150 m. But as it is shown in [4], upon use of double frequency GPS systems, using the frequencies L_1 and L_2, utilization of linear combination of these signals makes it possible to remove the ionospheric delay by 99.9%.

At the same time, the necessity of further increase of accuracy of GPS positioning systems requires more compensation of effect of ionospheric delay. The residual ionospheric delay, named also as an ionospheric delay of the second order, is generated as a result of interaction of ionosphere and the Earth's magnetic field, and depends on the total amount of electrones in declined direction, parameters of the magnetic field, the angle between the magnetic field and direction of signal's propagation.

As it is noted in the work [5], at the geographical middle-latitude zones the most ionospheric effects increases in direction of the north to the south.

As it was noted above, the main non-removable delay GPS signal is the wet delay.

According to the work [6], the zenith wet delay can be calculated by using the formula

$$ZWD = 10^{-6}\int_{r_s}^{r_a}\left[k_2'\left(\frac{e}{T}\right) + k_3\left(\frac{e}{T}\right)\right]Z_w^{-1}\mathrm{d}z\,, \tag{2}$$

where k_2' and k_3: the emipirical coefficients; e: pressure of water; Z_w: compressibility factor of water vapors; r_s: geocentrical radius of the site of installation of receiver's antenn; r_a: geocentrical radius of the upper part of neutral atmosphere ; $\mathrm{d}z$: differential of z measured length.

In model researches, the dimensionless coefficient Q is frequently used, determined as [6]

$$Q = \frac{ZWD}{PW}\,, \tag{3}$$

where PW: the amount of precipitated water, determined as

$$PW = \frac{1}{\rho_{\mathrm{H_2O}}}\int_{r_s}^{r_a}\rho_w \mathrm{d}z\,. \tag{4}$$

According to the work [6], there is the model of Q, where such parameters as the geographic latitude and day of year are taken into account

$$Q = a_0 + a_1\varphi + a_2\sin\left(\frac{2\pi T_D}{365}\right) + a_3\cos\left(\frac{2\pi T_D}{365}\right), \tag{5}$$

where T_D: the serial number of day of year;

$$a_0 = 5.882;\ a_1 = 0.01113;\ a_2 = 0.064;\ a_3 = 0{,}127 .$$

Obviously, concerning the chosen day T_D, we have

$$Q = a_0 + a_1\varphi + C(t_D) .$$

Taking into account the Formulas (3), (5), the ratio of signal/noise ψ_1 for determination of zenith wet delay may be calculated as

$$\psi_1 = \frac{ZWD}{\sigma(\varphi)_{ion}} , \tag{6}$$

where $\sigma(\varphi)_{ion}$: noises upon measurements, occurred due to ionospheric errors of the second order

$$\sigma(\varphi)_{ion} = \sigma_{0.ion} - q\cdot\varphi , \tag{7}$$

where $\sigma(\varphi)_{0.ion} = \sigma(\varphi)_{ion}$, upon $\varphi = 0$.

As it can be seen from Formula (7), $\sigma(\varphi)_{ion}$ increases by decrease of φ. Taking into account Formulas (3), (6) and (7) we get

$$\psi_1 = \frac{Q\cdot PW}{\sigma(\varphi)_{ion}} = \frac{\left[a_0 + a_1\varphi + C(t_D)\right]}{\sigma_{0.ion} - q\cdot\varphi} . \tag{8}$$

Now we consider the following optimization task. Assume that the measurements of ZWD are carried out and the authenticity of results of measurements is determined by Formula (8). The series of measurements are carried out at the different geographical latitudes φ. The number of measurements carried out of the latitude φ is determined as $N_\varphi = k\cdot\varphi$. The total amount of information received at the latitude φ can be determined as

$$M_\varphi = k\varphi\log_2\frac{\left[a_0 + a_1\varphi + C(t_D)\right]\cdot PW}{\sigma_{0.ion} - q\cdot\varphi} . \tag{9}$$

Integrating the Formula (9) along all the values of φ we get

$$M_{u_1} = \int_0^{\varphi_m} k\varphi\log_2\frac{\left[a_0 + a_1\varphi + C(t_D)\right]PW}{\sigma_{0.ion} - q\varphi}\mathrm{d}\varphi . \tag{10}$$

Let us introduce the searched function

$$PW = PW(\varphi) . \tag{11}$$

which can be determined alternatively as

$$PW_1(\varphi) = PW_{01} + d_1\varphi , \tag{12}$$

or

$$PW_2(\varphi) = PW_{02} - d_2\varphi , \tag{13}$$

where $PW_{01}, PW_{02} = \text{const}; d_1, d_2 = \text{const}$.

We assume that functions $PW_1(\varphi)$ and $PW_2(\varphi)$ meet following integral limitation condition

$$M_{u2} = \int_0^{\varphi_m} PW_i(\varphi)\mathrm{d}\varphi = C , \tag{14}$$

where $i = \overline{1,2}; C = \text{const}$.

Taking into account the Formulas (10), (11) and (14) we can compose the following functional of unconditional variation optimization

$$M_0 = \int_0^{\varphi_m} k\varphi \log_2 \frac{\left[a_0 + a_1\varphi + C\left(t_D\right)\right]PW\left(\varphi\right)}{\sigma_{0.ion} - q\varphi}\mathrm{d}\varphi + \lambda\int_0^{\varphi_m} PW\left(\varphi\right)\mathrm{d}\varphi\,, \tag{15}$$

where λ: Lagrange multiplier.

In order to determine the optimum function $PW\left(\varphi\right)$ we use the Euler's method, according to which following condition should be met

$$\frac{\mathrm{d}\left\{k\varphi\log_2\dfrac{\left[a_0 + a_1\varphi + C\left(t_D\right)\right]PW\left(\varphi\right)}{\sigma_{0.ion} - q\varphi} + \lambda\cdot PW\left(\varphi\right)\right\}}{\mathrm{d}PW\left(\varphi\right)}\,. \tag{16}$$

Taking into the Formula (16) we get

$$\left(N - k\varphi\right)\cdot\frac{1}{PW\left(\varphi\right)} + \lambda = 0\,. \tag{17}$$

From the Formula (17) we can find

$$PW\left(\varphi\right) = -\frac{\left(N_0 - k\varphi\right)}{\lambda}\,. \tag{18}$$

Taking into consideration the Formulas (14) and (18) we find

$$M_{u_2} = -\int_0^{\varphi_m}\frac{\left(N_0 - k\varphi\right)}{\lambda}\mathrm{d}\varphi = C \tag{19}$$

From the Formula (19) we get

$$-\frac{N_0\cdot\varphi_m}{\lambda} + \frac{k\varphi_m^2}{2\lambda} = C\,. \tag{20}$$

Using the Formula (20) we can get the value of the Lagrange multiplier

$$\lambda = \frac{k\varphi_m^2}{2C} - \frac{N_0\varphi_m}{C}\,. \tag{21}$$

Taking into consideration the Formulas (17) and (21) we get

$$\frac{\varphi_m^2}{2C} = \frac{N_0\varphi_m}{C} - \frac{k\varphi_m^2}{2C}\,. \tag{22}$$

From the Formula (22) we find

$$PW\left(\varphi\right) = \frac{2k\varphi C}{2N_0\varphi_m - k\varphi_m^2}\,. \tag{23}$$

Therefore, upon function (23) the functional (15) reaches its extremum value.

In order to determine the type of extremum, we should calculate the following second derivative

$$\frac{\mathrm{d}^2\left\{k\varphi\log_2\dfrac{\left[a_0 + a_1\varphi + C\left(t_D\right)\right]PW\left(\varphi\right)}{\sigma_{0.ion} - q\varphi} + \lambda\cdot PW\left(\varphi\right)\right\}}{\mathrm{d}PW\left(\varphi\right)^2}\,. \tag{24}$$

It is not difficult to check out that Formula (24) gains the negative value, *i.e.*, upon condition (23) the target functional (15) reaches its maximum. Hence, upon the functional dependence (12) the informativeness of held measurements can reach its maximum. But its well-known that the increase of φ cause the decrease of PW. Accordingly the type of the target functional (15) should be changed. Further, we assume, that the number of measurements in series is determined as

$$N_\varphi = N_0 - k\varphi\,, \tag{25}$$

where $N_0 = \text{const}$.

Taking into account (17) and (15) we get

$$M_0 = \int_0^{\varphi_m} (N_0 - k\varphi)\cdot \log \frac{\left[a_0 + a_1\varphi + C(t_D)\right] PW(\varphi)}{\sigma_{0.ion} - q\varphi} \mathrm{d}\varphi + \lambda \int_0^{\varphi_m} PW(\varphi)\mathrm{d}\varphi . \tag{26}$$

Using the above described method we can determine that the functional (26) will reach its maximum upon condition

$$PW(\varphi) = \frac{2C(N_0 - k\varphi)}{\left(2CN_0\varphi_m - k\varphi_m^2\right)\cdot k\varphi} . \tag{27}$$

In this case in order to determine the type of extremum we should compute the following second derivative

$$\frac{\mathrm{d}^2 \left\{ (N_0 - k\varphi)\cdot \log \dfrac{\left[a_0 + a_1\varphi + C(t_D)\right] PW(\varphi)}{\sigma_{0.ion} - q\varphi} + \lambda \cdot PW(\varphi) \right\}}{\mathrm{d}PW(\varphi)^2} \tag{28}$$

From Formula (28) we get

$$\frac{\mathrm{d}^2}{\mathrm{d}PW(\varphi)} = -\frac{(N_0 - k\varphi)}{PW^2(\varphi)\cdot \ln 2} .$$

Because Formula (28) reaches the negative value, the maximum informativeness could be reached upon condition (12), but the number of measurements in series can be determined in line with Formula (25).

Hence, informativeness of measurements carried out on geographical latitudes to determine the zenith wet delay can reach its maximum upon meeting of two conditions:

1. The total amount of precipitated water PW should decrease by the increase of geographical latitude;
2. The number of measurements in series should decrease by the increase of latitude.

References

[1] Schuler, T., Hein, G.W. and Eisfeller, B. (2000) On the Use of Numerical Weather Fields for Troposphere Delay Estimation in Wide Area Augmentation Systems. *Proceedings of GNSS* 2000, Royal Institute of Navigation, Edinburgh, 1-4 May 2000, 1077-1093.

[2] Stepniak, K., Wielgosz, P. and Paziewski, J. (2014) Accuracy Analysis of the Klobucher Ionosphere Model Transmitted by the GPS System. *The* 9*th International Conference "Environmental Engineering*", Vilnius, 22-23 May 2014, 1-6. http://leidykla.vgtu.lt/conferences/ENVIRO_2014/Articles/5/246_Stepniak.pdf

[3] Elsobeiey, M. and El-Rabbany, A. (2010) Ricorous Modeling of GPS Residual Errors for Precise Point Positioning. http://www.isprs.org/proceedings/XXXVIII/part1/09/09_04_Paper_136.pdf

[4] Bassiri, S. and Hajj, G. (1993) High-Order Ionospheric Effects of the Global Positioning System Observables and Means of Modeling Them. *Manuscripta Geodetica*, **18**, 280-289.

[5] Musman, S. (2015) Ionospheric Effects on GPS Surveing. http://www.ngs.noaa.gov/PUBS_LIB/IonosphericEffectsOnGPSSurveying.pdf

[6] Mendes, V.B., Prates, G., Santos, L. and Langley, R.B. (2000) An Evaluation of the Accuracy of Models for the Determination of the Weighted Mean Temperature of the Atmosphere. *Proceedings of the* 2000 *National Technical Meeting of the Institute of Navigation*, Anaheim, 26-28 January 2000, 433-438.