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Dynamic Based SMC of Nonholonomic Mobile Robots

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ABSTRACT

The aim of the paper is trajectory tracking control of a non-holonomic mobile robot whose centroid doesn't coincide to its rotation center in the middle of connecting axle of driving wheels. The nonholonomic dynamic model of the Wheeled Mobile Robot (WMR) is developed in global Cartesian coordinates where the WMR's forward and angular velocities are used as internal state variables. In order to include the effects of parameter uncertainties, measurement noises and other anomalies in the WMR system, a bounded perturbation vector is embedded to the developed dynamical model. Through defining the control inputs by computed torque method, a Dynamic Sliding Mode Controller (DSMC) is proposed to stabilize the sliding surfaces. Based on the proposed robust control system, the effect of uncertainties and noises in the robot performance is attenuated. By use of the WMR forward and angular velocities as internal state variables in the dynamic modeling, the developed model is relatively simple and mainly independent of the robot states. This makes the dynamical model more robust against measurement errors. Design of the DSMC based on such a model leads to perfect trajectory tracking and compensation for initial off-tracks even in the presence of disturbances and modeling uncertainties.

Keywords: Sliding Mode Control; Mobile Robot Dynamics; Robustness; Feedback Linearization

1. Introduction

Nowadays, Wheeled Mobile Robots (WMRs) have found many applications in industry, transportation, and inspection fields. Therefore, trajectory tracking control of nonholonomic WMRs has been an important problem in state of the art research works of recent literatures.

The assumption of pure rolling and not slipping motion leads to a non-integrable constraint in the kinematics of nonholonomic mobile robots. Since a non-holonomic system cannot be stabilized via smooth state feedback methods, the conventional linear control theories may not be applied to this class of systems [1]. Another important issue in real WMRs is arisen from parameter uncertainties, measurement noises and any other probable anomalies. In order to come over these problems and make the WMR converge to its reference trajectory, remarkable attempts have been done by researchers. For example, different controllers of time-invariant, time-varying and hybrid types based on Lyapunov control theories have been proposed by Kolmanovsky and McClamroch [2]. The global trajectory tracking problem has been discussed based on backstepping techniques [3]. In a research work done by Sun, the kinematic model of a mobile robot has been transformed into a linear time invariant system by state and input transformation, then, the pole-assignment method has been applied to design the controller [4]. Keighobadi, Menhaj and Kabganian have designed a mixed feedback-linearization and fuzzy controller for perfect trajectory tracking control of the WMR [5], see also [6-8] for more robust and intelligent applications.

As a robust control approach, sliding mode controller (SMC) is recently receiving increasing attentions. The advantages of using SMCs are fast response, good transient performance and significant robustness against perturbations and noises. This method is also respectively simple and doesn't have complexities. The trajectory tracking of a nonholonomic WMR based on an improved sliding mode control method has been done in which the switch function of the variable structure control is designed based on the back stepping technique [9]. An artificial neural network based sliding mode control for nonholonomic WMRs has been applied to reduce the effects of model uncertainty [10]. Shin, Kim and Koh have used a variable structure control law that makes the WMR converge to reference trajectories with bounded error of position and velocity components [11]. In trajectory tracking field, the controllers are designed com-

monly based on the kinematic model of the WMRs. Therefore, the control inputs of mobile robot are often considered its linear and angular velocities. In the proposed DSMC of this paper, the dynamical model of the WMR includes the total mass and inertia moment of WMR. Thus, the designed controller deals with the dynamical effects which make it more applicable in real world. In the dynamical control of a WMR, the imposed torques upon the driving wheels are considered the control inputs. Using the dynamic model of the WMR developed in polar coordinate, Yang and Kim have proposed a sliding mode control law for solving the trajectory tracking problem of the WMR in the presence of disturbances [12]. Hu and Huo have designed a robust controller based on a dynamic model in which the actuator dynamics is included [13].

In this paper, we propose a new control law that stabilizes the WMR around the given reference trajectory. The developed dynamic model includes inertia moment of driving wheels, the total mass and inertia moment of WMR which has a centroid offset from the connection center of its driving wheels. Following linearizing the error dynamics of WMR through the computed torque method, a sliding mode control law is applied for stabilizing the robot around its reference trajectory even in the presence of exogenous disturbances/noises and modeling uncertainties.

The recent controllers in the literatures have been limited for implementations in special kind WMRs; for instance, the heading direction of the WMR should be tangent to the circular trajectory around the origin of the global coordinate system. Furthermore, both the reference and the real trajectory should not cross through the origin after starting over the initial position; and the initial position of reference trajectory is always set to be the origin of coordinates; or the angular velocity of WMR is assumed to be nonzero during trajectory tracking. However, in this paper, due to design of the WMR control system in a global Cartesian coordinate frame, the proposed DSMC doesn't rely on the aforementioned limiting assumptions. Therefore, as it will be shown in the simulation results, the proposed control law results in smooth tracking and considerable robustness against perturbations and measurement noises. The rest of the paper is organized as follows:

In Section 2 using dynamics of nonholonomic mechanical systems and the kinematic model of the WMR, a new dynamic model of WMR is derived. Section 3 is devoted to present the design process of the proposed DSMC which stabilizes the WMR around its reference trajectory in the presence of perturbations and measurement noises. In Section 4, to show the performance of the proposed controller, tracking control of WMR on a circular path is simulated.

2. Dynamic Modeling of Nonholonomic WMR

2.1. Dynamics of Nonholonomic Mechanical Systems

As represented by Hu and Huo, many nonholonomic mechanical systems can be described by the following dynamic equations [13]:

$$M(q)\ddot{q} + V(q,\dot{q}) = B(q)\tau - A^{\mathrm{T}}(q)\lambda \qquad (1)$$

And the nonholonomic kinematic constraints are as:

$$A(q)\dot{q} = 0 \qquad (2)$$

where, $q \in R^n$ and $\tau \in R^r$ are respectively the generalized configuration and the control input vectors, respectively; $\lambda \in R^m$ is the constraint force vector; $M(q) \in R^{n \times n}$ is a positive definite matrix; $V(q,\dot{q}) \in R^n$ is the term which may include centripetal and Coriolis forces; $B(q)$ is a $n \times r$ full rank transformation input matrix; $A(q)$ is a $m \times n$ full rank matrix associated with the constraints. Let $N_i = N_i(q)$, $i = 1, \cdots, n-m$ is a set of smooth and linearly independent functions such that:

$$A(q)N_i = 0, i = 1, \cdots, n-m \qquad (3)$$

Considering Δ as the distribution spanned by the vectors $N_i, i = 1, \cdots, n-m$, then from (2) it follows that $\dot{q} \in \Delta$, that is, there exists an $(n-m)$ dimensional pseudo-velocity vector, $z = [z_1, \cdots, z_{n-m}]^{\mathrm{T}}$ such that:

$$\dot{q} = Nz \qquad (4)$$

where, $N = N(q) = [N_1(q), \cdots, N_{n-m}(q)]$.

Differentiating (4) results in:

$$\ddot{q} = N\dot{z} + \dot{N}z \qquad (5)$$

Substituting (5) into (1) and then pre-multiplying by N^{T} gives:

$$N^{\mathrm{T}}MN\dot{z} + N^{\mathrm{T}}M\dot{N}z + N^{\mathrm{T}}V = N^{\mathrm{T}}B\tau \qquad (6)$$

Through pre-multiplying (6) by $(N^{\mathrm{T}}B)^{-1}$, the nonholonomic mechanical system (1) and (2) reduces to:

$$\begin{cases} \dot{q} = Nz \\ \tilde{M}\dot{z} + \tilde{V}(q,z) = \tau \end{cases} \qquad (7)$$

where, $\tilde{M} = (N^{\mathrm{T}}B)^{-1} N^{\mathrm{T}}MN$ is a $(n-m) \times (n-m)$ matrix; $\tilde{V}(q,z) = (N^{\mathrm{T}}B)^{-1} N^{\mathrm{T}}(M\dot{N}z + V)$ is a $(n-m) \times m$ matrix and $\tau \in R^{(n-m) \times m}$. Now, the proposed system (7) is used to dynamic modeling of the WMR.

2.2. Dynamical Model

For the purpose of dynamical modeling, $\tilde{M}$ and $\tilde{V}$ should be determined. The driving wheels of the considered WMR rotate by independent actuator motors. Ac-

cording to the schematic model in **Figure 1**, indeed the robot is a plate body which is carried by two driving wheels and the other two caster wheels that prevents the robot from tipping over as it moves on a plane. In this paper, due to very little inertial parameters, the motion of caster wheels is not included in the dynamic modeling of the WMR. In **Figure 1**, a is the distance between the centroid, C of the robot and the connection center of the driving wheels. The WMR's centroid in the global coordinate system with axes X-Y is indicated as (X_c, Y_c), and x-y stand for a local coordinate system which its origin coincides to C and the axes are fixed to the robot body as **Figure 1** shows.

The symbol, θ_c is the angle between x-axis and X-axis representing the heading angle. v_c denotes the velocity of the robot along x-axis and ω_c denotes the angular velocity. $\dot{\varphi}_2$ and $\dot{\varphi}_3$ are the angular velocities of the right and left driving wheels, respectively. $2l$ and R denote the length of driving axel and the radius of every driving wheel, respectively.

The position vector of the WMR, q is defined as:

$$q = [X_c, Y_c, \theta_c]$$ (8)

According to the recent research works [5] and [8], considering the WMR kinematics, the holonomic and nonholonomic constraints are given as:

$$\dot{x}_c = \frac{R}{2}(\dot{\varphi}_2 + \dot{\varphi}_3)$$ (9)

$$\dot{y}_c = \frac{aR}{2l}(\dot{\varphi}_2 - \dot{\varphi}_3)$$ (10)

$$\dot{\theta}_c = \frac{R}{2l}(\dot{\varphi}_2 - \dot{\varphi}_3)$$ (11)

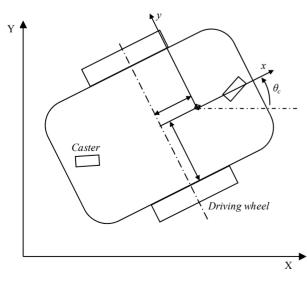The following rotation matrix transforming the veloc-

Figure 1. Schematics of WMR.

ity components between the global and local coordinate systems, play an important role in developing the kinematics of WMR:

$$\begin{bmatrix} \dot{X}_c \\ \dot{Y}_c \end{bmatrix} = \begin{bmatrix} \cos\theta_c & -\sin\theta_c \\ \sin\theta_c & \cos\theta_c \end{bmatrix} \begin{bmatrix} \dot{x}_c \\ \dot{y}_c \end{bmatrix}$$ (12)

Obtaining, $\dot{X}_c, \dot{Y}_c$ in terms of $\dot{x}_c, \dot{y}_c$ from (12) and using (9) and (10), the nonholonomic constraint of the WMR is represented as:

$$\dot{X}_c \sin\theta_c - \dot{Y}_c \cos\theta_c + a\dot{\theta}_c = 0$$ (13)

According to (8) and (13), $n = 3$ and $m = 1$; and therefore, z is a two dimensional vector. Considering v_c and ω_c as internal state variables gives:

$$z = [v_c, \omega_c]^T$$ (14)

Therefore, (4) can be constructed in the following form which is known the kinematic model of the WMR.

$$\dot{q} = \begin{bmatrix} \dot{X}_c \\ \dot{Y}_c \\ \dot{\theta}_c \end{bmatrix} = \begin{bmatrix} \cos\theta_c & -a\sin\theta_c \\ \sin\theta_c & a\cos\theta_c \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_c \\ \omega_c \end{bmatrix} = N(q)z$$ (15)

Now the following Lagrangian is considered for the WMR dynamic modeling [5,8].

$$L = \frac{1}{2}m(\dot{X}_c^2 + \dot{Y}_c^2) + \frac{1}{2}I_z\dot{\theta}_c^2 + \frac{1}{2}I(\dot{\varphi}_2^2 + \dot{\varphi}_3^2)$$ (16)

where, m is the total mass of the robot. I_z is the moment of inertia around the axis crossing C and perpendicular to X-Y plane. The moment of inertia of driving wheels is shown by I. By applying the Lagrangian approach, an equation like (1) is achieved, therefore, by comparing it with (1), M, V and B can be determined. Then, using (6) and (7), $\tilde{M}$ and $\tilde{V}$ can be easily computed. Finally, the dynamical model of the WMR is obtained as:

$$\tau = \begin{bmatrix} \dfrac{mR}{2} + \dfrac{I}{R} & \dfrac{md^2R}{l} + \dfrac{I_zR}{2l} + \dfrac{lI}{R} \\ \dfrac{mR}{2} + \dfrac{I}{R} & -\left(\dfrac{md^2R}{l} + \dfrac{I_zR}{2l} + \dfrac{lI}{R}\right) \end{bmatrix} \begin{bmatrix} \dot{v}_c \\ \dot{\omega}_c \end{bmatrix}$$

$$+ \begin{bmatrix} \dfrac{Rmd}{2}\left(\dfrac{v\dot{\theta}}{l} - \dot{\theta}^2\right) \\ -\dfrac{Rmd}{2}\left(\dfrac{v\dot{\theta}}{l} + \dot{\theta}^2\right) \end{bmatrix}$$ (17)

$\tau = [T_2, T_3]^T$ is the input torque vector in which T_2 and T_3 are the produced torques by the right and left driving wheels, respectively.

3.3. DSMC

3.1. Control System

The purpose of the DSMC is computation of control inputs $\tau = \left[T_2, T_3\right]^{\mathrm{T}}$ which make the WMR to track a feasible trajectory with bounded errors. The three dimensional posture variables of the reference trajectory are considered as, $q_r = \left[X_r, Y_r, \theta_r\right]$; the reference velocity and the acceleration vectors are derived by q_r as, $z_r = \left[v_r, \omega_r\right]^{\mathrm{T}}$ and, $\dot{z}_r = \left[\dot{v}_r, \ddot{\theta}_r\right]^{\mathrm{T}}$ respectively. The real world robotic systems have inherent system perturbations such as parameter uncertainties and external disturbances. Therefore, dynamical equations of the WMR are represented as:

$$\tilde{M}\dot{z} + \tilde{V}(q, z) + \tau_p = \tau \tag{18}$$

where, the perturbation vector, $\tau_p = \left[\tau_{p1}, \tau_{p2}\right]^{\mathrm{T}}$ considers the uncertainty and disturbance effects in the dynamical model. It is assumed that τ_p is energy bounded and satisfies the uncertainty matching condition as:

$$\tau_p = \tilde{M} \times p,$$
$$p = \left[p_1, p_2\right]^{\mathrm{T}}, |p_1| \le p_{1m}, |p_2| \le p_{2m}$$

p_{1m} and p_{2m} are the upper bounds of the perturbations.

3.2. Controller Design

First, the position and the orientation errors are defined as:

$$X_e = X_c - X_r$$
$$Y_e = Y_c - Y_r$$
$$\theta_e = \theta_c - \theta_r$$

To stabilize the tracking errors, the sliding surfaces are defined as:

$$s_1 = \dot{X}_e + K_1 X_e \tag{19}$$

$$s_2 = \dot{Y}_e + K_2 Y_e \tag{20}$$

where K_1 and K_2 are positive fixed parameters. If s_1 is asymptotically stable, then X_e and $\dot{X}_e$ will converge to zero asymptotically. Because if $s_1 = 0$ then $\dot{X}_e + K_1 X_e = 0$. Therefore, if $\dot{X}_e \le 0$ then $X_e \ge 0$ and if $\dot{X}_e \ge 0$ then $X_e \le 0$. Therefore, the equilibrium state of $X_e, \dot{X}_e$ is asymptotically stable. Similarly, if s_2 is asymptotically stable, Y_e and $\dot{Y}_e$ asymptotically converge to zero. Thus, if s_1 and s_2 become stabilized, the convergence of the WMR to the predetermined reference trajectory is guaranteed.

The control input vector is obtained through the computed-torque method as a feedback-linearization method [14]:

$$\tau = \tilde{M}\dot{z}_r + \tilde{V}(q, z) + \tilde{M}u \tag{21}$$

where $u = \left[u_1, u_2\right]^{\mathrm{T}}$ is the control law. Applying the control input (21) into the dynamic equation of WMR (18), the feedback-linearized dynamic equation is given as:

$$\dot{z} + p = \dot{z}_r + u \tag{22}$$

Hence, from (22):

$$\dot{v}_c + p_{1m} = \dot{v}_r + u_1 \tag{23}$$

$$\ddot{\theta}_c + p_{2m} = \ddot{\theta}_r + u_2 \tag{24}$$

The control actions u_1 and u_2 which stabilize the sliding surfaces s_1 and s_2 are proposed as:

$$u_1 = -D_1 \operatorname{sign}(s_1) - K_1 \dot{X}_e - \ddot{X}_e + \dot{v}_c - \dot{v}_r - E_1 X_e \operatorname{sign}(s_1 X_e)$$

$$u_2 = -D_2 \operatorname{sign}(s_2) - K_2 \dot{Y}_e - \ddot{Y}_e + \ddot{\theta}_c - \ddot{\theta}_r - E_2 Y_e \operatorname{sign}(s_2 Y_e)$$

where D_1, D_2 are greater than p_{1m}, p_{2m}, respectively; and K_1, K_2, E_1, E_2 are real positive constant values. Lyapunov's direct method is used to prove the stability of s_1 and s_2 by imposing u_1 and u_2. Substituting u_1 and u_2 in (22) and (23), respectively, yields $\ddot{X}_e$ and $\ddot{Y}_e$ as:

$$\ddot{X}_e = -D_1 \operatorname{sign}(s_1) - p_{1m} - K_1 \dot{X}_e - E_1 X_e \operatorname{sign}(s_1 X_e) \tag{25}$$

$$\ddot{Y}_e = -D_2 \operatorname{sign}(s_2) - p_{2m} - K_1 \dot{Y}_e - E_2 Y_e \operatorname{sign}(s_2 Y_e) \tag{26}$$

According to Lyapunov's direct method, the following Lyapunov function is introduced.

$$V = \frac{1}{2}\left(s_1^2 + s_2^2\right) \tag{27}$$

Taking the time derivative of V along the state trajectory gives:

$$\dot{V} = s_1 \dot{s}_1 + s_2 \dot{s}_2 = s_1\left(\ddot{X}_e + K_1 \dot{X}_e\right) + s_2\left(\ddot{Y}_e + K_2 \dot{Y}_e\right) \tag{28}$$

Replacing $\ddot{X}_e$ and $\ddot{Y}_e$ from (25) and (26) in (28) results in the following negative definite, $\dot{V}$.

$$\dot{V} = -\left(D_1 |s_1| + p_{1m} s_1 + E_1 |s_1 X_e|\right)$$
$$- \left(D_2 |s_1| + p_{2m} s_2 + E_2 |s_2 Y_e|\right)$$

Therefore, s_1 and s_2 are asymptotically stable and therefore, the WMR will converge to the desired reference trajectories. It should be noted that due to using the sign term in the designed controller, the chattering phenomenon may occur when the posture state errors are negligible values. To weaken the unwanted chattering phenomenon, some continuous functions, for example a saturation term could be used to approximate the sign term.

4. Simulation Results

To assess the effectiveness of the proposed DSMC, software simulations using MATLAB/SIMULINK are im-

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plemented. The simulations of tracking a circular trajectory are performed in which the desired position, orientation, velocities and acceleration of the WMR is monitored. **Figures 2-7** show the trajectory tracking performance of the designed control system of the WMR to compensate large initial off-tracks in the absence of uncertainties and measurement noises. According to **Figures 2-7**, by use of the proposed DSMC, the tracking errors of posture and velocity trajectories with respect to corresponding reference values smoothly converge to zero and therefore, a perfect trajectory tracking is obtained. On the other hand, the bounded control inputs of **Figure 8** show the significant performance of the proposed DSMC even under large posture off tracks.

Sliding mode control systems are known for robustness against parameter uncertainties and stochastic noises. To show this property of the proposed DSMC, Gaussian measurement noises are considered together with the measured posture variables. By the way, the simulated tracking performance of the WMR in the presence of measurement noises is shown in **Figures 8-14**. According to these figures, one can see that the DSMC keeps the posture and the velocity errors in a bounded small range and thus makes the WMR to track its reference trajectory
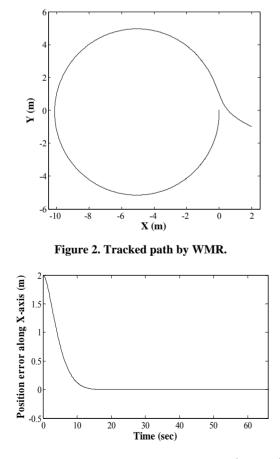

Figure 2. Tracked path by WMR.

Figure 3. Position tracking error along X axis $(X_c - X_r)$**.**
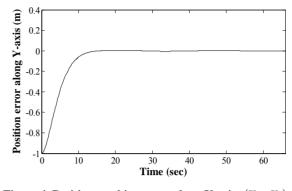

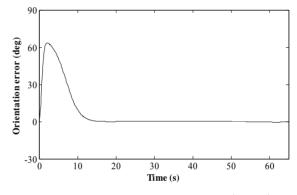**Figure 4. Position tracking error along Y axis** $(Y_c - Y_r)$**.**

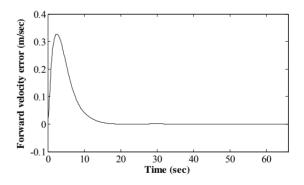**Figure 5. Tracking error of orientation** $(\theta_c - \theta_r)$**.**

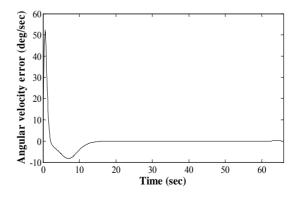**Figure 6. Tracking error of forward velocity** $(v_c - v_r)$**.**

Figure 7. Tracking error of angular velocity $(\omega_c - \omega_r)$**.**
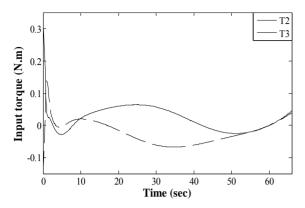
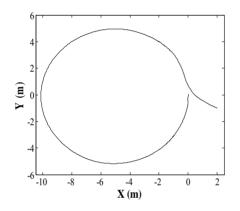**Figure 8. Produced torque by driving wheels.**

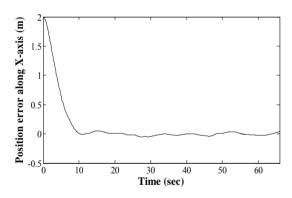**Figure 9. Tracked path of WMR under noises.**

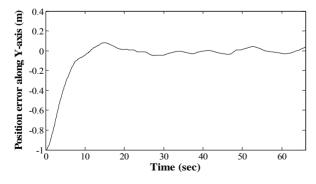**Figure 10. Position tracking error along X under noises.**

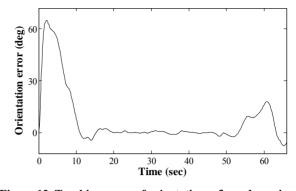**Figure 11. Position tracking error along Y under noises.**
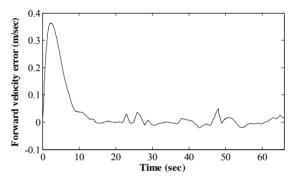

Figure 12. Tracking error of orientation, θ under noises.

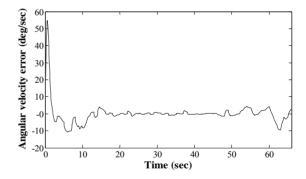**Figure 13. Tracking error of forward velocity under noises.**

Figure 14. Tracking error of angular velocity, ω under noises.

perfectly. For example, as **Figure 11** shows, after capturing the reference trajectory by the WMR, the tracking error along *Y* trajectory is bounded in the interval (−0.1, 0.1) which is negligible with respect to the large diameter of the circular trajectory $d = 10\,\text{m}$. Furthermore, according to **Figures 8** and **15**, the bounded input torques of the DSMC even under measurement noises shows its significance for real implementations.

5. Conclusion

A dynamic sliding mode controller for trajectory tracking control of a nonholonomic WMR has been proposed where the center of mass of the vehicle does not coincide to the middle point of connection center of driving wheels.
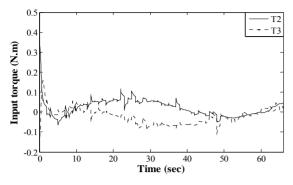
Figure 15. Produced torque by driving wheels.

The proposed controller is designed based on the developed dynamical model of the WMR in a global Cartesian coordinate system. According to simulation results, the designed DSMC keeps the WMR on the reference trajectory even in the presence of exogenous disturbances/noises. Furthermore, in the proposed DSMC, bounded control torques lead to compensation of large off track.

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