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Aplanatic and Telescopic Lens with a Radial Gradient of Refraction Index

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ABSTRACT

It is shown that aplanatic lens with a radial gradient of refraction index is simultaneously a telescopic lens, notably not only for an axial beam, but also for an off axis parallel beam. Consideration is carried out by an algebraic way on the basis of regularities of ray paths. It is also shown that aplanatic and telescopic properties of the lens are independent of the refracting surface shapes. Various versions of lens performance are shown below.

Keywords: Aplanatic Lens; Telescopic Lens; Radial Gradient; Refraction Index

1. Introduction

Aplanatic lenses with an axial and radial gradient of the refraction index are known by now. In these lenses, spherical aberration for a point on the optical axis is strictly eliminated.

Earlier possibility of creation of an aplanatic lens with an axial gradient of the refractive index was shown [1]. The homocentric beam leaving a point of M_1 on an optical axis, reaches to the first surface of a lens, refracts on it and then propagates parallel to an optical axis in the lens medium. Then, refracting on the second surface of a lens, the rays of the beam forms a homocentric dispersing beam again (**Figure 1**).

Virtual continuations of the ray forms the virtual image at the axial point M_2' on the optical axis. Various gradients of the refractive index and the related refracting surfaces can be used for formation of a lens. The calculations for lens parameters were provided with spherical and parabolic surfaces of revolution.

An aplanatic gradient lens [2] limited by the first and the second refracting surfaces of revolution with thickness d by the axis z, which is multiple of double nominal focal length, made of a material with a radial distribution of the refraction index $n(y)$ determined from the equation

$$n(y) = n_0 \operatorname{sech} ay = 2n_0 / \left(\exp(ay) + \exp(-ay)\right), \quad (1)$$

where n_0 is the refraction index value on the axis; a is the constant; having generatrix $y_1(z)$ of the 1st convex surface defined by the equation:

$$y_1^2(z) = \left(n^2(y) - 1\right) z^2 + 2 s_F \left(n(y) - 1\right) z, \quad (2)$$

where s_F is the front distance; and having generatrix $y_2(z)$ of the 2nd concave surface defined by the equation:

$$y_2^2(z) = \left(n^2(y) - 1\right)(z - d)^2 + 2 s'_{F'} \left(n(y) - 1\right)(z - d), \quad (3)$$

where $s'_{F'}$ is the rare distance; hence, $s'_{F'} = s_F$.

The shape of both refracting surfaces in the known lens is equal. Generatrixes of Equation (2) and Equation (3) may be conditionally called hyperbolas of the higher order.

It is known that the pitch (periodicity length) L for the refractive index (RI) distribution is determined by the Equation (1) and also called the hypersecans one equals [3]

$$L = 2\pi / a,$$

as a consequence, the half of the periodicity length is

$$L/2 = \pi/a$$

The nominal focal length of the lens with hypersecans RI distribution is

$$f_0' = L/4 = \pi/2a,$$

double nominal focal length will be equal to half the periodicity length

$$F = 2 f_0' = L/2 = \pi/a$$

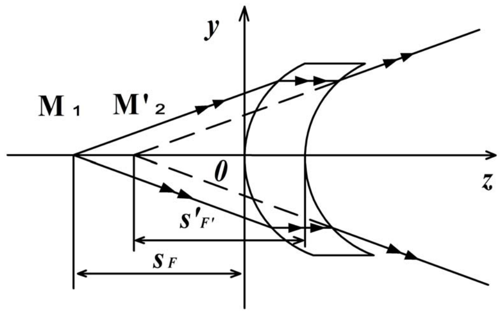

Figure 1. The known aplanatic lens with axial gradient.

When selecting certain thickness of the lens with the RI distribution of type Equation (1) and flat 1st and 2nd surfaces normal to the optical axis, focusing, diverging or telescopic lens can be obtained [3]. Hence, it is quite obvious that the telescopic lens is obtained in the case of selecting the thickness of the lens equal to twice the nominal focal length. As a result, the input parallel beam of rays, which is also parallel to the optical axis, becomes strictly parallel when leaving the lens.

The ray path equation in the medium with RI distribution Equation (1) is also known. For instance, if the initial height of a point, from which the ray is initiated, equals b_C, the initial coordinate is z_C, and the initial direction coefficient (the initial ray tangent) is φ_1, then the ray path equation becomes as follows [4]

$$y_R(z) = \frac{1}{a}\operatorname{arsh}\left(\operatorname{sh}ab_C\cos a(z-z_C)+\varphi_1\operatorname{ch}ab_C\sin a(z-z_C)\right) \quad (4)$$

As a consequence, considering that

$$\operatorname{arsh}x = \ln\left(x+\sqrt{x^2+1}\right)$$

and

$$\left[\operatorname{arsh}x\right]' = 1/\sqrt{x^2+1}$$

one can write down that

$$x = \operatorname{sh}ab_C\cos a(z-z_C)+\varphi_1\operatorname{ch}ab_C\sin a(z-z_C),$$

$$y_R(z) = \ln\left(x+\sqrt{x^2+1}\right)/a,$$

and the derivative is obtained from the expression

$$y_R'(z) = \frac{\varphi_1\operatorname{ch}ab_C\cos a(z-z_C)-\operatorname{sh}ab_C\sin a(z-z_C)}{\sqrt{\left[\operatorname{sh}ab_C\cos a(z-z_C)+\varphi_1\operatorname{ch}ab_C\sin a(z-z_C)\right]^2+1}} \quad (5)$$

Note that if the length between the coordinates of the ray exits point from the lens z_E and the ray entrance point of the lens z_D

$$d = z_E - z_D$$

equals to the periodicity length

$$L = 2\pi/a,$$

then according to Equation (4) the height of ray exit point from the lens y_E is equal to the height of ray entrance point to the lens y_D, or

$$y_E = y_D \quad (6)$$

As a result, the RI value at the ray entrance point to the lens will be equal to the RI value at the ray exit point from the lens

$$n(y_E) = n(y_D) \quad (7)$$

In this case, according to Equation (5), the ray tangent φ_2 occurring in the ray exit point from the lens before the refraction will equal to the initial tangent φ_1

$$\varphi_2 = \varphi_1 \quad (8)$$

and the derivative will correspond to the initial one

$$y_R'(z_D) = y_R'(z_E) \quad (9)$$

If the length d between coordinates of the ray exit point from the lens z_E and coordinates of the ray entrance point to the lens z_D equals to the half periodicity length,

$$L/2 = \pi/a,$$

then according to Equation (4) the height of the ray exit point from the lens y_E will be equal to the height of the ray entrance point to the lens y_D with the sign reversed

$$y_E = -y_D \quad (10)$$

The RI value at the ray entrance point to the lens will be equal to that at the ray exit point from the lens

$$n(y_E) = n(y_D)$$

The ray tangent φ_2 occurring at the ray exit point from the lens before the refraction will be equal to the initial tangent φ_1 with the sign reversed

$$\varphi_2 = -\varphi_1 \quad (11)$$

The value of derivative will be equal in the absolute magnitude to the initial one taken with the sign reversed

$$y_R'(z_D) = -y_R'(z_E) \quad (12)$$

According to the design, in the known aplanatic lens, every ray of the homocentric radiation beam exiting from a point on the optical axis, after the refraction on the 1st surface becomes parallel to the optical axis. Then spreading inside the lens, in a gradient medium by a curvilinear symmetrical path, each ray reaches the 2nd refractive surface. In the cross-point with the 2nd refractive surface the ray is also parallel to the optical axis.

After the refraction on the 2nd surface, each ray obtains the initial direction which allows re-forming a homocentric diverging beam with the center located on the optical axis, at a distance of $s'_{F'} = s_F$ from the 2nd surface center. Hence, $s'_{F'}$ is the rear distance (**Figure 2**).

Depending on the selected thickness d of the lens, which is multiple to the double nominal focal length, the refraction on the 2nd surface may happen both above and below the axis (if we conditionally assume the initial refraction on the 1st surface occurring above the optical axis). Thus, however causes no effect on the lens ability to form a homocentric diverging beam at the exit (**Figure 3**).

The known lens allows formation of a homocentric diverging radiation beam at the exit with the only help of two refractive surfaces of revolution of the same shape, which generatrix is rather complicated, Equation (2) and Equation (3), that makes its manufacture rather complex.

Thus, the fact that the other surfaces, including one simpler shape, cannot be used as the refractive surfaces is the disadvantage of the known lens restricting possibilities of its manufacture.

2. Analytical Treatment

Disadvantages of the known lens can be avoided, if we consider a remarkable, previously unknown property of the primary lens, which has a thickness multiple to the periodicity length, to preserve initial direction of the ray at the exit point, which appears on the 1st surface, independently of a chosen shape of the 1st and the 2nd surfaces, as well as front distance value and location of the radiation source, respectively.

The lens having a thickness multiple to half the periodicity length possesses a similar property of preserving the absolute value of the direction coefficient at the exit point for the entrance ray appearing on the 1st surface,

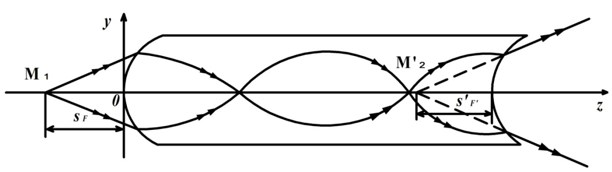

Figure 2. The known aplanatic lens with thickness L.

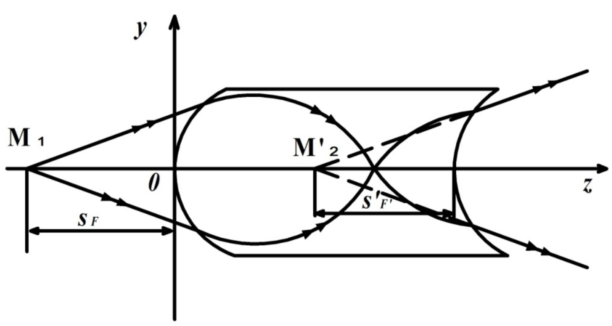

Figure 3. The known aplanatic lens with thickness $L/2$.

independently of the shapes selected for the 1st and 2nd surfaces, as well as of the front distance and location of the source of radiation, respectively. However, in this case the direction coefficient sign at the exit point for the refracted ray reverses.

Let us show the effect of these properties, firstly for the lenses of the minimal thickness with the thickness equal to the periodicity length L and half periodicity length $L/2$.

For the lenses of a greater thickness multiple to half the periodicity length, these properties also take effect.

A lens of thickness d, by the optical axis at a distance equal to the periodicity length L is made from a material with the radial RI distribution $n(y)$, which corresponds to (1) and is known for the given wavelength, and is limited by the refracting 1st and 2nd surfaces of the same shape with generatrixes $y_1(z)$ and $y_2(z)$ and disposed in a homogeneous medium (assume homogeneous medium RI $n_1 = 1$ (air)).

In the case under consideration, the same shape of the refracting surfaces of the lens d thickness, which is equal to the periodicity length L, means that the 2nd surface represents the 1st surface after parallel translation along the optical axis z by a distance d.

As a result, a distance along the axis z between any separate initial point on the 1st surface, which has a certain height above the axis z, and a point on the 2nd surface having the same height above the axis z and located on a straight line parallel to the axis z with the initial point of the 1st surface will be the same and equal to the thickness $d = L$.

In the simplest case, the refracting 1st and 2nd surfaces of the same shape may be the surfaces of revolution, but in more general case, this is optional.

The 1st surface vertex is located in the origin of the coordinate system. The axis z represents the optical axis of the lens. Assuming that the lens is axially symmetrical, consideration is made in the meridional plane.

The incident beam for the 1st surface originating from the point M_1 of the optical axis has a direction coefficient $u_{K1} = \varphi_1$ and is refracted in the point A having coordinates z_{C1}, y_{C1}. Let us denote: the angle of incidence of a ray on the 1st surface as ε_1, the angle of refraction on the 1st surface as ε'_1, normal to the 1st surface in a point A as N_1, the angle of incidence of a ray on the 2nd surface as ε_2, the angle of refraction on the 2nd surface as ε'_2, and normal to the 2nd surface in a point B as N_2.

Let us denote: direction coefficient of the entrance ray refracting on the 1st surface in point A as u_{T1}, direction coefficient of the ray refracted on the 1st surface in a point A as u_{E1}, direction coefficient of the ray refracting on the 2nd surface in a point B as u_{T2}, and direction coefficient of the ray refracted on the 2nd surface in a point B as u_{E2}.

Direction coefficient u_{N1} of the normal N_1 to the 1st surface in a point A can be expressed as follows:

$$u_{N1} = -1/y_1'(z), \quad (13)$$

where $y_1'(z)$ is the 1st derivative of $y_1(z)$.

Direction coefficient u_{N2} of the normal N_2 to the 2nd surface in a point B can be expressed as follows:

$$u_{N2} = -1/y_2'(z), \quad (14)$$

where $y_2'(z)$ is the 1st derivative of $y_2(z)$.

Let us consider the case of the 1st convex and the 2nd concave surfaces for a lens with the thickness multiple to L (**Figure 4**).

The refraction scheme in **Figure 3** relates to cases when after the refraction the direction coefficient u_{E1} of the exit ray refracted on the 1st surface is negative.

For this case, a general scheme of the ray path in the lens is shown in **Figure 5**.

For the refraction schemes shown in **Figure 3**, the angle of incidence ε_1 in the point A on the 1st surface can be determined from the following expression:

$$\tan\varepsilon_1 = (u_{T1} - u_{N1})/(1 + u_{T1}u_{N1}), \quad (15)$$

and the angle of refraction ε_1' is obtained from the expression

$$\tan\varepsilon_1' = (u_{E1} - u_{N1})/(1 + u_{E1}u_{N1}), \quad (16)$$

Moreover, according to Snell's refraction law

$$n_1 \sin\varepsilon_1 = n(y_{C1})\sin\varepsilon_1',$$

where $n(y_{C1})$ is the RI in the refraction point A.

Then, with regard to the fact that $n_1 = 1$, the refraction angle is expressed as

$$\sin\varepsilon_1' = \sin\varepsilon_1 / n(y_{C1}) \quad (17)$$

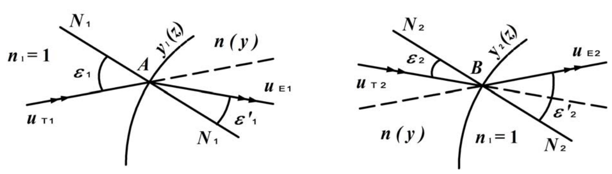

Figure 4. Schemes of refraction on the 1st convex and the 2nd concave surfaces for a lens with the thickness multiple to L.

Figure 5. The ray path scheme for the lens with the thickness multiple to L, having the 1st convex and the 2nd concave surfaces.

Since the ray refracted in the point A on the 1st surface, which has the initial direction u_{E1}, spreads after the refraction by a curvilinear path with a period equal to the periodicity length L, then in the point B, after completing the full period:

1) according to the condition Equation (6), the height y_{C2} of the refraction point B will be equal to the height y_{C1} of the refraction point A;

2) according to the condition Equation (8), direction coefficient $u_{T2} = \varphi_2$ of the ray incident to the 2nd surface will be equal to $u_{E1} = \varphi_1$ and, respectively,

$$u_{T2} = u_{E1}; \quad (18)$$

3) according to the condition Equation (8) and ratios Equation (9), Equation (13) and Equation (14), and since shapes of the 1st and 2nd surfaces are the same, direction coefficient u_{N2} of the normal N_2 to the 2nd surface in the point B will be equal to the direction coefficient u_{N1} of the normal N_1 to the 1st surface in the point A, and thus

$$u_{N2} = u_{N1} \quad (19)$$

The angle of incidence ε_2 in the point B will be equal to

$$\tan\varepsilon_2 = (u_{T2} - u_{N2})/(1 + u_{T2}u_{N2}) \quad (20)$$

Taking into account Equation (18) and Equation (19) and substituting them into Equation (20), we get:

$$\tan\varepsilon_2 = (u_{E1} - u_{N1})/(1 + u_{E1}u_{N1}) \quad (21)$$

Comparing Equation (16) and Equation (21), we get:

$$\varepsilon_1' = \varepsilon_2$$

On this basis, we can write down that

$$\sin\varepsilon_2 = \sin\varepsilon_1' \quad (22)$$

For the point B on the 2nd surface, according to Snell's refraction law,

$$n(y_{C2})\sin\varepsilon_2 = n_1\sin\varepsilon_2',$$

where $n(y_{C2})$ is the RI on the refraction point B.

Considering that $n_1 = 1$, the angle of refraction will be as follows:

$$\sin\varepsilon_2' = n(y_{C2})\sin\varepsilon_2$$

Taking into consideration relations Equation (17) and Equation (22), we get:

$$\sin\varepsilon_2' = n(y_{C2})\sin\varepsilon_1 / n(y_{C1})$$

Since $y_{C2} = y_{C1}$ in the point B, as mentioned above, and according to Equation (7), respectively, the expression $n(y_{C2}) = n(y_{C1})$ is true, and we get:

$$\sin\varepsilon_2' = \sin\varepsilon_1$$

Then, as a consequence,

$$\varepsilon_2' = \varepsilon_1$$

and

$$\tan\varepsilon_2' = \tan\varepsilon_1 \quad (23)$$

The refraction angle ε_2' in the point B may also be expressed as follows:

$$\tan\varepsilon_2' = (u_{E2} - u_{N2})/(1 + u_{E2}u_{N2})$$

If we use ratio Equation (19) in this expression, we get:

$$\tan\varepsilon_2' = (u_{E2} - u_{N1})/(1 + u_{E2}u_{N1}) \quad (24)$$

Based on Equation (23), and equating expressions Equation (15) and Equation (24) and making simple transformations, we get:

$$u_{E2} = u_{T1}$$

Thus, direction of the exit ray refracted in the point B will coincide with direction of the entrance ray in the point A.

Let us consider the case of the 1st convex and the 2nd concave surfaces for the lens with the thickness multiple to $L/2$ (**Figure 6**).

The refraction scheme shown in **Figure 6** also relates to the case, when after refraction the direction coefficient u_{E1} of the ray refracted on the 1st surface is negative.

For this case, a general scheme of the ray path in the lens is shown in **Figure 7**.

For the refraction scheme shown in **Figure 6**, the angle of incidence ε_1 in the point A on the 1st surface can be determined from the following expression:

$$\tan\varepsilon_1 = (u_{T1} - u_{N1})/(1 + u_{T1}u_{N1}), \quad (25)$$

and the angle of refraction ε_1' is obtained from the expression

$$\tan\varepsilon_1' = (u_{E1} - u_{N1})/(1 + u_{E1}u_{N1}) \quad (26)$$

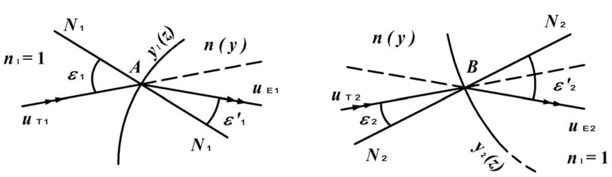

Figure 6. Schemes of refraction on the 1st convex and the 2nd concave surfaces for a lens with the thickness multiple to $L/2$.

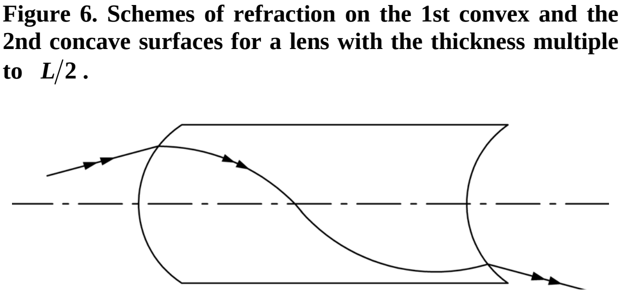

Figure 7. The ray path scheme for the lens with the thickness multiple to $L/2$, having the 1st convex and the 2nd concave surfaces.

Moreover, according to Snell's refraction law

$$n_1 \sin\varepsilon_1 = n(y_{C1})\sin\varepsilon_1'$$

Then, with regard to the fact that $n_1 = 1$, the refraction angle is expressed as

$$\sin\varepsilon_1' = \sin\varepsilon_1 / n(y_{C1})$$

For the lens with the periodicity length $L/2$, the ray refracted in the point A on the 1st surface, having the initial direction u_{E1}, spreads by periodical curvilinear path after the refraction and in the point B, after passing the half period:

1) the height y_{C2} of the refraction point B will be equal, by the absolute value, to the height y_{C1} of the refraction point A, but, according to the condition Equation (10), will be negative;

2) direction coefficient $u_{T2} = \varphi_2$ of the ray incident to the 2nd surface will be equal to $u_{E1} = \varphi_1$ by the absolute value but, according to the condition Equation (11), will be negative and, thus,

$$u_{T2} = -u_{E1}; \quad (27)$$

3) direction coefficient u_{N2} of the normal N_2 to the 1st surface in the point B will, by the absolute value, be equal to the direction coefficient u_{N1} of the normal N_1 to the 1st surface in the point A but, according to the condition Equation (12), will be negative:

$$u_{N2} = -u_{N1} \quad (28)$$

The angle of incidence ε_2 in the point B will be equal to

$$\tan\varepsilon_2 = (u_{N2} - u_{T2})/(1 + u_{T2}u_{N2}) \quad (29)$$

Taking into account Equation (27) and Equation (28) and substituting them into Equation (29), we get:

$$\tan\varepsilon_2 = (u_{E1} - u_{N1})/(1 + u_{E1}u_{N1}) \quad (30)$$

Comparing Equation (26) and Equation (30), we get:

$$\varepsilon_1' = \varepsilon_2$$

On this basis, we can write down that

$$\sin\varepsilon_2 = \sin\varepsilon_1'$$

For the point B on the 2nd surface, according to Snell's refraction law,

$$n(y_{C2})\sin\varepsilon_2 = n_1 \sin\varepsilon_2',$$

where $n(y_{C2})$ is the RI on the refraction point B. Subsequently, with respect to the fact that $n_1 = 1$, the refraction angle will be expressed as follows:

$$\sin\varepsilon_2' = n(y_{C2})\sin\varepsilon_2$$

Taking into account ratios Equation (18) and Equation (19), we get:

$$\sin\varepsilon_2' = n(y_{C2})\sin\varepsilon_1 / n(y_{C1})$$

As mentioned above, since in the point B $y_{C2} = -y_{C1}$, but at that $n(y_{C2}) = n(-y_{C1})$, we get:

$$\sin\varepsilon_2' = \sin\varepsilon_1$$

Then, as a consequence,

$$\varepsilon_2' = \varepsilon_1$$

and

$$\tan\varepsilon_2' = \tan\varepsilon_1 \quad (31)$$

The refraction angle ε_2' in the point B may also be expressed as follows:

$$\tan\varepsilon_2' = (u_{N2} - u_{E2}) / (1 + u_{E2}u_{N2})$$

If we use ratio Equation (28) in this expression, we get:

$$\tan\varepsilon_2' = (u_{E2} + u_{N1}) / (u_{E2}u_{N1} - 1) \quad (32)$$

Based on Equation (31), and equating expressions Equation (25) and Equation (32) and making simple transformations, we get:

$$u_{E2} = -u_{T1}$$

As a result, the direction coefficient u_{E2} of the exit ray refracted in the point B will, by the absolute value, be equal to the direction coefficient u_{T1} of the entrance ray in the point A, but will be negative.

Let us show execution of aplanatic properties of the lens on the example of the above-considered case of refraction.

It is commonly known that the two conjugated points in the space of objects and images are called aplanatic, if spherical aberration is absent in the image and the sine condition (or the Abbe sine law) is fulfilled.

The above proved property of the considered lenses having a thickness multiple to half the periodicity length, which is a preservation of direction of the initial ray incident to the 1st surface at the exit point independently of selected shape of the 1st and 2nd surfaces, provides the absence of the spherical aberration for a point on the optical axis at a finite distance from the lens.

Let us show now that for a pair of the conjugated points M_1 and M_2' on the optical axis the Abbe sine law is fulfilled.

In the general case, the Abbe sine law looks as follows:

$$n\sin\sigma = \beta n'\sin\sigma'$$

or

$$\beta = n'\sin\sigma' / n\sin\sigma,$$

where n is the RI of the medium, in which the object is located;

n' is the refractive index of the medium, in which the image is formed;

σ is the angle between the optical axis and the ray exiting from the axial object point;

σ' is the angle between the optical axis and the ray exiting from the optical system and passing through the axial image point;

β is linear magnification of the optical system.

For considered versions of the suggested lens, the ratio will become as follows:

$$\beta = \sin\sigma_2' / \sin\sigma_1 \quad (33)$$

where σ_1 is the angle between the optical axis and the ray exiting from the axial (object) point M_1;

σ_2' is the angle between the optical axis and the ray exiting from the lens and passing through the axial image point M_2' (the virtual image).

Let us consider the lenses with the 1st convex and the 2nd concave surfaces.

For the lens L thickness, with the 1st convex and the 2nd concave surfaces, the refraction scheme on the convex 1st surface for the case $u_{E1} < 0$ is shown in **Figure 8**, and for the 2nd surface—in **Figure 9**.

For the lens $L/2$ thickness, with the 1st convex and the 2nd concave surfaces, the refraction scheme on the convex 1st surface for the case $u_{E1} < 0$ is shown in **Figure 8**, and for the 2nd surface—in **Figure 10**.

For all cases considered in **Figures 8-10**, the ratio Equation (33) will become as follows:

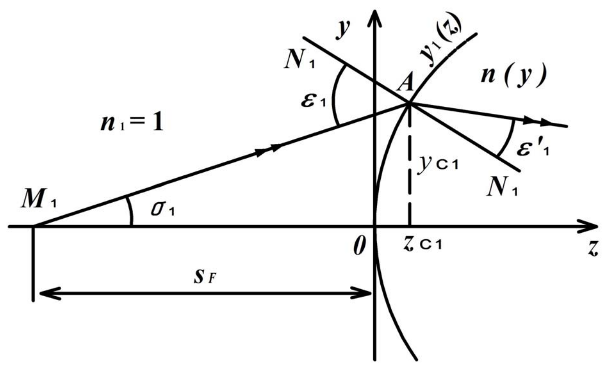

Figure 8. The refraction scheme on the 1st convex surface of the lens.

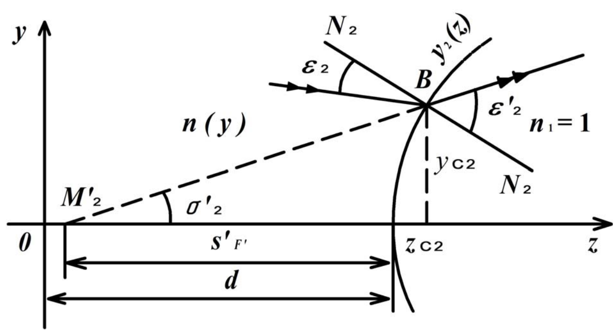

Figure 9. The refraction scheme on the 2nd concave surface for the lens with the thickness multiple to L.

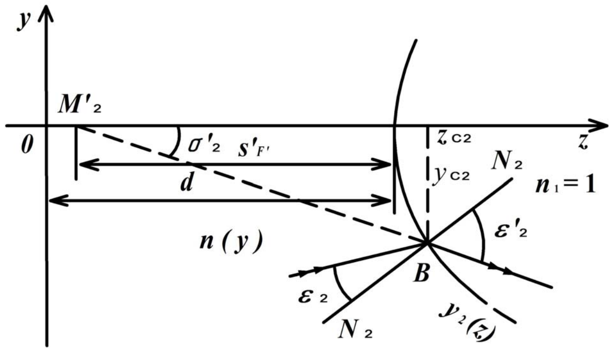

Figure 10. The refraction scheme on the 2nd concave surface for the lens with the thickness multiple to $L/2$.

$$\beta = \frac{\dfrac{y_{C2}}{\sqrt{y_{C2}^2 + \left(\left(z_{C2} - d\right) + s'_{F'}\right)^2}}}{\dfrac{y_{C1}}{\sqrt{y_{C1}^2 + \left(z_{C1} + s_F\right)^2}}}$$

Since generatrixes of the 1st and 2nd surfaces have the same shapes, then the deflections of the surfaces Δ_C for the same height are also equal, respectively:

$$\Delta_C = z_{C1} = z_{C2} - d$$

Moreover, according to the initial data, the front distance and the rear distance are equal, too:

$$s'_{F'} = s_F = s$$

As a result, we can write down that

$$\beta = \frac{\dfrac{y_{C2}}{\sqrt{y_{C2}^2 + \left(\Delta_C + s\right)^2}}}{\dfrac{y_{C1}}{\sqrt{y_{C1}^2 + \left(\Delta_C + s\right)^2}}}$$

As mentioned above, for the lenses L thickness:

$$y_{C1} = y_{C2},$$

and for the lenses of the thickness $L/2$

$$y_{C1} = -y_{C2}$$

As a result, linear magnification of the lenses will be constant and, for the lenses with the thickness multiple to L, will be equal to:

$$\beta = 1,$$

and for the lenses with the thickness multiple to $L/2$:

$$\beta = -1$$

Then in the lens under consideration, for a pair of the conjugated points M_1 and M'_2 on the axis, spherical aberration will also be absent, and for any ray exiting from an axial object point, the Abbe sine law will be fulfilled. Subsequently, points M_1 and M'_2 will form a pair of aplanatic points, and the lens in this case may also be called aplanatic.

The indicated property of the suggested lenses to preserve direction of the entrance ray at the exit point (or preserve the absolute value of direction coefficient of the ray with reversing the coefficient sign), as a consequence, leads to the fact that the entrance homocentric diverging beam of rays (with the center that may locate both on the optical axis and outside of it) will preserve homocentricity at the exit of the lens, and virtual extensions of the rays at the exit point will concur forming an virtual image of the object point.

Conditions for the lenses providing the proved property can be formulated:

1) as the lens is thickness multiple to F, the lens thickness projection on the optical axis in any plane containing the optical axis, for any two points of the 1st and 2nd surfaces having the same absolute height relative to the optical axis but located on different sides of it, equals to F; or

2) as the lens is thickness multiple to $2F$, the lens thickness projection on the optical axis in any plane containing the optical axis, for any two points of the 1st and 2nd surfaces having the same height relative to the optical axis and located on the same side of the optical axis, equals to $2F$.

Thus, the lens having refractive surfaces, which meet the above conditions, will possess the proved properties.

It is notable that the 1st consequence of this property will be the absence of the necessary fixed location of an object point on the optical axis. This means that the object point may be located on the optical axis at any distance from the lens, and the homocentricity of the beam at the entrance of the lens will not be disturbed.

The 2nd consequence of this property will be the absence of the necessary fixed location of an object point directly on the optical axis. This means that the object point may be located outside the optical axis at any distance from the lens (with respect to restrictions applied to the refraction conditions on the 1st surface associated with full internal reflection and finite diameter of the lens), and the homocentricity of the beam at the exit point of the lens will not be disturbed.

The third consequence of this property will be simultaneous telescopic properties of the lens, since the entrance parallel beam of rays at the exit point of the lens will preserve its direction, and all rays of the exit beam will remain parallel, hence, the entrance parallel beam is not necessarily parallel to the optical axis of the suggested lens.

As a result, the suggested lenses will possess aplanatic and telescopic properties simultaneously, which has not been known before.

Thus, for the known lens [2], the presence of aplanatic properties was proved before, but the simultaneous telescopic properties were not known yet.

For another known lens with the refracting 1st and 2nd flat surfaces, which are normal to the optical axis [3], the telescopic properties were indicated, hence, exclusively for the exit ray parallel to the entrance ray. However, the aplanatic properties were not known simultaneously.

Consideration of other cases and refraction versions on the 1st and 2nd surfaces, in particular, for the 1st concave and the 2nd convex surfaces, for flat 1st and 2nd surfaces etc., also allows proving the indicated property of the lenses.

Numerical computation performed proves the above indicated property of the lenses.

The indicated property of the suggested lenses with a thickness multiple to L and $L/2$ can be used for forming gradient lenses possessing both aplanatic and telescopic properties simultaneously, as well as various refractive surfaces.

It is natural to use surfaces of revolution with various simpler generatrixes than type Equation (2) generatrix - a straight line, a circle, etc. In this case, refractive surfaces will be symmetrical relative to the optical axis. One may also note that the refractive surfaces may have generatrixes with inflection points.

However, this is optional.

In the general case, the refractive surfaces may be unsymmetrical relative to the optical axis. There is a possibility to use the refractive surfaces as the inclined planes, for example, as well as combined refractive surfaces as a combination of an inclined plane and a plane normal to the optical axis, two inclined planes with different inclined angles, etc.

Note that if two inclined planes, 1st and 2nd, are used, aplanatic and telescopic properties of the lens can be proved using the above-mentioned approach, if each ray path of the homocentric or parallel beam entrance in the plane containing this ray and the optical axis of the lens is considered. Then consideration may be reduced to one of the above-considered cases (not shown here).

It is possible to prove aplanatic and telescopic properties of a lens that has the refractive surfaces of revolution with generatrixes having inflection points using the above-mentioned approach, if we consider a path of each ray of the homocentric or parallel beam entrance separately for generatrix sections with different curvature signs. As a consequence, consideration for every section can be reduced to one of the above-considered cases (not shown here, either).

3. Possible Versions of the Lens Performance

Obviously, the simplest version of the lens performance will be a lens with flat 1st and 2nd surfaces normal to the optical axis.

Other versions of suggested lens performance with the thickness multiple to L and $L/2$ are also possible:

with spherical refractive surfaces (**Figures 11**, **12**);

with conic refractive surfaces (**Figure 13**);

with flat refractive surfaces (**Figure 14**);

with flat inclined refractive surfaces (**Figure 15**);

with combined refractive surfaces having flat and inclined flat surfaces (**Figure 16**).

In the context of considered versions of the suggested lens, the known lens [2] is a particular case, for which

1) surfaces of revolution are selected as the refractive ones;

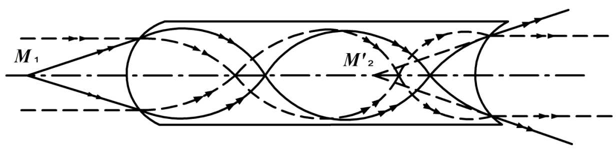

Figure 11. Parallel and homocentric beam path scheme for the lens with the thickness multiple to L, having the 1st convex and the 2nd concave spherical surfaces.

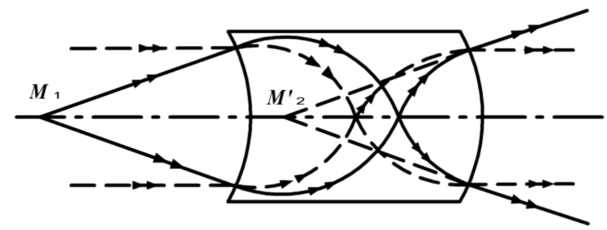

Figure 12. Parallel and homocentric beam path scheme for the lens with the thickness multiple to $L/2$, having the 1st concave and the 2nd convex spherical surfaces.

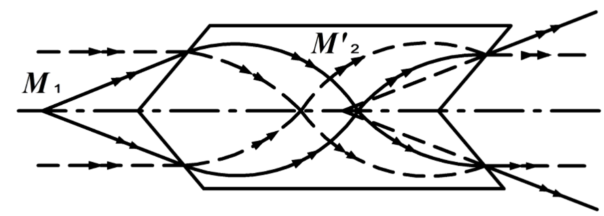

Figure 13. Parallel and homocentric beam path scheme for the lens with the thickness multiple to $L/2$, having the 1st convex and the 2nd concave conic surfaces.

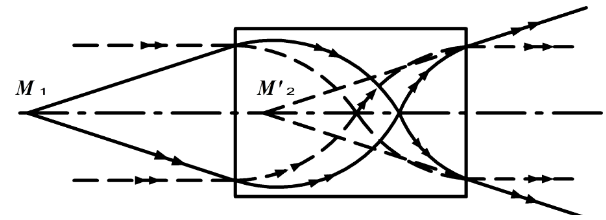

Figure 14. Parallel and homocentric beam path scheme for the lens with the thickness multiple to $L/2$, having the 1st and the 2nd flat surfaces normal to the optical axis.

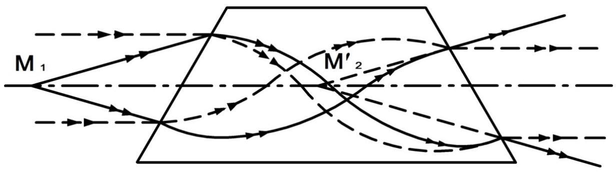

Figure 15. Parallel and homocentric beam path scheme for the lens with the thickness multiple to $L/2$, having the 1st and the 2nd flat inclined surfaces.

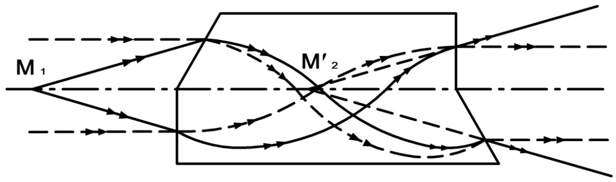

Figure 16. Parallel and homocentric beam path scheme for the lens with the thickness multiple to $L/2$, having the 1st and the 2nd combined refractive surfaces with flat inclined and flat surfaces normal to the optical axis.

2) a version with the 1rst convex and the 2nd concave surfaces is selected;

3) for generatrixes for surfaces of revolution, special functions Equation (2) and Equation (3) are selected; for the entrance ray, after the refraction on the 1st surface, application of these functions provides direction parallel to the optical axis.

It may also be noted that the maximum attainable numerical aperture for all considered aplanatic lenses will obviously be principally lower than that for the known lens [2], all other conditions being equal and the same lens diameters, because after refraction on the 1st surface, rays are inclined to the optical axis. This means that then, when passing the lens material, maximal height of the ray on the path will exceed the entrance height by a certain value depending on this angle. According to the design, in the known lens, after refraction on the 1st surface all rays are parallel to the optical surface. Then, while passing through the lens material, the maximum ray height on the path does not exceed the ray entrance height.

4. Conclusions

As shown, aplanatic lens with a radial gradient of refraction index is simultaneously a telescopic lens, notably not only for an axial beams, but also for an off axis parallel beams.

Maximum reachable numerical aperture of the lens is principally lower than that of the known aplanatic lens. Aplanatic and telescopic properties of the lens are independent of the refracting surface shapes, which simplifies production of the lens.

Various suggested versions of the lens performance may be applied to fiber optics and optical instrument-making, creation of boroscopes, objectives, condensers, couplers for fiber-optic communication lines with sources of radiation and photodetectors, etc.

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