

A Comparison of Active and Passive Metamaterials from Equivalent Lumped Elements Modes

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ABSTRACT

With ever-increasing operating frequencies and complicated artificial structures, loss effects become more and more important in applications of metamaterials. Based on circuit theory and transmission line principle, the design equations for effective electromagnetic (EM) parameters (attenuation constant α , phase constant β , characteristic impedance Z_0) of general active and passive metamaterial are compared and derived from the equivalent lumped circuit parameters (R, G, L_L, C_L, L_R, C_R). To verify the design equations, the α , β and Z_0 in different cases, including balanced, unbalanced, lossless, passive and active, are shown by numerical simulations. The results show that using the active method can diminish the loss effects. Meantime, it also has influence on phase constant and real part of characteristic impedance.

Keywords: Metamaterials; Equivalent Lumped Circuit elements Modes; Loss Compensation

1. Introduction

The electromagnetic (EM) material parameters, such as propagation constant (including attenuation constant and phase constant), characteristic impedance, permittivity, permeability, refractive index, phase velocity and conductivity, of material always used to describe the EM response in certain frequency whenever EM wave exist in or propagation (transmit/evanescent) across media [1]. Based on transmission line theory, where EM wave propagation is considered in media the EM behaviors could be entirely represented with equivalent lumped circuit parameters, i.e., resistance, conductance, inductance and capacitance, which are also frequency dependent parameters like EM parameters of material [1]. It provided the relationship between the equivalent lumped circuit parameters and the EM material parameters. These equivalent lumped circuit parameters also serve as bridges between complicated EM field calculations and straightforward circuit analysis. Hence, EM material parameters can be expressed in terms of the equivalent lumped circuit parameters. Similarly, the equivalent lumped circuit parameters can be determined by EM material parameters [1,2].

Metamaterial, which control the electromagnetic waves

with their structures rather than the compositions [3-6], such as zero index materials, negative refractive index metamaterials, plasmonics, have received much attention owing to their extraordinary properties not readily achievable in nature [7-21]. Left-handed material with negative refractive index was first investigated theoretically by Veselago in 1968 [3], which can exhibits lots of unusual physical properties, like reversed Doppler shift, inverted Snell's law (negative refraction) and reversed Cerenkov radiation, compared with conventional righthanded material (RHM), tremendous attention has been focused on theory and engineering applications of this novel type of artificial material [7-21]. There are two main artificial methods to realize metamaterials: first, based on the resonant-type structures; second, using quasi-lumped elements developed on transmission lines. With the first method, Shelby et al. constructed the first LHM with periodical array of copper split-ring resonators (SRRs) and thin copper wires in the microwave regime [4]. By using the second method which usually termed composite right/left-handed transmission line (CRLH-TL) method and utilizes transmission line (TL) principle [15-17], the EM behavior can be easily controlled by equivalent lumped circuit parameters. It is not easy to acquire purely LHM with the second method since unavoidable RH parasitic effects in nature, but the CRLH-TLs may have

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advantages of planar structure, broader bandwidth, adjustable phase response and compatible with microwave integrated circuits. Many applications of CRLH-TL to RF/ Microwave/Terahertz (THz)/ Optical components have been proposed, such as filter, antenna, power divider, phase shifter, coupler, balun and some other active components [2]. However, most of them are based on ideal lossless CRLH-TL method which ignores the losses effects. With ever-increasing operating frequencies and complicated artificial structures, losses effects must have paid much attention for engineering application of metamaterials [18-21].

In this paper, we introduce the design equations of effective EM material parameters of general active and passive metamaterial from equivalent lumped elements modes. And then detailed analysis of ten examples, including lossless/passive/active metamaterial cases, based on these equations will be presented.

2. Theory and Design Equations

The equivalent circuit of the unit-cell of general active and passive metamaterial is shown in **Figure 1**. The unit-cell is modeled with six lumped element circuit parameters, i.e., R , G , C_L , L_L , C_R , and L_R . These parameters are normalized to length and may be detailed interpreted as following: a series resistance (R , unit in $[\Omega/m]$) and a shunt conductance (G , unit in $[S/m]$) to account for the effects of losses (conductive loss and dielectric loss, respectively); a series capacitance (C_L , unit in $[F \cdot m]$) and a shunt inductance (L_L , unit in $[H \cdot m]$) to obtain the left-handed transmission line (LH-TL) by using the dual principle of circuit theory; as well as a series inductance (L_R , unit in $[H/m]$) and a shunt capacitance (C_R , unit in $[F/m]$) from unavoidable practice effects.

The period, or termed lattice constant, of the unit-cell is P . For satisfying the principle of ideal effective homogenous TL, it should be chosen to meet the electrically small condition:

$$P \ll \lambda_g, \text{ at least } P < \frac{\lambda_g}{4}, \text{ typically } P \approx \frac{\lambda_g}{10} \quad (1)$$

where λ_g is the guided wavelength.

From **Figure 1**, the series impedance Z and shunt admittance Y for the unit-cell of general active and passive metamaterial can be written as:

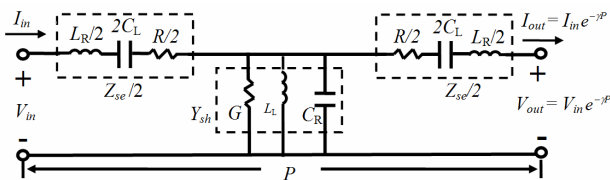


Figure 1. Equivalent circuit for unit-cell of general active and passive metamaterial.

$$Z = R + j \left(\omega L_R - \frac{1}{\omega C_L} \right) \quad (2)$$

$$Y = G + j \left(\omega C_R - \frac{1}{\omega L_L} \right) \quad (3)$$

Based on the classical TL principle [1], the propagation constant γ and characteristic impedance Z_0 of the general active and passive metamaterial are given by:

$$\gamma = \alpha + j\beta = \sqrt{ZY} \quad (4)$$

$$Z_0 = \sqrt{\frac{Z}{Y}} \quad (5)$$

where α is attenuation constant (unit in Np/m) and β is phase constant (unit in rad/m).

By inserting (2) and (3) into (4) and separating the real and imaginary parts, the equations of α and β are distilled and simplified as (6) and (7), respectively. When $R = G = 0$, it become ideal lossless case, which has already been widely used in former publications and reports for calculation simplicity [2]. From (6) and (7), it is clear that the attenuation constant $\alpha = 0$ and the propagation constant $\gamma = j\beta$ in this ideal case. Similarly, using (2) and (3) into (5) the characteristic impedance Z_0 is given as (8).

As we have stated above, since the electrical size of the unit-cell of general active and passive metamaterial is small enough to suppress all diffraction scattering effects, the general active and passive metamaterial can be characterized with the some other effective constitutive parameters, such as permittivity, permeability and refractive index. The effective values of them can be easily extracted from the EM material parameters (γ and Z_0) [13-15].

3. Numerical Simulations and Comparison Analysis

Ten numerical examples, including lossless ($R=G=0$), passive lossy ($R > 0, G \neq 0$) and active lossy ($R < 0, G \neq 0$) cases, are presented in this section to study losses effects in general active and passive metamaterial. The equivalent lumped circuit parameters are presented in **Table 1**.

Conductance generally represents the dielectric loss and it has the relationship with dielectric loss tangent ($\tan \delta$) is:

$$G = \omega \epsilon_r \epsilon_0 \tan \delta \quad (9)$$

where ϵ_r is the relative permittivity (dielectric constant), and $\epsilon_0 (= 8.854 \times 10^{-12})$ is permittivity in free space. Take the low cost substrate material Duroid ($\epsilon_r = 2.2, \tan \delta = 0.0009$) as an example, which has been widely used for RF/Microwave engineering, the corresponding

conductance is 0.001 (unit in [S/m]) at 10 GHz.

Table 1 shows balanced case (example 1-5), unbalanced case (example 6-10), lossless case (example 1 and 6), passive lossy case (example 2, 3, 7 and 8) and active lossy case (example 4, 5, 9 and 10).

In the balanced case (example 1-5), from the parameters, it is easy to calculate the transition frequency: $f_0 = 5 \text{ GHz}$. The attenuation constant curve is shown in **Figure 2**. As can be seen in **Figure 2**, the attenuation constant is increasing with resistance in passive case

($R \geq 0$), but decreasing with the enhanced active (which can be represented by the absolute value of negative resistance) metamaterial. Hence, included active elements may be a convenient way to overcome the major drawback of losses in metamaterials.

The phase constant curve in the balanced case is shown in **Figure 3**. The sign of phase constant (β) is crucial important for metamaterial, because it determines the backward wave, forward wave and broadside (in the balanced case) or stopband (in the unbalanced case) frequencies.

$$\alpha = \sqrt{\frac{-(\omega^2 C_L L_R - 1)(\omega^2 C_R L_L - 1) + \sqrt{[\omega^4 C_R^2 L_L^2 + \omega^2 (G^2 L_L^2 - 2C_R L_L) + 1][\omega^4 C_L^2 L_R^2 + \omega^2 (R^2 C_L^2 - 2C_L L_R) + 1]}}{2\omega^2 C_L L_L}} \quad (6)$$

$$\beta = \sqrt{\frac{(\omega^2 C_L L_R - 1)(\omega^2 C_R L_L - 1) + \sqrt{[\omega^4 C_R^2 L_L^2 + \omega^2 (G^2 L_L^2 - 2C_R L_L) + 1][\omega^4 C_L^2 L_R^2 + \omega^2 (R^2 C_L^2 - 2C_L L_R) + 1]}}{2\omega^2 C_L L_L}} \quad (7)$$

$$Z_0 = \sqrt{\frac{L_L}{C_L} \left(\frac{\omega^4 C_L^2 L_R^2 - 2\omega^2 C_L L_R + \omega^2 C_L^2 R^2 + 1}{\omega^4 C_R^2 L_L^2 - 2\omega^2 C_R L_L + \omega^2 L_L^2 G^2 + 1} \right)^{1/4}} \exp \left\{ \frac{1}{2} j \text{Arg} \left[\frac{L_L [-j + \omega C_L (R + j\omega L_R)]}{C_L [-j + \omega L_L (G + j\omega C_R)]} \right] \right\} \quad (8)$$

Table 1. Equivalent lumped circuit parameters of active and passive metamaterials.

Example	R [Ω / m]	L _R [nH / m]	C _L [pF · m]	L _L [nH · m]	C _R [pF / m]	G [S/m]
1	0	1	1	1	1	0
2	0.1	1	1	1	1	0.001
3	1	1	1	1	1	0.001
4	-0.1	1	1	1	1	0.001
5	-1	1	1	1	1	0.001
6	0	1	2	5.5	1	0
7	0.1	1	2	5.5	1	0.001
8	1	1	2	5.5	1	0.001
9	-0.1	1	2	5.5	1	0.001
10	-1	1	2	5.5	1	0.001

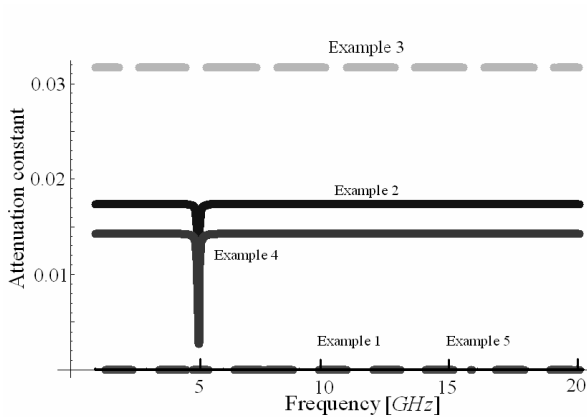


Figure 2. Attenuation constant curve for balanced case (example 1-5).

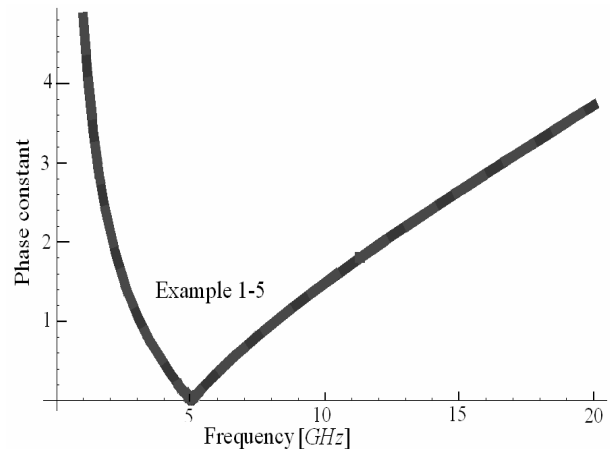


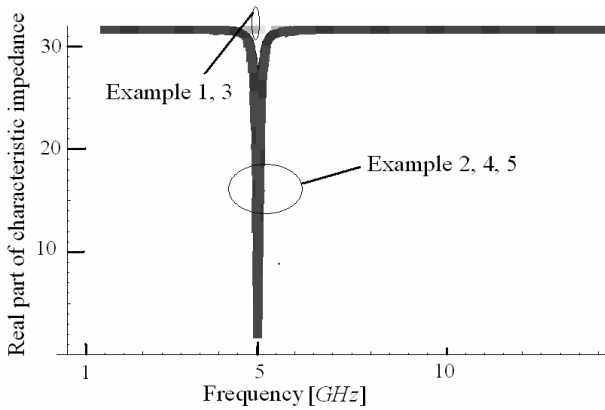
Figure 3. Phase constant curve for balanced case.

Figure 3 shows the zero phase constant $\beta = 0$ at the transition frequency f_0 of 5 GHz, usually termed Zeroth-Order Resonator (ZOR, the physical dimensions can be arbitrary but not limited by the conventional wavelength [2]). It also can be observed the continuous leakage/fast frequency band (between two air lines $\omega = \pm\beta c$) from LH (phase constant $\beta < 0$, $0.5 < f < 5$ GHz) to RH ($\beta > 0$, $5 \text{ GHz} < f < 20$ GHz) state through the transition frequency point in all of the numerical calculations including lossless, passive and active states.

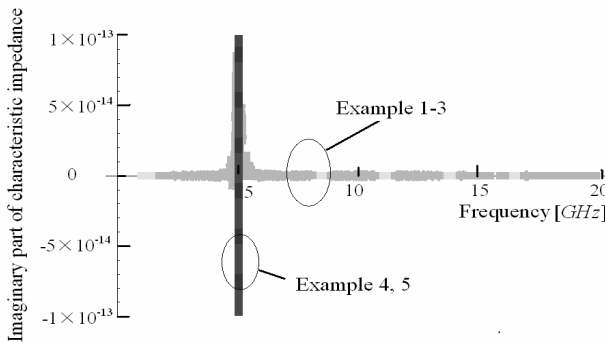
In **Figure 4(a)** and **(b)**, the real part and imaginary part of the characteristic impedance are plotted respectively for balanced case.

Like the balanced case analysis, the attenuation constant, phase constant, real part and imaginary part of characteristic impedance as function of frequency are calculated and plotted in **Figures 5-7(a)** and **(b)** for unbalanced case (Example 6-10), respectively.

In the unbalanced case (Example 6-10), the stopband can be calculated from 2.33 GHz to 3.56 GHz by using the design equations of Section II and the parameters presented in the **Table 1**. The results show that using the active method can diminish the loss effects. Meantime, it also has influence on phase constant and real part of characteristic impedance.



(a) Real part



(b) Imaginary part

Figure 4. Real and imaginary parts of Z_0 as function of frequency for balanced case.

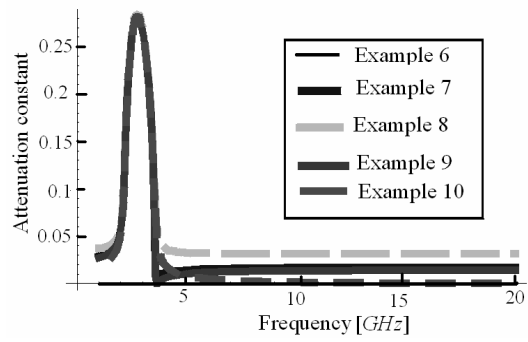


Figure 5. Attenuation constant curve for unbalanced case (example 6-10).

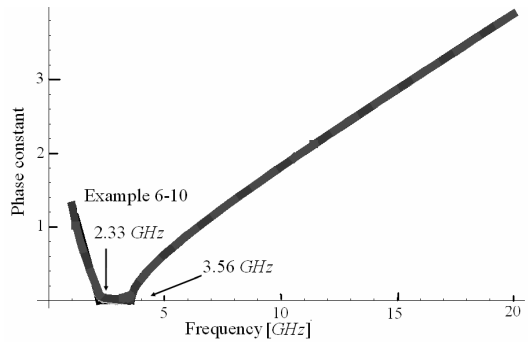
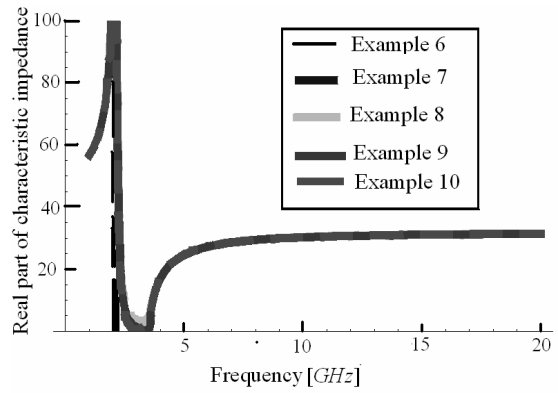
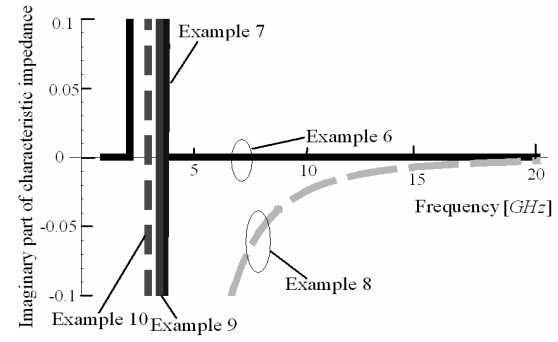


Figure 6. Phase constant curve for unbalanced case.



(a) Real part



(b) Imaginary part

Figure 7. Real and imaginary part of Z_0 as function of frequency for unbalanced case.

4. Conclusions

The EM material parameters equations for general active and passive metamaterial are calculated and compared from equivalent lumped elements modes. In the comparison analysis of balanced and unbalanced lossless ($R = G = 0$), passive lossy ($R > 0, G \neq 0$) and active lossy ($R < 0, G \neq 0$) cases, different losses effects (from the changes of resistance in this paper) have some effects on phase constant and the real part of characteristic impedance, as well as a big influence on attenuation constant and the imaginary part of characteristic impedance. One can use included active element to metamaterial to improve the attenuation characteristics and then to overcome the major drawback of losses effects in metamaterials.

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