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Comparison of Iterative Wavefront Estimation Methods*

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ABSTRACT

The iterative reconstruction methods of the wavefront phase estimation from a set of discrete phase slope measurements have been considered. The values of the root-mean-square difference between the reconstructed and original wavefront have been received for Jacobi, Gauss-Seidel, Successive over-relaxation and Successive over-relaxation with Simpson Reconstructor methods. The method with the highest accuracy has been defined.

Keywords: Wavefront Estimation; Phase; Reconstruction; Iterative Methods

1. Introduction

Now methods of adaptive optics become more and more widespread in optics. That appears to be particularly important for ultra-high intensity lasers as the number of optical elements that this lasers include is rather high so the appearance of static and thermo-induced aberrations of the wavefront are possible [1], for diagnostic medical systems for correction of the crystalline lens and corneal aberrations[2] and for ground-based telescopes to correct for the atmospheric aberrations. In this paper we consider the comparison research of the iterative methods of estimating wavefront phase from a set of discrete phase slope measurements that was obtained from a Hartmann sensor.

2. Shack-Hartmann Wavefront Sensor

2.1. Operation Principle

The Shack-Hartmann wavefront sensor contains a lenslet array that consists of a two-dimensional array of a few hundred lenslets all with the same diameter and the same focal length. The light ray spatially sampled into many individual beams by the lenslet array and forms multiple spots in the focal plane of the lenslets. A CCD camera placed in the focal plane of the lanslet array records the spot array pattern for wavefront calculation. For a wave with plane wavefront the Shack-Hartmann spots are formed along the optical axis of each lenslet, resulting in a regularly spaced grid of spots in the focal plane of the lenslet array. In contrast, individual spots formed by wavefront with aberrations are displaced from the optical axis of each lenslet. The displacement of each spot is proportional to the wavefront slope.[2]

2.2. Specific Features

Modern Shack-Hartmann sensors consist of a large number of lenslets, this number can exceed 10000. While using such sensors it's possible to face the problem of crosstalk, limited aperture and the loss of some points. Not all methods of wavefront estimation are suitable under these conditions.

3. Wavefront Reconstruction Methods

As an example of reconstruction methods can be taken modal and zonal wavefront reconstruction methods.

- In the modal approach the wavefront is expanded into a set of orthogonal basis functions, and the coefficients of the set of basis functions are estimated from the discrete phase-slope measurements. The appearance of the large reconstruction error is possible.
- In the zonal approach, the wavefront is estimated directly from a set of discrete phase-slope measurements. Estimated with this method wavefront has less reconstruction error then that in modal approach. For the zonal reconstruction method the iterative algorithms may be used.

4. Iterative Methods

4.1. Jacobi and Gauss-Seidel Methods

One of the first iterative methods are Jacobi and Gauss-Seidel methods. These methods were proposed by Southwell for wavefront reconstruction. The idea behind this method is to take into account the wavefront deformation for some vertical and horizontal adjacent spots in the calculation of the wavefront deformation for all spots. Southwell proposed an iterative solution where for any point (n,m) the wavefront is calculated with (1), (2) and

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(3) integrating from four neighboring points, one above, one below, one at the left, and one to the right of the point being considered, as it's illustrated in **Figure 1**. [3]

$$\phi_{n,m}^{k+1} = \left(\frac{\overline{\phi_{n,m}}}{g_{n,m}} \right)^{k} + \frac{S_{n,m}}{g_{n,m}} \quad (1)$$

$$S_{n,m}=g_{n-1,m}Sx_{n-1,m}- g_{n+1,m}Sx_{n+1,m} +g_{n,m-1}Sy_{n,m-1}- g_{n,m+1}Sy_{n,m+1} \quad (2)$$

$$\overline{\phi_{n,m}} = \phi_{n-1,m} g_{n-1,m} + \phi_{n+1,m} g_{n+1,m} + \phi_{n,m-1} g_{n,m-1} + \phi_{n,m+1} g_{n,m+1} \quad (3)$$

where $\overline{\phi_{n,m}}$ is the nearest-neighbor phase average; $S_{n,m}$ -slope measurement; $g_{n,m}$-matrix of weights defined for all spots.

If the left-hand side of (1) is held in a separate array until all points are evaluated, this procedure is the Jacobi method. If, however, the left hand side updates the wavefront array directly, such that succeeding evaluations of the right-hand side could use phase points that have already been updated, then the procedure is called the Gauss-Seidel method. Generally, the Gauss-Seidel method converges faster and is easier to implement. [4]

4.2. Successive Over-relaxation (SOR) and SOR with Simpson Iterator

None of these methods, however, updates a phase point based on the previous value at the point. By adding and subtracting $\phi_{n,m}^{k}$ on the right-hand side of the (1) and introducing the relaxation parameter ω (5), we get (4)

$$\phi_{n,m}^{k+1} = \phi_{n,m}^{k} + w\left(\left(\frac{\overline{\phi_{n,m}}}{g_{n,m}} \right)^{k} + \frac{S_{n,m}}{g_{n,m}} - \phi_{n,m}^{k} \right) \quad (4)$$

This iterative technique is the SOR method. The SOR method generally promises improvement in convergence, however, it is necessary to determine the value of the relaxation parameter ω which maximizes the rate of convergence. Fortunately the phase reconstruction problem belongs to the class of matrices for which the optimal value of ω is known. [4]

$$w = \frac{2}{1 + \sin\left(\frac{\pi}{N+1} \right)} \quad (5)$$

The further development of the SOR method is the use of the Simpson scheme to make differential equations. In this case, to calculate the phase of the point, values in 8 neighboring points are used. As it is shown on **Figure 2**.

The iteration formula for SOR will be modified to (6). More information about Simpson iterator, and how the equations for slope measurements and phase average are received, can be found in [5].

5. Wavefront Reconstruction using Described Algorithms

5.1. All Points are Known

Described iterative methods have been applied to the reconstruction of the simulated wavefront that contains the astigmatism (7) **Figure 3**.

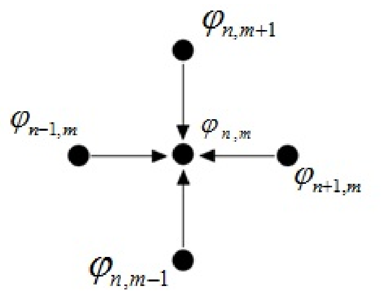

Figure 1. Wavefront calculation for point (n,m).

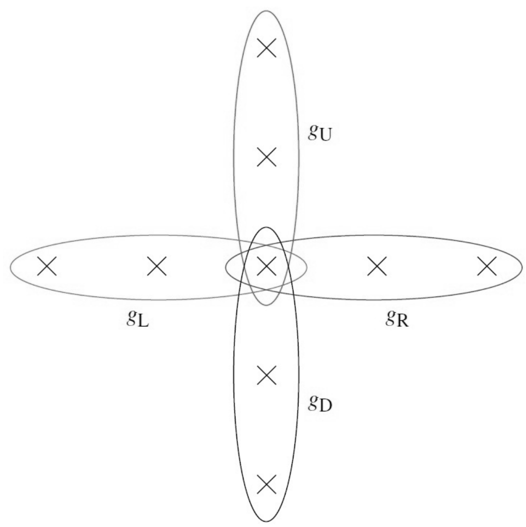

Figure 2. Simpson iterator geometry.

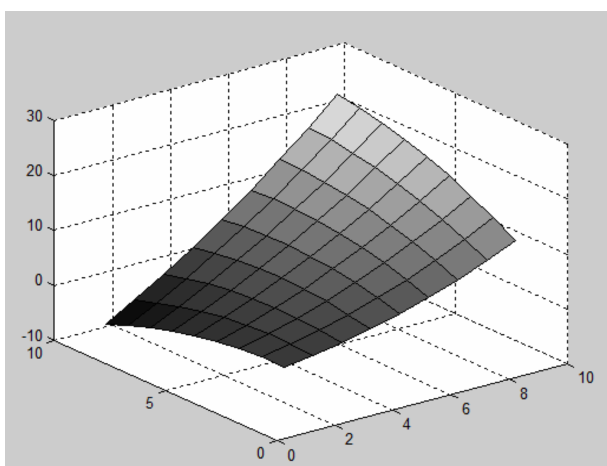

Figure 3. Simulated wavefront.

$$\phi_{n,m} = \frac{(\phi 0_{n,m} + \Delta S_{n,m} + \lambda \Delta \phi_{n,m})}{[(1+\lambda)(g_L + g_R + g_U + g_D) + 4\lambda(g_{LR} + g_{UD})]} \quad (6)$$

where λ-value less 0.08; g_L, g_R,g_U,g_D -are flags with values 0'or 1.; g_{LR},g_{UD} -is 1 only if two points exist.

$$\phi = 2.3717 \cdot \frac{x^2 - y^2}{(N/2)^2} + \frac{6xy}{(N/2)^2} \quad (7)$$

This wavefront was reconstructed using 4 described above methods for several values of iterations performed. As a parameter characterizing the degree of accuracy of reconstruction of the wave front was chosen the root mean square difference of the reconstructed wavefront from original one. According to the obtained values, it was concluded that the SOR method using Simpson scheme provides the greatest accuracy and speed of recovery.

Received data is given in the **Table 1**, reconstructed with SOR Simpson method wavefront is given on the **Figure 4**.

5.2. Case of Data Point Loss

Besides the behavior of the reconstruction algorithm under the conditions of absence for some reasons (for example due to speckle modulation [6]) of data point at one or more nodes of the array has been studied. In this case according to the SOR Simpson method features, each of four groups of points can be used in the equation only when all of its elements exist. This strategy is realized by using g parameters as shown in eq.(6). Even under such conditions method gives rather high accuracy of wavefront reconstruction. The contour plots received in case of all points existence **Figure 5** and in case of loss of one of the points **Figure 6** are given below.

Table 1. Received rms values.

Number of iterations	Iterative methods			
	Jacobi	*Gauss-Seidel*	*SOR*	*SOR Simpson*
16	0.4008	0.3340	0.1666	$4.78 * 10^{-5}$
64	0.3385	0.1822	0.0181	$6.28 * 10^{-16}$
128	0.2014	0.0717	0.0019	$7.06 * 10^{-16}$

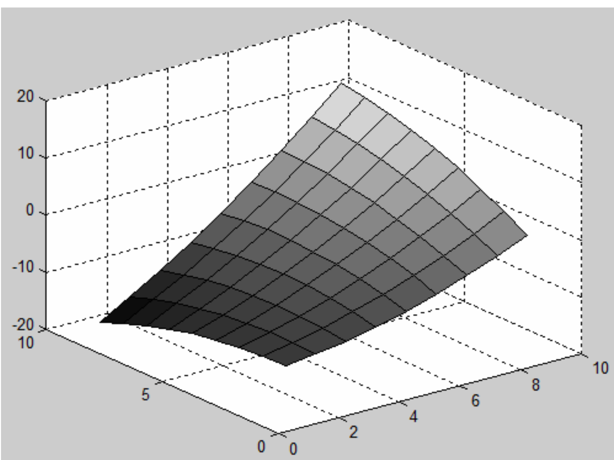

Figure 4. Reconstructed wavefront.

6. Reconstruction Method using Fourier Transform

In unit III we mentioned only two methods, that are used for wavefront estimation - zonal and modal, but still there is one more method that provides rather high level of accuracy. The Fourier method.

In [7] offered a method for using the 2D fast Fourier transform (FFT) twice, for acquiring both gradient field components from a Hartmann-Shack measurement.

The algorithm of such a method comprises of the following steps:

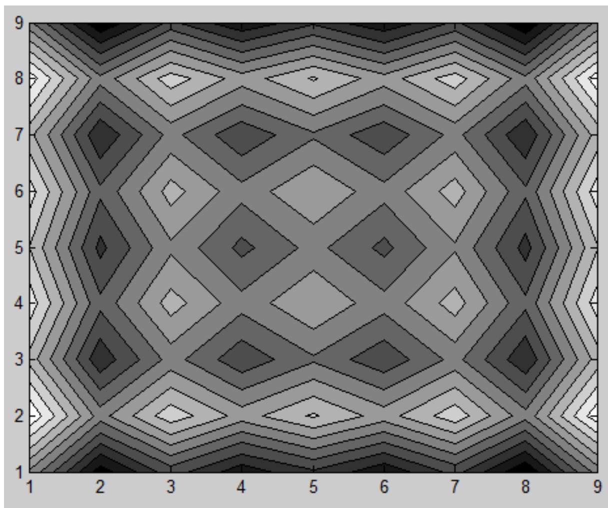

Figure 5. Contour plot in case when all points are known.

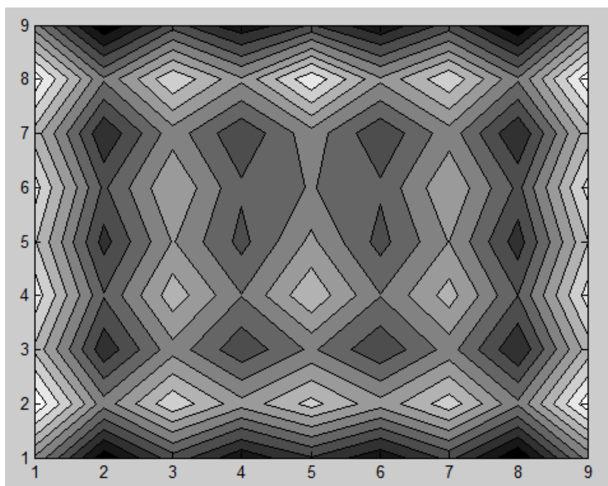

Figure 6. Contour plot in case when one point is lost.

- Zero-pad the image to twice the aperture size;
- Fourier transform the image;
- Multiply the transform by a band-pass filter around the x sidelobe;
- Center the result of the Fourier origin;
- Inverse transform the filter x lobe;
- Extract the phase of the result, to yield the x slope of the wavefront
- Repeat previous steps for y lobe;
- Reconstruct the phase from it's x and y slopes;

Expression for the irradiance function of the Hartmann-Shack pattern is written in the next form:

$$I(r) = 1/2\{2V(r) - C_x(r)e^{ik_x x} - C_x^*(r)e^{-ik_x x} - C_y(r)e^{ik_y y} - C_y^*(r)e^{-ik_y y}\} \quad (6)$$

$$C_x(r) = V(r)e^{-iF\varphi_x}, C_y(r) = V(r)e^{-iF\varphi_y} \quad (7)$$

where V(r) is the pattern amplitude at location r(x,y), and is assumed to be constant within the optical aperture, zero without. k_x and k_y are two k vector components: $k_x = 2\pi/P_x$ and $k_y = 2\pi/P_y$, where the lenslet array pitch is $P_x \times P_y$. F is the lenslet array focal length, and ϕ_x, ϕ_y are the phase x and y derivatives at r and are to be determined.

Than the Fourier transform of this expression is taken. And all described above steps are made.

This method considered to be rather accurate.

The comparison of two described above methods was held for the same simulated wavefront that contained astigmatism over a square aperture of area 17^2. Table II contains received rms values for different number of points where data don't exist.

After looking at the Table II it becomes obvious that SOR Simpson method provides higher level of accuracy then the FFT method. Even thou the RMS for SOR Simpson method provides non-obvious increase in accuracy for a small number of absent data point in comparison with RMS when all points are known. But this could be explained with features of Simpson method of compilation of differential equations.

7. Conclusions

A number of iterative methods of wavefront estimation were studied and data that reflects the level of accuracy of each method received. After having the results of numerical simulations analyzed it was concluded that the highest accuracy of reconstruction, the highest rate of convergence and resistance to the loss of some points of array only the SOR with Simpson scheme has. So it can be recommended for usage in conjunction with high resolution Shack-Hartmann sensors in variety of applications.

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