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An Exact Analysis of the Fundamental and First Higher Order Mode in Graded Index Fibers with Direct Power Series Method

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ABSTRACT

An exact and fast analytic method based on power series is established to predict the modal field distributions, Petermann-2 spot size, the normalized propagation constant corresponding to fundamental and first higher order mode in graded index fibers with any arbitrary power law profile. The variation of normalized cut-off frequencies of some LP_{lm} modes in graded index fibers with different profile exponents are also shown here and an empirical relation between them is determined.

Keywords: Power Series Method; Chebyshev Series Method; Petermann-2 Spot Size; Cut-Off Frequency

1. Introduction

First few modes of graded index fiber find some important applications in the field of optical communication in recent times. LP_{11} mode was used to design fiber sensors to separate temperature variation from longitudinal strain by intermodal interference [1]. Optical fiber mode filters in dual mode transmission system have also been developed using LP_{01}-LP_{11} modes [2]. As a wavelength filter, the LP_{01}-LP_{11} mode coupler offers much narrower spectral width than a coupler whose constituent fibers are single-mode. This coupler may also be used as a mode converter for dispersion compensation application [3]. The transmission characteristics of dual mode and dual polarization of CO-OFDM system are studied using LP_{01}-LP_{11} mode [4,5]. In recent times, there is considerable effort on developing few mode fibers to enhance the system capacity based on mode division multiplexing and multiple-input multiple-output digital signal processing [6-8].

Earlier, graded index fibers were analyzed numerically with beam propagation method [9], finite difference method [10], matrix methods [11] etc. to study their modal properties and various propagation characteristics. Beam propagation method, finite difference, finite element methods involve complex algorithms; moreover, these are inaccurate as one approaches cut-off region. Matrix methods are problematic for profiles with sharp index difference. In the literature, there are some approximate analytic methods also like variational analysis [12,13], Chebyshev power series method [14,15] etc. The analytic methods involve cumbersome algebraic calculations followed by some computation; due to approximations these methods are not so accurate. There are some works based on direct power series method [16,17] solution but those were confined to the fundamental mode only.

In this paper, our approach is to apply the power series method (PSM) in a most general way to solve the wave equation for graded index fiber. For step index fiber (SIF), the modal solutions of Helmholtz equation are Bessel's functions arising from recursion relations in power series method. We have shown that for graded index fiber also, one can find a simple series solution with the recursion relation slightly modified and the derived series is also convergent. Depending on the power exponent value, one part of the recursion relation is different; the rest is the same as in Bessel function.

We have solved scalar Helmholtz equation governing the power flow in a graded index waveguide by deriving general recursion relations for any mode, any arbitrary profile exponent and developing a simple algorithm. Since the current interest is in Two Mode Fibers where

LP_{01} and LP_{11} modes both exist, it is important to find the range within which these modes can exist. We have compared our data with one analytic method like Chebyshev Power Series (CPS) technique, as it is already established as an approximate analytic method for the calculation of different propagation characteristics of graded index fiber in LP_{01} and LP_{11} mode [14,15]. We found that Chebyshev technique results are in close accordance with our values. It is clear from our results and comparison of computation time with Chebyshev technique, that, PSM is one accurate, fast and simple method for the determination of modal fields and related quantities of a graded index fiber with any arbitrary geometric profile function.

We have studied modal field distributions, Petermann-2 spot size, normalized propagation constants, cut-off frequencies for LP_{01}-LP_{11} modes. Earlier works on cut-off frequency calculations based on matrix method [18], perturbation method [19] and variational method [20], are compared with our approach. The calculation of cut-off frequency by PSM shown in this paper proves to be an exact method in comparison to earlier methods.

In Section 2, we have shown the necessary recursion relations while solving the Helmholtz equation with PSM, incorporating the role of the profile exponent. Results are shown in Section 3. In Section 3.1, we have shown the modal field distribution of LP_{01} and LP_{11} modes for a particular normalized frequency and also presented the values of Petermann-2 spot size. In Section 3.2, the dependence of normalized propagation constant on normalized frequencies has been illustrated for LP_{01}-LP_{11} modes. In Section 3.3, we have shown the variation of normalized cut-off frequencies with profile exponent values using PSM. In Section 3.4, we have compared our LP_{11} mode cut-off frequency data with results existing in literature and in Section 3.5, we have derived a convenient empirical formula relating the above two quantities.

2. Theory

2.1. Modal Solution in an Optical Fiber

In the weakly-guiding approximation, Helmholtz Equation governing light propagation in an optical fiber is [21]

$$\frac{d^2\Psi}{dR^2}+\frac{1}{R}\frac{d\Psi}{dR}+a^2\left[k_0^2n^2(R)-\beta^2\right]\Psi-\frac{l^2}{R^2}\Psi=0 \quad (1)$$

where $R=r/a$, r is radial co-ordinate, a is core radius; $n(R)$ is the refractive index distribution in the fiber; $k_0=2\pi/\lambda$, λ is free space wavelength; β is the propagation constant; l is a parameter coming from the azimuthal part after separation of variables. Ψ is any transverse field component, either *E*-field or *H*-field.

In fibers with graded index core, $n(R)$ falls from n_1—the refractive index value on the axis of the fiber to n_2—the refractive index value at the core-cladding interface. Then

$$n^2(R)=n_1^2-\left(n_1^2-n_2^2\right)f(R) \quad (2)$$

$f(R)=R^q$ is called profile function. The exponent q in $f(R)$ gives the shape of the core index profile. The distribution $q=1$ is called triangular profile; $q=2$ is called parabolic profile. $q=\infty$ stands for uniform core with index n_1; when the cladding index is uniform (having the refractive index value n_2) as in our case, it is a step index fiber. The power or exponent q of R setting the profile shape is very important in fibers allowing more than one mode to propagate. It controls numerical aperture, intermodal dispersion, zero dispersion wavelength etc. in multimode fibers.

$n(R)$ is defined as in (2), (1) becomes

$$\frac{d^2\Psi}{dR^2}+\frac{1}{R}\frac{d\Psi}{dR}+\left[U^2-V^2f(R)\right]\Psi-\frac{l^2}{R^2}\Psi=0 \quad (3)$$

where $U=a\left(k_0^2n_1^2-\beta^2\right)^{1/2}$ and $V=k_0a\left(n_1^2-n_2^2\right)^{1/2}$. V is called normalized frequency and U is called normalized propagation constant. Equations (1) or (3) has two linearly independent solutions. For a particular l, the solutions are denoted as LP_{lm}.

The mode with highest β is the fundamental mode, denoted as LP_{01} mode. LP_{11} is the next higher order mode. Within the core, one has to find the solution of (3) with $f(R)=R^q$.

Outside the core *i.e.* in the cladding, $f(R)=1$; Equation (3) becomes

$$\frac{d^2\Psi}{dR^2}+\frac{1}{R}\frac{d\Psi}{dR}-W^2\Psi-\frac{l^2}{R^2}\Psi=0 \quad (4)$$

where $W^2=V^2-U^2$. The solution of (4) is standard modified Bessel function. In region I, that is within the core let the solution be $\Psi_{lm}(R)\big|_{I}$ and in region II, that is in the cladding, the solution is $\Psi_{lm}(R)\big|_{II}=K_{lm}(WR)$.

$\Psi_{lm}(R)$ satisfies the following boundary conditions at the interface between region I and region II that is at $R=1$:

$$\begin{aligned}\Psi_{lm}(R)\big|_{I}&=K_{lm}(WR)\big|_{II}\\ \Psi'_{lm}(R)\big|_{I}&=WK'_{lm}(WR)\big|_{II}\end{aligned} \quad (5)$$

2.2. Power Series Method

Using power series technique, $\Psi_I(R)$, the solution of (3) in region I, can be expanded in the following form

$$\Psi_I(R)=\sum_{n=0}^{\infty}a_nR^{k+n} \quad (6)$$

As in the solution of Bessel's Equation, we take $a_0\neq 0$, then $k=l$ and $a_1=0$.

For any q, $a_0 = 1/(2^l l!)$. Here we have used the same normalization factor used in the solution of Bessel's equation, because for $q = \infty$, the series must converge to Bessel function.

The recursion relation consists of two parts

$$1)\quad a_n = \frac{-U^2 a_{n-2}}{n(2l+n)}, n \le (q+1) \tag{7}$$

$$2)\quad a_n = \frac{-U^2 a_{n-2} + V^2 a_{n-2-q}}{n(2l+n)}, n > (q+1) \tag{8}$$

where $U^2 = V^2 - W^2$. Evidently, for even q, all the odd coefficients are zero. For odd q, the odd coefficients are zero up to $n = q$; all the coefficients are non-zero for $n > q$.

Using the standard recurrence relation for the modified Bessel function, one gets at $R = 1$:

$$\frac{\Psi_1'(R)}{\Psi_1(R)} = \frac{lK_l(W) - WK_{l+1}(W)}{K_l(W)} \tag{9a}$$

or

$$\frac{\Psi_1'(R)}{\Psi_1(R)} = \frac{-lK_l(W) + WK_{l-1}(W)}{K_l(W)} \tag{9b}$$

To find out the value of unknown W, the above equation is to be solved at the interface for a given value of V, starting with an initial value of W. Then the values of all coefficients are known from (7)-(8). Putting these values in (6) we get $\Psi(R)$:

$$\begin{aligned}\Psi(R) &= \sum_{n=0}^{\infty} a_n R^{k+n} \quad \text{for } R \le 1 \\ &= \left(\sum_{n=0}^{\infty} a_n\right)\frac{K_l(WR)}{K_l(W)} \text{ for } R > 1\end{aligned} \tag{10}$$

The Petermann-2 spot size is given by:

$$W_{P2} = 2\frac{\int_0^\infty \Psi^2(R) R\mathrm{d}R}{\int_0^\infty (\Psi'(R))^2 R\mathrm{d}R} \tag{11}$$

The normalized propagation constant is defined as

$$b = W^2/V^2 \tag{12}$$

The normalized cutoff frequency V_c is an important characteristic of a fiber as it demarcates between the range of one mode and its next higher mode. The cut-off condition of a mode is $U = V$ or $W = 0$.

Applying this condition in (9b) which gives proper limiting values of $K_l(W)$, one obtains

$$\frac{\Psi_1'(R)}{\Psi_1(R)} = -l \tag{13}$$

Using this relation, one can get the values of V_c for different q corresponding to different LP_{lm} modes.

3. Results and Discussions

Using PSM, we first solved the second order Helmholtz Equation (3) keeping l non-zero so as to get a general solution for any mode and considering different profile exponents both odd and even. Then using this solution we obtained different waveguide parameters like Petermann-2 spot size, normalized propagation constant, normalized cutoff frequency etc.

To find unknown W, we start with an initial value of W for a normalized frequency V and a particular profile exponent q. We evaluate the coefficients from recursion relation (7)-(8); that gives $\Psi(R)$ and $\Psi'(R)$ at $R = 1$. This is matched with $K(WR)$ at $R = 1$. It was observed that 500 terms are sufficient even for q as high as 200. However, we kept about 2000 terms in the series, matching is done with accuracy of the order of 10^{-8} and the run time is found to be less than a second.

In Section 3.1, we have shown the modal field distributions for LP_{01}-LP_{11} modes and determined Petermann-2 spot size which is a measure of the spread of the fields around the axis.

In Section 3.2, we have shown the variation of the normalized propagation constant b with normalized frequency V; this gives an idea about the dispersion in the fiber.

To find normalized cut-off frequencies, we evaluate $\Psi(R)$ and $\Psi'(R)$ at $R = 1$ using recursion relations (7)-(8) and putting $U = V$.

We have compared our results with those obtained by Chebyshev method already reported in our earlier paper [15]. It was shown there that the values of propagation characteristics of LP_{11} mode in parabolic and triangular index fibers obtained by CPS technique using four Chebyshev points closely matched with standard numerical values.

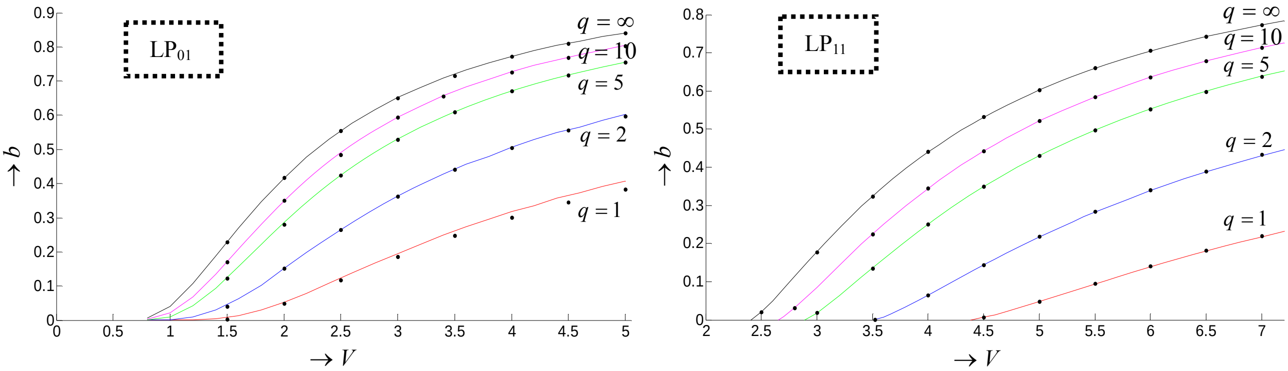
In our graphs, following line properties are used for different q values:

Solid red line (–) for PSM with $q = 1$;
Solid blue line (–) for PSM with $q = 2$;
Solid green line (–) for PSM with $q = 5$;
Solid magenta line (–) for PSM with $q = 10$;
Solid black line (–) for PSM with $q = \infty$.

Dots (•) represent the values obtained by Chebyshev power series method for the corresponding profile (in **Figures 1(a)** and **(b)**).

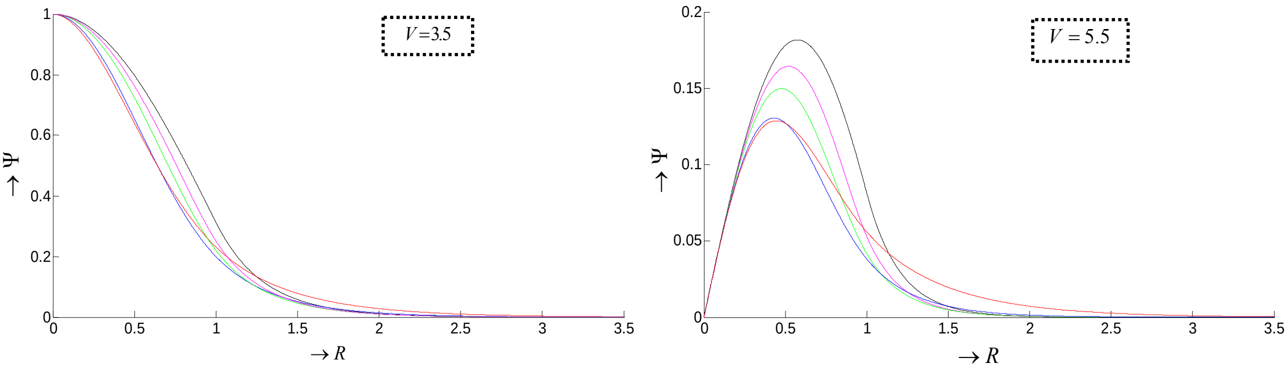
3.1. Modal Field Distributions of LP_{01} and LP_{11} Mode

The variations of fundamental and first higher order modal fields for various profile functions with normalized radius are shown in **Figures 2(a)** and **(b)**.



(a) (b)

Figure 1. (a) Variation of normalized propagation constant *b* with normalized frequency *V*: LP_{01} mode; (b) Variation of normalized propagation constant *b* with normalized frequency *V*: LP_{11} mode.



(a) (b)

Figure 2. (a) Variation of Ψ with *R* for LP_{01} mode for different profile exponents; (b) Variation of Ψ with *R* for LP_{11} mode for different profile exponents.

From the figures, it can be concluded that as the value of q increases the radial fields of both the modes fall more rapidly inside the core for lower values of q. For both the modes, the curves are found to spread outward in the cladding region as q value decreases.

The Petermann-2 spot size is one of the important characteristics of graded index fiber as it can be used to determine some quantities like splice loss etc.; it is obtained using the modal fields. **Tables 1(a)** and **(b)** show the values of W_{P2} for different q corresponding to LP_{01} and LP_{11} modes in graded index fiber.

Petermann-2 spot size decreases more rapidly with increase in V for lower q values. In LP_{01} mode, this parameter is almost independent of q at V = 3.5; for LP_{11} mode, it is almost independent at V = 6.

Petermann-2 spot size was determined with Chebyshev Power Series technique in [15] for LP_{11} mode; it was also compared with an accurate numerical calculation. Those results are very close to our results with Power Series technique in this paper. W_{P2} consistently falls with increasing V. However, results of [22] are quite contradictory; their W_{P2} values oscillates with V.

3.2. Variation of Normalized Propagation Constant with the Normalized Frequency

An important parameter that is connected with the normalized propagation constant b is the cut-off normalized frequency V; the condition b = 0 (*i.e.* when $\beta = k_0 n_2$) is known as cut-off of the mode. The study of b is also required to derive waveguide dispersion. **Tables 2(a)** and **(b)** and **Figures 1(a)** and **(b)** describe the variation of b with V for different values of profile exponent q including the step index case also. For a particular q, b increases gradually with V and for a particular value of V, it also increases with q.

It was also observed that even for q as high as 200; the profile is far from step index. To get the results for SIF with power series, one has to put $V = 0$ in (8) *i.e.* to convert the series to Bessel function.

3.3. Cut-Off Frequencies (V_c) for Different Profile Exponents (q) with PSM

Tables 3(a)-(d) show the cut-off V values for some LP_{lm} modes for different q. Applying PSM we have deter-

Table 1. (a) W_{P2}-V values for different profile exponents corresponding to LP_{01} mode with PSM; (b) W_{P2}-V values for different profile exponents corresponding to LP_{11} mode with PSM.

(a)

V	SIF	*q* = 10	*q* = 5	*q* = 2	*q* = 1
1.5	1.693311	1.786419	1.932361	2.607155	4.390023
2.5	1.053490	1.029151	1.026754	1.092383	1.316376
3.5	0.885940	0.844047	0.819666	0.808168	0.873235
4.5	0.808222	0.759407	0.725133	0.684378	0.700277

(b)

V	SIF	*q* = 10	*q* = 5	*q* = 2	*q* = 1
4.5	0.690709	0.678510	0.687513	0.774498	1.153140
5.0	0.659583	0.641401	0.642257	0.694095	0.883329
5.5	0.636086	0.613903	0.609247	0.639964	0.763316
6.0	0.617664	0.592592	0.583831	0.600094	0.688334

Table 2. (a) b-V values: Direct power series and Chebyshev power series values for different profile exponents corresponding to LP_{01} mode; (b) b-V values: Direct power series and Chebyshev power series values for different profile exponents corresponding to LP_{11} mode.

(a)

V	*b* × 10									
	q = 1		*q* = 2		*q* = 5		*q* = 10		SIF	
	PSM	CPS	PSM	CPS	PSM	CPS	PSM	CPS	PSM	CPS
1.5	0.08003	0.01978	0.44982	0.39781	1.23977	1.21801	1.70903	1.70616	2.29247	2.29363
2.0	0.52833	0.48246	1.51312	1.51521	2.85799	2.80607	3.48971	3.50600	4.16163	4.17226
2.5	1.22910	1.17283	2.65096	2.65315	4.24024	4.24106	4.90687	4.90635	5.53917	5.53937
3.0	1.95427	1.86334	3.63123	3.62508	5.29965	5.29614	5.93874	5.93676	6.51471	6.50876
3.5	2.48001	2.61020	4.42868	4.41503	6.09932	6.09548	6.70157	6.69554	7.21408	7.20588
4.0	3.00773	3.17750	5.07116	5.05030	6.70890	6.70673	7.27130	7.26328	7.72734	7.71876
4.5	3.66228	3.45257	5.55778	5.56318	7.18134	7.18160	7.70549	7.69641	8.11308	8.10499

(b)

V	*b* × 10									
	q = 1		*q* = 2		*q* = 5		*q* = 10		SIF	
	PSM	CPS	PSM	CPS	PSM	CPS	PSM	CPS	PSM	CPS
4.5	0.06315	0.06811	1.43795	1.44307	3.49219	3.49274	4.42809	4.43333	5.30955	5.31843
5.0	0.47897	0.48561	2.17288	2.18286	4.30371	4.30392	5.21179	5.21798	6.02413	6.03408
5.5	0.94260	0.95310	2.82141	2.83590	4.97320	4.97185	5.84482	5.85111	6.59208	6.60212
6.0	1.38940	1.40350	3.38566	3.40447	5.52854	5.52486	6.36082	6.36686	7.04917	7.05878
6.5	1.80248	1.81903	3.87569	3.89882	5.98919	5.98651	6.78579	6.78575	7.42163	7.42760
7.0	2.17860	2.19636	4.30259	4.33006	6.38564	6.37542	7.13940	7.14474	7.72867	7.73685

Table 3. (a) q-V_c values: Cut-off frequencies for LP_{0m} modes; (b) q-V_c values: Cut-off frequencies for LP_{1m} modes; (c) q-V_c values: Cut-off frequencies for LP_{2m} mode; (d) q-V_c values: Cut-off frequencies for LP_{3m} mode.

(a)

Mode	$q = \infty$	$q = 200$	$q = 10$	$q = 5$	$q = 2$	$q = 1$
LP_{01}	0	0	0	0	0	0
LP_{02}	3.8317	3.8508	4.1743	4.4394	5.0675	5.9483
LP_{03}	7.0155	7.0505	7.5912	8.0245	9.1576	10.773
LP_{04}	10.173	10.224	10.958	11.567	13.197	15.535

(b)

Mode	$q = \infty$	$q = 200$	$q = 10$	$q = 5$	$q = 2$	$q = 1$
LP_{11}	2.4048	2.4168	2.6492	2.8861	3.5180	4.3815
LP_{12}	5.5200	5.5477	6.0267	6.4271	7.4514	8.9330
LP_{13}	8.6537	8.6969	9.3781	9.9396	11.425	13.575
LP_{14}	11.791	11.850	12.718	13.449	15.408	18.248

(c)

Mode	$q = \infty$	$q = 200$	$q = 10$	$q = 5$	$q = 2$	$q = 1$
LP_{21}	3.8317	3.8510	4.2429	4.6534	5.7439	7.2180
LP_{22}	7.0155	7.0508	7.6758	8.2127	9.6450	11.715
LP_{23}	10.173	10.225	11.042	11.733	13.590	16.301
LP_{24}	13.324	13.390	14.390	15.244	17.555	20.930

(d)

Mode	$q = \infty$	$q = 200$	$q = 10$	$q = 5$	$q = 2$	$q = 1$
LP_{31}	5.1356	5.1616	5.7139	6.3008	7.8475	9.9188
LP_{32}	8.4172	8.4598	9.2287	9.9079	11.760	14.415
LP_{33}	11.620	11.678	12.632	13.455	15.702	18.981
LP_{34}	14.796	14.870	16.001	16.982	19.661	23.588

mined the exact values of V_c for any arbitrary profile exponent. In our previous work with CPS technique [15] V_c (LP_{11}) was obtained as 3.5180 for $q = 2$ and 4.3816 for $q = 1$. Some of our results of V_c are compared with earlier published results [18-20].

3.4. Comparison of Cut-Off Frequency Data

In this section, we compare our cut-off frequency results of LP_{11} mode for different q with the results existing in literature.

From all these data in **Table 4**, it is evident that this direct power series method provides accurate results for all power-law profiles in graded index fiber.

3.5. Empirical Fit of q-V_c Data

We have tried an empirical form of the following type to

Table 4. q-V_c values: Comparison of cut-off frequencies for LP_{11} modes Ref. [18]-Matrix method; Ref. [19]-Perturbation method; Ref. [20]-Variational method.

q	V_c			
	Our results	Ref. [18]	Ref. [19]	Ref. [20]
1	4.3815	4.381	4.390	4.572
2	3.5180	3.518	3.518	3.613
3	3.1808	3.181	3.184	3.238
5	2.8861	2.886	2.900	2.910
10	2.6492	2.650	2.686	2.650
20	2.5268	2.529	2.583	2.520
∞	2.4048	2.405	2.487	2.397

describe the nature of dependence of V_c on q,

$$V_c\left(q\right)=\sum_{m=1}^{N}\frac{a_m}{q^{m-1}} \quad (13)$$

N is the number of parameters used. We have fitted q-V_c data keeping three and four parameters respectively. The results of different fits are shown in **Tables 5(a)** and **(b)** and **Figures 3(a)** and **(b)**.

We have evaluated the values of V_c using the empirical form (13) considering three and four parameters separately for two LP_{lm} modes taking different q. From **Tables 5(a)** and **(b)**, it is clear that the results obtained fit best with three parameters for LP_{11} mode and with four parameters for LP_{02} mode.

For LP_{11} mode:

Three parameter fit: a_1 = 2.403844, a_2 = 2.0504123, a_3 = −0.527509; maximum error = 0.1714%

For LP_{02} mode:

Four parameter fit: a_1 = 3.831020, a_2 = 3.788971, a_3 = −3.848937, a_4 = 2.180391; maximum error = 0.6252%

The error in this calculation of V_c using (13) lies within 0.2% in case of LP_{11} and within 0.7% in case of LP_{02}, which is far better in comparison to previous works [17,18]. Moreover, from **Figures 3(a)** and **(b)** it is clear that the estimated values of V_c fit fairly well with those obtained from PSM. So, relation (13) can be considered to give a reasonable estimate of V_c for different q corresponding to different LP_{lm} modes.

4. Conclusion

We have established a general power series method for graded index fibers with a power law profile. It is based on modification of the recursion relations in power series solution of Bessel's equation and an algorithm to find the co-efficients of the series. Finding the solution comes down to constructing the series with only a simple summation. Here the method has been employed to calculate the propagation characteristics and the cut-off frequencies for graded index fibers of different core index distributions. This method is, no doubt, an accurate one—it has no approximations. It does not pose problems near cut-off or in case of sharp discontinuity. The only care

Table 5. (a) q-V_c values: Cut-off frequencies of LP_{11} mode for different q; (b) q-V_c values: Cut-off frequencies for LP_{02} mode for different q.

(a)

q	V_c (PSM)	3-parameter		4-parameter	
		V_c (empirical)	Error %	V_c (empirical)	Error %
200	2.4168	2.4164	0.0185	2.4215	0.1956
30	2.4859	2.4867	0.0334	2.4876	0.0691
10	2.6492	2.6490	0.0082	2.6434	0.2185
5	2.8861	2.8836	0.0877	2.8763	0.3391
3	3.1808	3.1799	0.0270	3.1810	0.0055
2	3.5180	3.5240	0.1714	3.5429	0.7079
1	4.3815	4.3805	0.0238	4.3768	0.1075

(b)

q	V_c (PSM)	3-parameter		4-parameter	
		V_c (empirical)	Error %	V_c (empirical)	Error %
200	3.8508	3.8722	0.5565	3.8499	0.0242
30	3.9559	3.9563	0.0091	3.9531	0.0702
10	4.1743	4.1481	0.6270	4.1736	0.0166
5	4.4394	4.4205	0.4250	4.4523	0.2906
3	4.7357	4.7550	0.4076	4.7471	0.2408
2	5.0675	5.1269	1.1726	5.0358	0.6252
1	5.9483	5.9348	0.2269	5.9514	0.0529

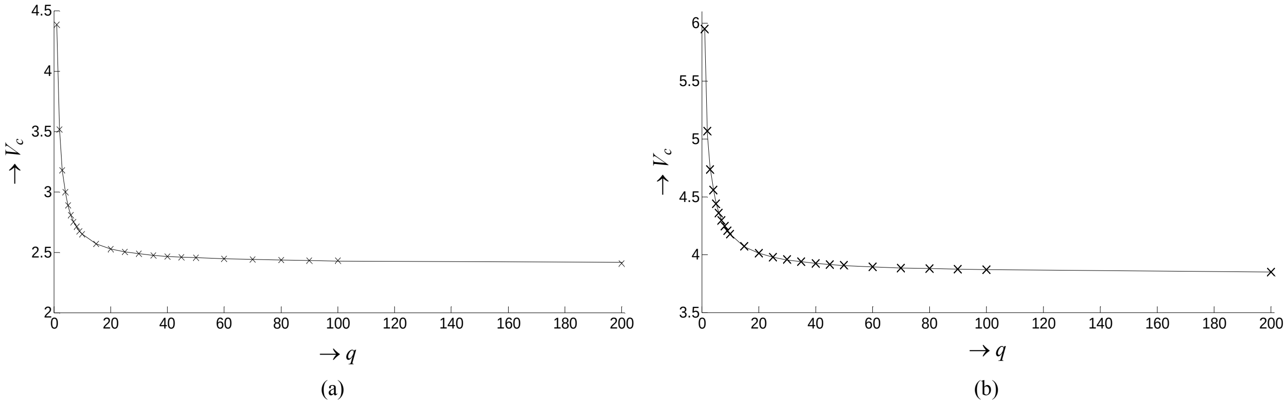


Figure 3. (a) Variation of cut-off frequencies for LP_{11} modes. Cross: [q-V_c] data; Solid line: 3-parameter; (b) Variation of cut-off frequencies for LP_{02} modes. Cross: [q-V_c] data; Solid line: 4-parameter.

one has to take is the convergence of the series for different q value. As q increases, number of terms to be kept in the series expansion required for convergence increases.

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