

Properties of Time-Varying Causality Tests in the Presence of Multivariate Stochastic Volatility*

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Abstract

This paper compares the statistical properties of time-varying causality tests when errors of variables have multivariate stochastic volatility (SV). The time-varying causality tests in this paper are based on a logistic smooth transition autoregressive model. The compared time-varying causality tests include asymptotic tests, heteroskedasticity-robust tests, and tests using wild bootstrap. Our simulation results show that asymptotic tests and heteroskedasticity-robust counterparts have size distortions under multivariate SV, whereas tests using wild bootstrap have better size properties regardless of type of error. In particular, the time-varying causality test with first-order Taylor approximation using wild bootstrap has better statistical properties.

Keywords

Time-Varying Causality Tests, Wild Bootstrap, Multivariate Stochastic Volatility

1. Introduction

Granger causality is one of most representative methods to analyze causality between economic variables. It is based on linear vector autoregressive (VAR) models and investigates whether past information is effective for prediction. Although Granger causality is used for various studies, it can be applied to examine only stable linear relationships in the long run. The relationship between economic variables is not necessarily stable in the long run and frequently has time-varying properties. This implies that a causality relationship can also be time-varying, and hence we should take into account the time-varying properties when analyzing a causality relationship.

One method to introduce time-varying properties to Granger causality is through the

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use of a logistic smooth transition (LST) function. By using an LST function with time as the transition variable, we can test for both smooth and abrupt causalities. When a causality has such nonlinearity, the usual Granger causality tests based on a linear VAR model have low power and tend to give the misleading result of having no causalities in the system. [1] [2] and [3] proposed nonlinear causality tests. Their analyses also showed significant nonlinear causality.

While time-varying causality is significant for the precise analysis of variables, heteroskedastic variances influence the tests for causality and nonlinearity such as time-varying properties. For example, [4] provided Monte Carlo evidence that causality tests have size distortions under heteroskedastic variances. In addition, [5] and [6] showed that heteroskedastic variances lead to spurious nonlinearity. Several economic variables investigated using Granger causality have heteroskedastic variances such as stochastic volatility (SV) (e.g., [7] and [8]). Therefore, if we do not deal appropriately with heteroskedastic variances in the tests for causality, we would not be able to obtain reliable results when examining for time-varying causality. However, previous studies have not clarified the influences of heteroskedastic variances on time-varying causality tests.

This paper investigates the statistical properties of time-varying causality tests when the disturbance terms have SV. The investigated tests include asymptotic tests based on first-order and third-order Taylor approximation and their counterparts with the heteroskedasticity-consistent covariance matrix estimators (HCCME) as introduced by [9]. As pointed out by [10], the order of Taylor approximation affects the performance of linearity tests. We reveal the impact of the order of Taylor approximation on time-varying causality tests in the presence of SV. We also examine the time-varying causality tests using wild bootstrap. Wild bootstrap was proposed by [11] and replicates a sampling that does not depend on the form of heteroskedastic variances. [12] and [13] examine the properties of tests using wild bootstrap. We show which tests perform well even under SV by analyzing the size and power of the tests.

Our simulation results provide evidence that asymptotic time-varying causality tests and their counterparts with HCCME over-reject the null hypothesis of no causality in the presence of SV. This implies that their tests tend to yield misleading and unreliable results. In particular, their tests based on third-order Taylor approximation have larger distortions than those based on first-order Taylor approximation. In contrast, we find that time-varying causality tests using wild bootstrap have reasonable empirical sizes and sufficient power. The results of this paper would enable appropriate and reliable time-varying causality tests.

The rest of this paper is organized as follows. Section 2 presents time-varying causality tests. Section 3 provides the size and power properties of tests. Finally, Section 4 concludes the paper.

2 Time-Varying Causality Tests

We consider the following bivariate vector autoregressive system to test for time-varying causality relationship.

$$y_t = \alpha_{01} + \alpha_{11}y_{t-1} + (\beta_0 + \beta_1x_{t-1})F(\gamma, t, c) + u_{1t}, \quad (1)$$

$$x_t = \alpha_{02} + \alpha_{12}x_{t-1} + u_{2t}, \quad (2)$$

where u_{1t} and u_{2t} are zero mean errors and $F(\gamma, t, c)$ is a logistic smooth transition function to model time-varying causality. The transition function $F(\gamma, t, c)$ can be given by

$$F(\gamma, t, c) = \frac{1}{1 + \exp\{-\gamma(t-c)\}} - 1/2, \quad (3)$$

where γ is a parameter determining the function's smoothness, t is a transition variable, and c is the point where a regime changes from one to another. We assume that $\gamma > 0$, $t > 0$, and $c > 0$. The system has a causality from x_t to y_t when $\beta_1 \neq 0$ and $F(\gamma, t, c) \neq 0$. $\beta_0 \neq 0$ means a time-varying constant. When $t = c$, the causality from x_t to y_t does not appear because $F(\gamma, t, c)$ becomes zero. The value of the logistic smooth transition function is bounded between $-1/2$ and $1/2$. When $t < c$, we have $\frac{1}{1 + \exp\{-\gamma(t-c)\}} < 1/2$ and $-1/2 < F(\gamma, t, c) < 0$. Meanwhile, when $t > c$, we have $\frac{1}{1 + \exp\{-\gamma(t-c)\}} > 1/2$ and $0 < F(\gamma, t, c) < 1/2$. $F(\gamma, t, c)$ moves toward $-1/2$ when $t < c$ and small $\gamma(t-c)$, and toward $1/2$ when $t > c$ and large $\gamma(t-c)$. The causality is time-varying when depending on the value of $F(\gamma, t, c)$. In addition, the time-varying causality using $F(\gamma, t, c)$ includes abrupt structural changes of causality because $F(\gamma, t, c)$ is the indicator function taking the value of $-1/2$ or $1/2$ when $\gamma = \infty$.

The null and alternative hypotheses to test for time-varying causality in the system are

$$H_0 : \gamma = 0, \quad H_1 : \gamma > 0. \quad (4)$$

If $\gamma = 0$, Equation (1) has no causality from x_t to y_t . However, the test is not simple and easy because the null hypothesis has an identification problem about β_0 and β_1 . They are identified only under the alternative hypothesis with $\gamma > 0$. The identification problem was considered by [14] and [15]. To conduct the test in the presence of the identification problem, [16] proposed a Taylor series approximation. We use first-order and third-order Taylor series approximation around $\gamma = 0$ because the performance of the tests depends on the order of Taylor series approximation (e.g., [10]).

The regression models for (1) using the first-order and third-order Taylor series approximation are given by

$$\text{First-order : } y_t = a_0 + a_1y_{t-1} + b_0t + b_1tx_{t-1} + e_t, \quad (5)$$

$$\text{Third-order : } y_t = a_0 + a_1y_{t-1} + c_{10}t + c_{11}tx_{t-1} + c_{20}t^2 + c_{21}t^2x_{t-1} + c_{30}t^3 + c_{31}t^3x_{t-1} + e_t, \quad (6)$$

where e_t is an error term including a remainder term of Taylor series approximation. $\gamma = 0$ implies $b_0 = b_1 = 0$ in (5) or $c_{10} = c_{11} = c_{20} = c_{21} = c_{30} = c_{31} = 0$ in (6). Since we cannot test for $\gamma = 0$ directly, we instead test for $b_0 = b_1 = 0$ or for

$c_{10} = c_{11} = c_{20} = c_{21} = c_{30} = c_{31} = 0$. Denoting \mathbf{a} , \mathbf{b} , and \mathbf{c} as $\mathbf{a} = (a_0, a_1)'$, $\mathbf{b} = (b_0, b_1)'$, and $\mathbf{c} = (c_{10}, c_{11}, c_{20}, c_{21}, c_{30}, c_{31})'$, (5) and (6) can be rewritten respectively as

$$\text{First-order : } y_t = \mathbf{a}'\mathbf{y}_t + \mathbf{b}'\mathbf{x}_{bt} + e_t, \tag{7}$$

$$\text{Third-order : } y_t = \mathbf{a}'\mathbf{y}_t + \mathbf{c}'\mathbf{x}_{ct} + e_t, \tag{8}$$

where $\mathbf{y}_t = (1, y_{t-1})'$, $\mathbf{x}_{bt} = (t, tx_{t-1})'$, and $\mathbf{x}_{ct} = (t, tx_{t-1}, t^2, t^2x_{t-1}^2, t^3, t^3x_{t-1}^3)'$.

Testing for time-varying causality is expressed as

$$\text{First-order : } H_0 : \mathbf{b} = 0, \quad H_1 : \mathbf{b} \neq 0, \tag{9}$$

$$\text{Third-order : } H_0 : \mathbf{c} = 0, \quad H_1 : \mathbf{c} \neq 0. \tag{10}$$

The Wald statistics to test for time-varying causality are derived as

$$\text{First-order : } F1 = \frac{1}{\hat{\sigma}^2} \hat{\mathbf{b}}' \left[R_1 \left(\sum_{t=1}^T \mathbf{Y}_{1t} \mathbf{Y}_{1t}' \right)^{-1} R_1' \right]^{-1} \hat{\mathbf{b}}, \tag{11}$$

$$\text{Third-order : } F3 = \frac{1}{\hat{\sigma}^2} \hat{\mathbf{c}}' \left[R_3 \left(\sum_{t=1}^T \mathbf{Y}_{3t} \mathbf{Y}_{3t}' \right)^{-1} R_3' \right]^{-1} \hat{\mathbf{c}}, \tag{12}$$

where $\mathbf{Y}_{1t} = (\mathbf{y}_t', \mathbf{x}_{bt}')'$ and $\mathbf{Y}_{3t} = (\mathbf{y}_t', \mathbf{x}_{ct}')'$, $\hat{\mathbf{b}}$ and $\hat{\mathbf{c}}$ are estimates of \mathbf{b} and \mathbf{c} , and $\hat{\sigma}^2$ is the estimate of the residual variance in each regression. R_1 and R_3 are matrixes that satisfy $R_1 \mathbf{d}_1 = \mathbf{b}$ and $R_3 \mathbf{d}_3 = \mathbf{c}$, where $\mathbf{d}_1 = (\mathbf{a}', \mathbf{b}')'$ and $\mathbf{d}_3 = (\mathbf{a}', \mathbf{c}')'$. Under the null hypothesis of no time-varying causality, (11) and (12) follow F distributions with degrees of freedom $(2, T - 2)$ and $(6, T - 6)$, respectively. When we use HCCME for statistics (11) and (12), they are given by

$$\text{First-order : } HC1 = \hat{\mathbf{b}}' \left[R_1 \left(\sum_{t=1}^T \mathbf{Y}_{1t} \mathbf{Y}_{1t}' \right)^{-1} \left(\sum_{t=1}^T \hat{e}_t^2 \mathbf{Y}_{1t} \mathbf{Y}_{1t}' \right) \left(\sum_{t=1}^T \mathbf{Y}_{1t} \mathbf{Y}_{1t}' \right)^{-1} R_1' \right]^{-1} \hat{\mathbf{b}}, \tag{13}$$

$$\text{Third-order : } HC3 = \hat{\mathbf{c}}' \left[R_3 \left(\sum_{t=1}^T \mathbf{Y}_{3t} \mathbf{Y}_{3t}' \right)^{-1} \left(\sum_{t=1}^T \hat{e}_t^2 \mathbf{Y}_{3t} \mathbf{Y}_{3t}' \right) \left(\sum_{t=1}^T \mathbf{Y}_{3t} \mathbf{Y}_{3t}' \right)^{-1} R_3' \right]^{-1} \hat{\mathbf{c}}, \tag{14}$$

where \hat{e}_t represents the residual in each regression. Statistics (13) and (14) using HCCME asymptotically have the same distributions as (11) and (12).

Wild bootstrap is also used for regression models with heteroskedastic variances to obtain reliable results. The method can simply resample heteroskedastic variances like SV. This paper employs the recursive-design wild bootstrap. The testing procedure is as follows.

Step 1. Compute test statistics (11) and (12) by applying (7) and (8) to the data.

Step 2. Estimate the system using the restricted model with $\mathbf{b} = 0$ in (7) and $\mathbf{c} = 0$ in (8) and obtain the estimate of \mathbf{a} and residuals denoted as \hat{e}_{rt} .

Step 3. Obtain the estimates $\hat{\alpha}_{02}$ and $\hat{\alpha}_{12}$ and the residual \hat{u}_{2t} , where \hat{u}_{2t} is the residual of (2).

Step 4. Generate the bootstrapped sample as

$$y_t^* = \hat{\mathbf{a}}' \mathbf{y}_t^* + e_{yt}^*, \quad (15)$$

$$x_t^* = \hat{\alpha}_{02} + \hat{\alpha}_{12} x_{t-1}^* + e_{xt}^*, \quad (16)$$

where $\hat{\mathbf{a}}$ is the estimate of \mathbf{a} in (7) or (8), $e_{yt}^* = \varepsilon_{yt} \hat{e}_{yt}$, $e_{xt}^* = \varepsilon_{xt} \hat{u}_{2t}$, and ε_{yt} and ε_{xt} are i.i.d. $N(0, 1)$. y_t^* and x_t^* are data generated recursively.

Step 5. Compute test statistics (11) and (12), denoted as WB1 and WB3, by applying (7) and (8) to the generated bootstrap sample.

Step 6. Repeat the bootstrap iterations M for steps 4 and 5. We obtain M statistics WB1 and WB3.

Step 7. Compute the bootstrap p -values as follows:

$$\text{First-order: } P(\text{WB1}) = \frac{1}{M} \sum_{m=1}^M I(\text{WB1} > F1), \quad (17)$$

$$\text{Third-order: } P(\text{WB3}) = \frac{1}{M} \sum_{m=1}^M I(\text{WB3} > F3), \quad (18)$$

$I(\cdot)$ is an indicator function such that $I(\cdot)$ is 1 if (\cdot) is true and 0 otherwise. The null hypothesis is rejected if the p -value is smaller than a significant level.

3. Size and Power Properties

This section conducts Monte Carlo simulations to compare the size and power properties of causality tests under multivariate SV. The nominal size of the tests is 0.05, and we consider sample sizes $T = 200$ and 400. Causality tests using wild bootstrap have 1000 bootstrap replications. The number of replications of simulations for all the tests is 10,000. We generate data with $T + 100$ and use the data with sample size T . The initial 100 samples are discarded to avoid the effect of initial conditions. We denote the tests compared in this section as F1, F3, HC1, HC3, WB1, and WB3; we also denote the linear Granger causality test as F0, its test with HCCME as HC0, and its test using wild bootstrap as WB0.

We first investigate the size properties based on data generating process (DGP) given as

$$y_t = 1 + \alpha y_{t-1} + u_{1t}, \quad (19)$$

$$x_t = 1 + 0.5x_{t-1} + u_{2t}, \quad (20)$$

where u_{1t} and u_{2t} are error terms. We set u_{1t} and u_{2t} with normal error to the following.

$$\begin{pmatrix} u_{1t} \\ u_{2t} \end{pmatrix} = N \left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix} \right). \quad (21)$$

The correlation parameter ρ between u_{1t} and u_{2t} is set to $\rho = 0$ and $\rho = 0.5$. DGP do not have any causality from x_t to y_t in the system from (19) to (21).

Table 1(a) presents the size properties of tests for normal error. We investigate two cases of $\alpha = 0.2$ and $\alpha = 0.8$. The results in **Table 1(a)** indicate that for all the tests, the correlation parameter ρ does not have any influence on size. Linear Granger

causality test F0 and its time-varying causality version F1 perform well regardless of the value of α . Their rejection frequencies are near the nominal size 0.05. However, we find that F3 has small over-rejections. For example, the size of F3 for $\rho = 0$, $\alpha = 0.8$, and $T = 200$ is 0.090. Compared with the results between $\alpha = 0.2$ and $\alpha = 0.8$, F3 has a larger over-rejection for $\alpha = 0.8$ than for $\alpha = 0.2$. The high persistence of y_t affects the empirical size of F3. The property that F3 has additional regression parameters may lead to size distortions. We find that HC0, HC1, and HC3 perform worse than F0, F1, and F3. The rejection frequencies are larger than 0.05. In particular, the rejection frequency of HC3 is more than 0.1. These results imply that causality tests with HCCME are not useful under normal error. In contrast, the empirical sizes of WB0, WB1, and WB3 using wild bootstrap are close to the nominal size 0.05. While WB3 has small under-rejections, small size distortions are acceptable. Causality tests with wild bootstrap have reasonable empirical sizes.

We next examine the empirical sizes of tests under multivariate SV. The property of SV is that volatility is influenced by an error and changes stochastically. Multivariate SV allows for the correlation between errors of volatilities. u_{1t} and u_{2t} in (21) with SV are generated by

$$\begin{pmatrix} u_{1t} \\ u_{2t} \end{pmatrix} = \Omega_t \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix}, \tag{22}$$

where $\varepsilon_{it} \sim \text{i.i.d.}N(0,1)$ and

$$\Omega_t = \begin{pmatrix} \exp(h_{1t}/2) & 0 \\ 0 & \exp(h_{2t}/2) \end{pmatrix}. \tag{23}$$

Here, h_{1t} and h_{2t} are given by

$$h_{1t} = \phi_1 h_{1t-1} + \kappa_{11} u_{1t-1} + \kappa_{12} |u_{1t-1}| + \eta_{1t}, \tag{24}$$

$$h_{2t} = \phi_2 h_{2t-1} + \kappa_{21} u_{1t-2} + \kappa_{22} |u_{1t-2}| + \eta_{2t}, \tag{25}$$

where

$$\begin{pmatrix} \eta_{1t} \\ \eta_{2t} \end{pmatrix} = N \left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix} \right). \tag{26}$$

The regression parameter α in (19) is set to 0.2. ϕ_1 and ϕ_2 describe the size of volatility persistence. High ϕ_1 and ϕ_2 indicate persistent volatility. κ_{ij} is a parameter to determine the asymmetry of SV. While asymmetric volatility has $\kappa_{ij} \neq 0$, symmetric multivariate volatility has the restriction of all $\kappa_{ij} = 0$. For example, the volatility with $\kappa_{11} < 0$ and $\kappa_{21} > 0$ increases when u_{1t} is minus. The multivariate stochastic model is based on [17] and [18].

Table 1(b) presents the size performance of tests under symmetric multivariate SV with $(\phi_1, \phi_2) = (0.2, 0.2)$ and $(0.8, 0.8)$. All κ_{ij} are set to zero in (24) and (25). Multivariate SV clearly leads to over-rejection. For example, the empirical sizes of F1, F3, HC1, and HC3 for $\alpha = 0.2$, $\rho = 0$, $\phi_1 = \phi_2 = 0.8$, and $T = 200$ are respectively 0.059, 0.090, 0.078, and 0.153 in **Table 1(a)** but 0.088, 0.127, 0.129, and 0.225 in **Table 1(b)**.

In addition, we observe that the correlation between errors affects the size performance of F0, F1, and F3. When compared with the empirical sizes of F0, F1, and F3 for

Table 1. (a) Size properties under normal error; (b) Size properties under symmetric multivariate stochastic volatility; (c) Size properties under asymmetric multivariate stochastic volatility.

(a)									
	F0	F1	F3	HC0	HC1	HC3	WB0	WB1	WB3
$\rho = 0$									
$\alpha = 0.2$									
$T = 200$	0.046	0.053	0.066	0.068	0.071	0.138	0.048	0.042	0.030
$T = 400$	0.051	0.048	0.063	0.063	0.065	0.112	0.050	0.043	0.032
$\alpha = 0.8$									
$T = 200$	0.049	0.059	0.090	0.074	0.078	0.153	0.046	0.042	0.031
$T = 400$	0.048	0.062	0.076	0.061	0.071	0.123	0.053	0.040	0.033
$\rho = 0.5$									
$\alpha = 0.2$									
$T = 200$	0.043	0.053	0.061	0.068	0.074	0.135	0.052	0.042	0.032
$T = 400$	0.046	0.046	0.055	0.060	0.067	0.108	0.050	0.045	0.032
$\alpha = 0.8$									
$T = 200$	0.051	0.059	0.095	0.068	0.071	0.151	0.047	0.040	0.030
$T = 400$	0.051	0.059	0.076	0.060	0.069	0.120	0.048	0.044	0.028
(b)									
	F0	F1	F3	HC0	HC1	HC3	WB0	WB1	WB3
$\rho = 0$									
$\phi_1 = \phi_2 = 0.2$									
$T = 200$	0.048	0.054	0.072	0.075	0.074	0.145	0.050	0.041	0.027
$T = 400$	0.049	0.052	0.067	0.069	0.071	0.119	0.052	0.047	0.031
$\phi_1 = \phi_2 = 0.8$									
$T = 200$	0.050	0.088	0.127	0.109	0.129	0.225	0.049	0.068	0.062
$T = 400$	0.050	0.086	0.125	0.108	0.134	0.235	0.052	0.073	0.066
$\rho = 0.5$									
$\phi_1 = \phi_2 = 0.2$									
$T = 200$	0.057	0.053	0.068	0.076	0.073	0.148	0.053	0.041	0.029
$T = 400$	0.063	0.055	0.066	0.066	0.064	0.114	0.056	0.049	0.033
$\phi_1 = \phi_2 = 0.8$									
$T = 200$	0.131	0.140	0.203	0.083	0.124	0.235	0.039	0.058	0.053
$T = 400$	0.160	0.159	0.229	0.067	0.111	0.223	0.041	0.064	0.047

(c)

	F0	F1	F3	HC0	HC1	HC3	WB0	WB1	WB3
$\rho = 0$									
$\phi_1 = \phi_2 = 0.2$									
$T = 200$	0.049	0.053	0.066	0.062	0.068	0.134	0.048	0.039	0.028
$T = 400$	0.050	0.049	0.063	0.065	0.061	0.108	0.049	0.049	0.032
$\phi_1 = \phi_2 = 0.8$									
$T = 200$	0.051	0.066	0.093	0.077	0.092	0.190	0.051	0.051	0.041
$T = 400$	0.054	0.071	0.088	0.043	0.091	0.166	0.053	0.057	0.051
$\rho = 0.5$									
$\phi_1 = \phi_2 = 0.2$									
$T = 200$	0.038	0.044	0.045	0.062	0.063	0.114	0.046	0.044	0.028
$T = 400$	0.036	0.043	0.046	0.067	0.064	0.100	0.050	0.049	0.037
$\phi_1 = \phi_2 = 0.8$									
$T = 200$	0.072	0.078	0.099	0.067	0.095	0.189	0.045	0.057	0.041
$T = 400$	0.076	0.083	0.097	0.061	0.084	0.157	0.046	0.054	0.046

$\phi_1 = \phi_2 = 0.8$ and $\rho = 0$, they have larger over-rejections for $\phi_1 = \phi_2 = 0.8$ and $\rho = 0.5$. This is different from the results in **Table 1(a)**. Thus, the correlation between errors increases the over-rejections when the errors have multivariate SV. The results imply that multivariate SV causes size distortions in time-varying causality tests. Possibly, they provide misleading results that indicate a time-varying causality relationship. However, note that WB0, WB1, and WB3 perform better even under SV. The empirical sizes of WB0, WB1, and WB3 for $\phi_1 = \phi_2 = 0.8$, $\rho = 0.5$, and $T = 200$ are 0.039, 0.058, and 0.053, respectively.

Asymmetric multivariate SV also results in size distortions for causality tests. We set κ_{11} and κ_{21} to $\kappa_{11} = \kappa_{21} = -0.5$ and κ_{12} and κ_{22} to $\kappa_{12} = \kappa_{22} = 0.3$. From **Table 1(c)**, while HC0, HC1, and HC3 have larger rejection frequencies than F0, F1, and F3, their size distortions are smaller than those for symmetric multivariate SV. WB0, WB1, and WB3 show reasonable size performances, as in **Table 1(a)** and **Table 1(b)**. A comparison of **Table 1(b)** and **Table 1(c)** shows that asymmetry of SV does not have a large impact on time-varying causality tests. From the results of empirical sizes, time-varying causality tests using HCCME have a negative influence on empirical sizes. Furthermore, time-varying causality tests F3 and HC3 based on third-order Taylor approximation is inferior to tests F1 and HC1 based on first-order Taylor approximation. Although WB3 tends to have slight under-rejection, WB0, WB1, and WB3 are superior to other tests. In particular, WB0 and WB1 perform best irrespective of type of error.

We next investigate the power properties based on DGP, given as

$$y_t = 1 + 0.2y_{t-1} + (\beta_0 + \beta_1 x_{t-1})F(\theta, t, c) + u_{1t}, \tag{27}$$

$$x_t = 1 + 0.5x_{t-1} + u_{2t}, \tag{28}$$

$$F(\theta, t, c) = \frac{1}{1 + \exp\{-\theta(t - c)\}} - 1/2, \quad (29)$$

where c is the point at which a regime changes from one to another. We set c to $c = T/2$. We compare the cases of $\theta = (0.01, 0.1, 1)$ and $(\beta_0, \beta_1) = \{(0.4, 0), (0, 0.2), (0.4, 0.2)\}$.

Table 2(a) reports the power performance of tests under normal errors u_{1t} and u_{2t} with $\rho = 0$ in (21). All the tests have a larger power when $(\beta_0, \beta_1) = (0, 0.2)$ or $(0.4, 0.2)$ than when $(\beta_0, \beta_1) = (0.4, 0)$. All the tests find it more difficult to detect changes only in a constant β_0 than only in AR parameter β_1 or in both a constant and AR parameter. In addition, the power of most of the tests increases when θ is large, because a large θ provides a sharp change in the smooth transition function. F0, HC0, and WB0 have smaller power compared to other tests regardless of the value of θ , β_0 , and β_1 . This shows that it is difficult for linear Granger causality tests to detect time-varying causality. F1, HC1, HC3, and WB1 apparently outperform other tests. However, HC1 and HC3 have over-rejections, as shown in **Table 1(a)**. The better power performance of HC1 and HC3 is attributed to over-rejection of the null hypothesis; moreover, they tend to lead to spurious time-varying causality. Although F3 has size distortions under the null hypothesis with normal error, F3 has similar or lower power compared to F1. WB3 has a small under-rejection of the null hypothesis and lower power. These results indicate that time-varying causality tests with third-order Taylor approximation are not advantageous. Accordingly, F1 and WB1 obtain reasonable empirical sizes and better power performance when a variable has time-varying causality.

Table 2(b) presents the power properties under multivariate SV. SV is generated by (22) with $\phi_1 = \phi_2 = 0.8$ and $\rho = 0.5$. HC0 and WB0 are naturally inferior to other tests. F0 performs well, unlike the results of **Table 2(a)**. This performance is attributed to the size distortions under SV, as in **Table 1(b)**. The same is true of the results of F1, F3, HC1, and HC3. They outperform other tests, but over-reject the null hypothesis. When DGP have stochastic volatility, they are likely to reject the null hypothesis of no time-varying causality and yield misleading results. It is important to have reasonable and acceptable empirical sizes in order to avoid misleading results. Although the power of WB1 and WB3 are lower than that of F1, F3, HC1, and HC3, they have reasonable and acceptable empirical sizes and lead to reliable results. We see that WB1 has higher power than WB3. The simulation results provide clear evidence that WB1 is more reliable from the perspective of controlling the size and obtaining sufficient power to find time-varying causality regardless of the presence of SV.

4. Conclusion

This paper investigated the statistical properties of time-varying causality tests when the errors of variables have multivariate SV. It is important to clarify the statistical properties of time-varying causality tests under SV, because economic variables often have SV and the relationship between them is time-varying. The tests we compared

include the standard linear Granger causality and the time-varying causality tests, their tests with HCCME, and their tests using wild bootstrap. Simulation results indicate that time-varying causality tests and their counterparts with HCCME have size distortions

Table 2. (a) Power properties under normal error; (b) Power properties under multivariate stochastic volatility.

		(a)								
		F0	F1	F3	HC0	HC1	HC3	WB0	WB1	WB3
$(\beta_0, \beta_1) = (0.4, 0)$										
$\theta = 0.01$										
$T = 200$		0.045	0.071	0.079	0.068	0.081	0.140	0.048	0.054	0.037
$T = 400$		0.049	0.162	0.117	0.065	0.168	0.182	0.048	0.150	0.077
$\theta = 0.1$										
$T = 200$		0.046	0.239	0.175	0.070	0.248	0.290	0.048	0.212	0.110
$T = 400$		0.050	0.413	0.310	0.062	0.435	0.418	0.049	0.416	0.262
$\theta = 1$										
$T = 200$		0.047	0.246	0.183	0.068	0.259	0.303	0.047	0.218	0.122
$T = 400$		0.049	0.423	0.316	0.065	0.438	0.425	0.054	0.420	0.262
$(\beta_0, \beta_1) = (0, 0.2)$										
$\theta = 0.01$										
$T = 200$		0.052	0.108	0.096	0.073	0.100	0.181	0.046	0.076	0.041
$T = 400$		0.063	0.368	0.295	0.075	0.380	0.429	0.056	0.341	0.218
$\theta = 0.1$										
$T = 200$		0.097	0.509	0.471	0.113	0.556	0.698	0.056	0.471	0.359
$T = 400$		0.110	0.818	0.829	0.106	0.831	0.913	0.065	0.802	0.755
$\theta = 1$										
$T = 200$		0.104	0.519	0.525	0.109	0.563	0.732	0.065	0.486	0.394
$T = 400$		0.104	0.811	0.827	0.099	0.831	0.916	0.066	0.793	0.760
$(\beta_0, \beta_1) = (0.4, 0.2)$										
$\theta = 0.01$										
$T = 200$		0.058	0.184	0.138	0.078	0.187	0.251	0.050	0.148	0.073
$T = 400$		0.067	0.737	0.612	0.080	0.752	0.741	0.056	0.733	0.531
$\theta = 0.1$										
$T = 200$		0.106	0.866	0.805	0.116	0.887	0.924	0.067	0.856	0.732
$T = 400$		0.111	0.991	0.988	0.104	0.990	0.996	0.068	0.988	0.971
$\theta = 1$										
$T = 200$		0.112	0.874	0.847	0.120	0.896	0.945	0.065	0.860	0.755
$T = 400$		0.117	0.991	0.989	0.109	0.991	0.996	0.069	0.989	0.972

(b)									
	F0	F1	F3	HC0	HC1	HC3	WB0	WB1	WB3
$(\beta_0, \beta_1) = (0.4, 0)$									
$\theta = 0.01$									
$T = 200$	0.129	0.148	0.200	0.083	0.133	0.243	0.041	0.062	0.052
$T = 400$	0.159	0.204	0.255	0.062	0.168	0.262	0.039	0.085	0.062
$\theta = 0.1$									
$T = 200$	0.132	0.229	0.255	0.085	0.236	0.356	0.043	0.118	0.088
$T = 400$	0.162	0.300	0.334	0.069	0.281	0.391	0.040	0.153	0.115
$\theta = 1$									
$T = 200$	0.134	0.241	0.272	0.089	0.250	0.361	0.043	0.126	0.093
$T = 400$	0.149	0.294	0.346	0.072	0.283	0.409	0.044	0.156	0.116
$(\beta_0, \beta_1) = (0, 0.2)$									
$\theta = 0.01$									
$T = 200$	0.145	0.174	0.227	0.097	0.148	0.275	0.042	0.069	0.058
$T = 400$	0.196	0.330	0.366	0.100	0.279	0.422	0.059	0.173	0.126
$\theta = 0.1$									
$T = 200$	0.236	0.410	0.464	0.176	0.413	0.615	0.083	0.256	0.215
$T = 400$	0.282	0.561	0.660	0.169	0.531	0.713	0.101	0.401	0.378
$\theta = 1$									
$T = 200$	0.252	0.423	0.489	0.190	0.435	0.645	0.098	0.275	0.238
$T = 400$	0.300	0.581	0.665	0.176	0.551	0.718	0.099	0.394	0.397
$(\beta_0, \beta_1) = (0.4, 0.2)$									
$\theta = 0.01$									
$T = 200$	0.141	0.214	0.250	0.094	0.190	0.330	0.042	0.108	0.077
$T = 400$	0.202	0.474	0.479	0.107	0.451	0.555	0.059	0.324	0.218
$\theta = 0.1$									
$T = 200$	0.253	0.605	0.629	0.194	0.620	0.767	0.095	0.472	0.381
$T = 400$	0.286	0.791	0.824	0.175	0.761	0.855	0.105	0.660	0.582
$\theta = 1$									
$T = 200$	0.265	0.626	0.657	0.208	0.631	0.791	0.095	0.487	0.410
$T = 400$	0.301	0.785	0.827	0.182	0.754	0.860	0.111	0.652	0.585

under highly persistent SV. Standard linear Granger causality tests perform relatively well under SV but has low power under time-varying causality. In contrast, time-varying causality tests using wild bootstrap have better size properties regardless of type of error. In particular, the time-varying causality test with first-order Taylor approximation and wild bootstrap has better statistical properties. These results indicate that the time-varying causality test with first-order Taylor approximation and wild bootstrap is

reliable and useful to test for time-varying causality.

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