

A Toy Model of Universe

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ABSTRACT

In this paper, we suggest that a toy model of our universe is based on FRW bulk viscous cosmology in presence of modified Chaplygin gas. We obtain modified Friedman equations due to bulk viscosity and Chaplygin gas and calculate time-dependent energy density for the special case of flat space.

KEYWORDS

FRW Cosmology; Modified Chaplygin Gas; Bulk Viscosity

1. Introduction

It is found that our universe expands with acceleration [1-5]. The accelerating expansion of the universe may be explained in context of the dark energy [6]. Due to negative pressure, the simplest way for modeling the dark energy is the Einstein's cosmological constant. On the other hand, the study of the cosmological constant is one of the important subjects in the theoretical and experimental physics [7-10]. Another candidate for the dark energy is scalarfield dark energy model [11-19]. However, presence of a scalar field is not only requirement of the transition from a universe filled with matter to an exponentially expanding universe. Therefore, Chaplygin gas is used as an exotic type of fluid, which is based on the recent observational fact that the equation of state parameter for dark energy can be less than -1 .

On the other hand, we know that the viscosity plays an important role in the cosmology [20]. In another word, the presence of viscosity in the fluid introduces many interesting pictures in the dynamics of homogeneous cosmological models, which is used to study the evolution of universe. In Ref. [21], the exact solutions of the field equations for a five-dimensional space-time with viscous fluid were obtained. Also in Ref. [22] a cosmological model with viscous fluid in higher-dimensional space-time was constructed. Then, in Ref. [23] the exact solutions of the field equations for a five-dimensional cos-

mological model with variable bulk viscosity were obtained. The isotropic homogeneous spatially flat cosmological model with bulk viscous fluid was constructed in Ref. [24]. The bulk viscous cosmological models with constant bulk viscosity coefficient were constructed in Ref. [25]. In the recent work [26] the FRW bulk viscous cosmology was considered and bulk viscous coefficient was obtained in the flatspace, and then extended to non-flat space [27]. In this work, we consider both bulk viscous effect and Chaplygin gas in FRW cosmology in flat space.

2. Equations

The Friedmann-Robertson-Walker (FRW) universe in four-dimensional space-time is described by the following metric [28,29],

$$ds^2 = -dt^2 + a^2(t) \left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2 \right) \quad (1)$$

where $d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2$, and $a(t)$ represents the scale factor. The θ and ϕ are the usual azimuthal and polar angles of spherical coordinates. Also, constant k denotes the curvature of the space. In this paper we consider the case of $k=0$ only, which is corresponding to flat space. In that case the Einstein equation is given by,

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = T_{\mu\nu} + g_{\mu\nu} \Lambda \quad (2)$$

where we assumed $c=1$ and $8\pi G=1$. Also the energy-momentum tensor corresponding to the bulk viscous fluid and modified Chaplygin gas [30-35] is given by the following relation,

$$T_{\mu\nu} = (\rho + \bar{p})u_\mu u_\nu - \bar{p}g_{\mu\nu} \quad (3)$$

where ρ is the energy density and u_μ is the velocity vector with normalization condition $u^\mu u_\mu = -1$. Also, the total pressure and the proper pressure involve bulk viscosity coefficient ζ and Hubble expansion parameter $H = \dot{a}/a$ are given by the following equations [36-42],

$$\bar{p} = p - 3\zeta H \quad (4)$$

and,

$$p = \gamma\rho - \frac{B}{\rho^\alpha} \quad (5)$$

with $B > 0$ and $0 < \alpha \leq 1$. The equation of state γ is one of the most important quantity to describe the features of dark energy models. It is clear that the parameter ζ shows bulk viscosity and B shows effect of Chaplygin gas. In the Ref. [43] the dynamics of FRW cosmology with modified Chaplygin gas as the matter formulated. Then the nature of the critical points are studied by evaluating the eigenvalues of the linearized Jacobi matrix for the special case of $\alpha = 0.6$. In this paper we consider special case with $\alpha = 0.5$ and extend the Ref. [43] to including bulk viscous coefficient.

In that case the Friedmann equations are given by,

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{\rho}{3} \quad (6)$$

and,

$$\left(\frac{\dot{a}}{a}\right)^2 + 2\frac{\ddot{a}}{a} = -\bar{p} \quad (7)$$

where dot denotes derivative with respect to cosmic time t . The energy-momentum conservation law obtained as the following,

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + \bar{p}) = 0 \quad (8)$$

In the next section we try to obtain time-dependent

density by using above equations.

3. Solutions

Using the Equations (4)-(6) in the conservation relation (8) we have,

$$\dot{\rho} + \sqrt{3}(\gamma + 1)\rho^{\frac{3}{2}} - 3\zeta\rho - \sqrt{3}B = 0 \quad (9)$$

If we set $\zeta = 0$, then one can extract energy density depend on scale factor [43],

$$\rho(a) = \left[\frac{1}{\gamma + 1} \left(B + \frac{c}{\sqrt{a^{9(1+\gamma)}}} \right) \right]^{\frac{2}{3}} \quad (10)$$

where c is an integration constant. Here we also consider bulk viscous coefficient and would like to obtain energy density depend on time. In order to solve Equation (9) we use the following ansatz,

$$\rho = \frac{A}{t^2} + \frac{E}{t} + ht + Ce^{bt} \quad (11)$$

where constants A , E , h , C and b should be determined. Substituting relation (11) in the Equation (9) gives us the following coefficients,

$$h = \sqrt{3}B \quad (12)$$

$$A = \frac{4}{3(1+\gamma)^2} \quad (13)$$

$$E = \frac{2\zeta}{(1+\gamma)^2} \quad (14)$$

$$C = \frac{(1+\gamma)^2}{4} \left[\frac{8\sqrt{3}\zeta^2}{(1+\gamma)^3} - \frac{3(1+\gamma)^4}{16\zeta^2} \right] \quad (15)$$

If we neglect both bulk viscosity and presence of Chaplygin gas then,

$$\rho = \frac{4}{3(1+\gamma)^2 t^2} \quad (18)$$

which is agree with results of the Refs. [27,43] where $\rho \propto t^{-2}$ established. On the other hand for the large bulk viscosity coefficient one can find that $b < 0$ and hence $\rho \propto \zeta/t$ obtained. Also for the case of infinitesimal ζ

$$b = \frac{\zeta \left[\sqrt{3}\zeta(1+\gamma) \left(B(1+\gamma) - \frac{9}{2}\zeta^3 \right) + \frac{27}{32} \left(\frac{1}{8} + \gamma \right) + \frac{9}{16}\zeta^4 + O(\gamma^n) \right]}{8(1+\gamma) \left(\sqrt{3}\zeta^4 - \frac{3}{128}(1+\gamma)^7 \right)} \quad (16)$$

where,

$$O(\gamma^n) \equiv \frac{27}{256}\gamma^8 + \frac{27}{32}\gamma^7 + \frac{189}{64}\gamma^6 + \frac{189}{32}\gamma^5 + \frac{945}{128}\gamma^4 + \frac{189}{32}\gamma^3 + \frac{189}{64}\gamma^2 \quad (17)$$

one can obtain constant negative energy density. In the general case, Equation (11) with coefficients (12)-(16) tells us that the energy density is decreasing function of time. Such behavior happen for the Hubble expansion parameter which is discussed below.

By using time-dependent density in the relation (6) one can obtain Hubble expansion parameter. In that case we draw plot of Hubble expansion parameter in the **Figure 1** for $\gamma = 1/3$.

In that case the modified Chaplygin gas model describes the evolution of the universe from the radiation regime to the Λ -cold dark matter scenario, where the fluid behaves as a cosmological constant, so there is an accelerated expansion of the universe.

It is possible to study deceleration parameter of this theory which obtained by the following relation,

$$q = -\left(1 + \frac{\dot{H}}{H^2}\right) \tag{19}$$

Numerically we draw deceleration parameter in terms of time in the **Figure 2**.

4. Conclusions

In this work, we studied the FRW bulk viscous cosmology with modified Chaplygin gas as the matter contained. We obtained the modified Friedmann equations due to bulk viscous and Chaplygin gas coefficients. Then tried to solve equations and found time-dependent energy density. Therefore, we could extract Hubble expansion and deceleration parameters.

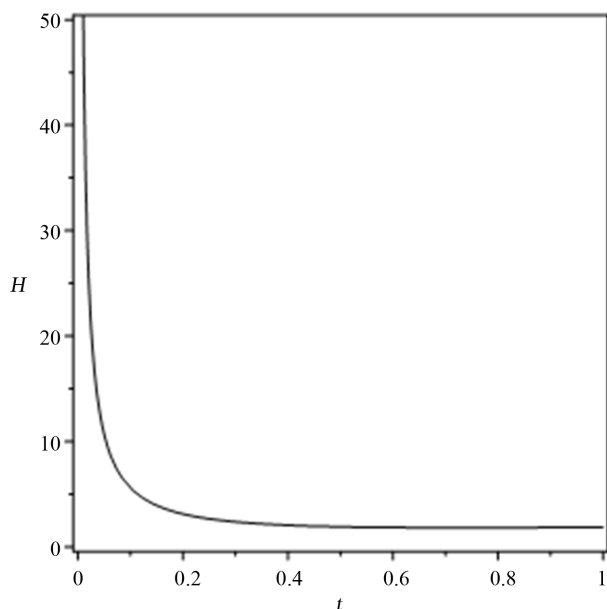


Figure 1. Hubble expansion parameter in terms of time for $B = 3.4$, $\zeta = 1$, and $\gamma = 0.3$.

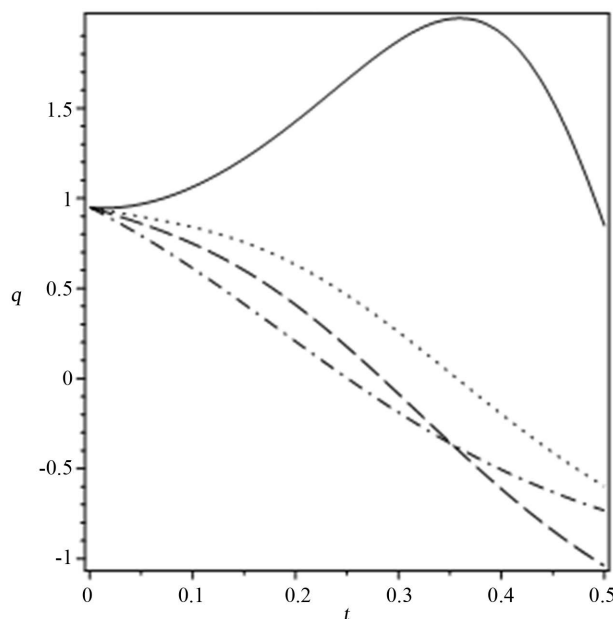


Figure 2. Deceleration parameter in terms of time for $B = 3.4$ and $\gamma = 0.3$. Solid, dotted, dashed and dash dotted lines represent $\zeta = 0.2, 0.4, 0.6, 1$ respectively.

For the future work, it is possible to repeat calculation of this paper for the case of arbitrary α or non-flat universe where $k \neq 0$. In that case one deals with the following equation,

$$\dot{\rho} + 3H\left(\rho(\gamma + 1) - \frac{B}{\rho^\alpha} - 3\zeta H\right) = 0 \tag{20}$$

where $H^2 = \frac{\rho}{3} - \frac{k}{a^2}$.

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