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Effect of Residential Quarters Opening on Urban Traffic from the View of Mathematical Modeling

Manlin Kang*, **Peiru Bao**, **Yihua Cai**

Jinan University, Guangzhou, China
Email: *2232432861@qq.com

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Abstract

With the great prosperity of national economy, there has been a dramatic rise of vehicles on city road, which makes increasing pressure of road transportation. Currently, many countries are confronting the severe situation of traffic jam in different degrees. Nevertheless, there are many triggers contributing to this congestion, one of which is the blocking of residential quarters towards vital traffic line. Therefore, it is extremely necessary to study whether the opening of residential quarters can improve the road capacity of the entire city and remit the traffic pressure. Our paper is based on graph theory, density theory and random utility theory. First of all, we demonstrate a mathematical model of road traffic. Secondly, we explore the influence of residential quarters opening on urban traffic, taking three factors into account listed as road traffic capacity, road network density and network average running time. On the basis of above contents, the impact analysis of vehicle traffic caused by pedestrians is added afterwards. Finally, our paper takes three different types of residential areas into account as an example to empirically analyze the tangible impact of the opening, and finally come to the benefit of the traffic system after the opening.

Keywords

Urban Road Network, Network Diagram, Traffic Capacity, Average Running Time

1. Introduction

At present, there has been a rapid development of the urban road systems among countries all over the world. However, due to the rapid increase in the number of urban population, the total quantity of urban vehicles also grows quickly. From 2005 to 2014, the average urbanization rate increased from 45% to 54%, the amount of private car ownership rose from 18.48 billion to 1223.39 billion and the amount of highway operation cars improved from 7.33 million to 15.38 million, which caused increasing pressure on city road. In order to alleviate traffic pressure, roadways in many big cities have been widened at the present stage, and the number of the overpasses has been increased continuously. Regrettably, road traffic has not been effectively improved.

Residential quarters opening can increase the road network density, optimize the structure of the road network and improve the capacity of road traffic. As for the travelers, the most common and direct benefit is to improve the speed of driving and shorten the average travel time. However, the impact of the opening act on the surrounding road traffic, which is based on the location of the area, the acreage and the conditions of internal roads, should be discussed specifically and accurately. Among them, traffic capacity is an important indicator to measure the effectiveness of the road. Traffic capacity refers to the maximum number of vehicles in a certain section of a roadway in unit time under a certain road and traffic conditions.

In this paper, we use the theory of graph theory to establish three mathematical models, and then explore the influence of residential quarters opening on urban traffic, considering three factors listed as road traffic capacity, road network density and network average running time. Next, through the analysis of authentic examples, we get a preliminary conclusion: it is bound to bring great benefits about opening the areas that block the main traffic road. However, the way to open will determine the whole effectiveness. According to the results of the actual impact of the opening of the district, the government can formulate a reasonable and effective policy to ease the problem of local traffic tension.

2. Establishment of Road Traffic Model

The complexity of the internal road structure of the residential quarters differs from each other, the roadways which can make cars pass through in residential quarters consist of the road network. According to the knowledge of graph theory, each intersection point in the road network is considered as a node, and each path is considered as an arc (directional or non-directional) to make up a graph (directed graph or undirected graph). The residential quarter's internal road network includes multiple origins and end points, which is showed as follows: (**Figure 1**).

BPR function: the service quality provided by the transportation system for the get-

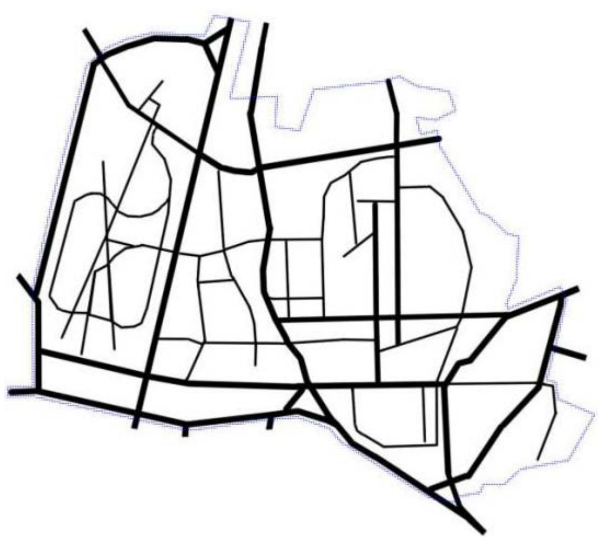

Figure 1. Map of residential quarter's internal network road.

around people generally measured by impedance. Less service quality will be offered when the impedance is much higher, and vice versa. The impedance function [1] refers to the relationship between the travel time of vehicles on the road and the traffic conditions of the roadway.

2.1. Road Capacity Analysis

1) Modeling foundation—Graph theory

When the road is abstracted as a network, the most common method for solving the capacity of the road network used to be Cut Set Method [2]. This method is mainly used to calculate the capacity of the road network to reach the maximum value. In the target area, when the road network capacity reaches limit closely, the maximum capacity of the network can be equated to the sum of all the traffic capacity of roads in each cut with minimality, that is to say, the process of solving the minimum cut set capacity is the process of calculating the road network capacity.

2) Theoretical basis

The maximum flow and the minimum cut set theorem: according to the capacity of the network, the maximum flow from the beginning point to the receiving point is equal to the flow of the minimum cut set.

Method for determining the maximum flow of a network with multiple transmittable and receivable points [3]:

If there are multiple transmittable and receivable points in a network, then we can switch it into merely one sending and receiving network, which can be done by just adding a newly total beginning point and a newly total ending point. Furthermore connect arcs weighted A from the beginning point to the other points in the network, and connect arcs weighed $+\infty$ from all the other points to the ending point in the same way. After that, the difficulty is converted into the issue about network flow with single sending point and single receiving point.

3) Solving method

Ford-Fulkerson labeling method:

Firstly find a feasible flow and label it to acquire an augmenting chain from sending point to receiving point; secondarily increase the capacity of the feasible flow after the adjustment along the augmenting chain, we get a new feasible flow. This process is repeated until no augmenting chain of the feasible flow can be obtained, and that is to say, the maximum flow comes into being. Steps are listed as follows [4]:

a) Mark the sending point v_s, labeling for $(-,+\infty)$. At this moment, v_s is a labeled sending point but has not been checked. Inversely, the rest of the points have not been marked jet.

b) Select a sending point which is labeled but has not been checked such as v_i, and if it has unsaturated arc like (v_i,v_j) with unmarked point v_j, we then mark v_j for $(v_i,l(v_j))$, in which a formula showed as $l(v_j)=\min\{l(v_i),c_{ij}-f_{ij}\}$. Then v_j becomes the labeled point which has not been checked according with v_i. Later on, as for other unmarked points like v_j which has nonzero arc (v_i,v_j), we now remark it as $(-v_i,l(v_j))$, in which $l(v_j)=\min\{l(v_i),c_{ij}-f_{ij}\}$. Then v_j turns into the labeled point that has not checked. However, v_i becomes the point that has been both labeled and checked.

c) Repeat last step constantly, until any of the points such as v_t becomes labeled or all labeled points have been checked. If v_t is a labeled point, it means an augmented chain from the sending point to the receiving point has been gained, with that we successfully get into the adjusting segment. Besides, it is indicated that the maximum feasible flow is current feasible one when all the points labeled have been checked. Finally, increase the amount of the front arc with the quantity of $l(v_t)$, and meanwhile reduce the amount of the latter arc with the same quantity of $l(v_t)$ among each accomplished augmented chains.

2.2. Road Network Density Analysis

1) Modeling foundation——Sketch of road network density

The road network density is defined as the ratio of the total length of the road network and the total area of the road network (unit: kilometer per square kilometer). The total length is calculated based on the road center line in the road network, and the total area is calculated according to the scope of the service provided by the road network.

2) Discussion on the rationality of road network density

Reasonable density of the urban road network will largely reflect the overall level of service of the urban road, and subsequently determine the rationality about the design of the entire traffic network.

Provided that the road network density is too small, the total capacity of urban road network will lead to insufficient, and there will be hidden dangers of long-term traffic congestion. Oppositely, if the road density is too large, the resources will not be able to make full use of, and the terrible situation will also give rise to the additional increase of the intersection in the whole road traffic network, which is bound to enhance the conflict among the traffic streams, and the total efficiency of the traffic transportation will reduce dramatically as well.

Therefore, in order to display the scale benefit of the urban road traffic network, we must design a reasonable density range to maximize the traffic efficiency.

3) Model building

The definition of the road network density under standard refers to the ratio of the total length of the unified road network and the total area of the network (unit: normative road length per square kilometers). The standard length of the road is obtained by the real roadway in the road network after conversion.

Nevertheless, different types of roads have different effects on road density due to their different purposes of design. The specific influence coefficients are shown in the following Table 1.

Table 1 lists the influence coefficients of different types of roadway, which will serve as an independent indicator of the impact of diverse road network density. Therefore, the calculation formula of the standard road network density can be obtained

Table 1. Influence coefficients of variously classified roadway.

Road classification	Expressway	Main road	Secondary road	branch
influence coefficient	1.2	1.1	1.0	0.9

as follows:

$$\gamma = \frac{\sum h_i \times \mathrm{d}_i / \mathrm{d}_s}{S_i} \alpha_i \tag{1}$$

Among them:

γ —Standard road network density; d_i —Actual cross profile width of a road;

d_s —Standard road width indifferent types; h_i —Actual road length of a road;

S_i —Corresponding area of network acreage; α_i —Influence coefficients of classified roadway.

2.3. Analysis of Average Running Time in Road Network

1) Modeling foundation——Random utility theory

a) Set correlative variable [5]

Set $t_{l,u}^{se}$ as the ideal running time through the path l from the starting spot s to the ending spot e, in which $l \in L^{se}$ and $t_{l,u}^{se}$ is regarded as random variables.

Set $t_{l,v}^{se}$ as the actual running time through the path l from the starting spot s to the ending spot e, in which $l \in L^{se}$.

Set $t_{l,u}^{se} = t_{l,v}^{se} + \xi_l^{se}$, where ξ_l^{se} is the random error term and $E\left(\xi_l^{se}\right) = 0$ (mean).

b) Determine the path selection probability and average running time between a pair of OD

Model hypothesis:

1) The choice preference of vehicles on the road is reflected by the utility value of the road.

2) Vehicles tend to choose the largest utility value among the roads as the travel decision.

3) The traffic impedance of each pair of OD on the path is deemed to be random variables.

4) Random error term follows Gumbel distribution.

A great mass of relevant contents in the law of large numbers [6] illustrates that the ratio caused by the amount of vehicles using the path l between the starting spot and the ending spot and the total number of vehicles for the destination can be expressed as follows:

$$\begin{aligned} P_l^{se} &= P(-t_{l,u}^{se} \geq -t_{j,u}^{se}, \forall j \in L^{se} / k) \\ &= P(-t_{l,v}^{se} + \xi_l^{se}) \\ &\geq -(t_{j,v}^{se} + \xi_j^{se}) \\ &= P(t_{l,v}^{se} + \xi_l^{s} \leq t_{j,v}^{se} + \zeta_j^{se}) \\ &= P[(\xi_l^{s} - \xi_j^{se}) \leq (t_{j,v}^{se} - t_{l,v}^{se})] \end{aligned} \tag{2}$$

As is showed in the formula ahead, can be measured. Thus, it is solely necessary to determine the distribution of the stochastic error which is vital to work out the proportion above. In addition, according to the error term that follows the Gumbel distribution, the running time function can comply with the Gumbel distribution as well. Then with the aid of the utility-maximizing rule, the selection probability of the Logit model can be derived from the model [6]:

$$P_l^{se} = \mathrm{e}^{t_{l,v}^{se}} \Big/ \sum_{m \in L^{se}} \mathrm{e}^{t_{m,v}^{se}} \quad (3)$$

As is showed in the formula ahead, P_l^{se} represents the selection probability when vehicles choose the path l traveling from the starting spot to the ending spot. $t_{l,v}^{se}$ represents the observable time when vehicles are traveling on the path l. Therefore, the average running time of vehicles between the starting spot and the ending spot is following:

$$t^{se} = \sum_{i=1}^{n} t_i^{se} \times P_i^{se} \quad (4)$$

As is showed in the formula ahead, t^{se} represents the average traveling time while vehicles are traveling between the starting spot and the ending spot. The total number of all the roads is equal to n. P_i^{se} is also the probability of traveling vehicles selecting path i for their destinations from the starting spot.

c) Calculate the average running time of vehicles in the road network in a certain time period

The average running time between each pair of OD can be calculated through last formula:

$$t = \sum_{i=1}^{m} t_i \times \eta_i \Big/ \sum_{i=1}^{m} \eta_i \quad (5)$$

As is showed in the formula ahead, t represents for the average running time of vehicles in the whole road network under saturated condition. m is the number of OD on the network. t_i is the average running time in the road network of the OD marked i. η_i is the preference of vehicle towards the ending point marked i.

2) Model implementation

a) Select a typical road network for the field observation, and determine a quantity of different average running time of the vehicles as each road is in a saturated state. According to the consistency principle of average running time which focus on reaching the certain saturation state, we can calculate the average running time of vehicles in the road network:

$$t_{ij} = C_{ij} / v_{\infty} \quad (6)$$

As is showed in the formula ahead, t_{ij} is the average running time of the vehicle traveling on any of the road in the limit capacity (saturation capacity) state. C_{ij} is the length between the pathmarked i and j. v_{∞} is the critical speed of the vehicle when the path reaches its maximum capacity.

b) Establish fuzzy correlation matrix [6] of road network.

c) Compute transitive closure matrix [5].

d) Find the OD matrix which can be reached in one step on the basis of the correlation matrix, and then calculate the travel time of the road under saturation condition.

e) Find the OD matrix which can be reached in two steps on the basis of the correlation matrix, then calculate the average travel time of each pair of OD and the travel time of the road under saturation condition.

f) Find the OD matrix which can be reached in three or four steps on the basis of the correlation matrix, and then calculating the travel time of the road under saturation condition.

g) Put each path into Equation (6) to obtain the selection probability for each path (Table 2).

3. Model Development Analysis—Pedestrian Factor

1) Characteristics of walking

When it comes to walking, it is common that the speed is slow, the goal is flexible, the randomness is strong, the space occupied by the road is little, and the entire walking distance is relatively short.

Walking speed is generally different from the crowds, and adult's walking speed is approximately concentrated among 1.0 - 1.3 m/s.

2) Effect of residential quarters opening on pedestrian traffic mode

Pedestrian density [7] is defined as: the number of people in unit area on one road which is selected with certain length and width (Table 3).

On the one hand, there will be residents emerging within the public places for activity and the no-parking avenues original in residential quarters. Nevertheless, part of the public facilities will be removed, and a number of avenues merely for walking will greatly allow vehicles to travel through after the opening, which results in tremendous decrease in the number of residents who are fond of taking a walk. Thus, the opening of the area will reduce the pressure of the external road traffic.

On the other hand, in the course of the rush hours and collective periods for activity, there will be a growing number of pedestrians in the exoteric roads. When the pedestrian density goes up to 3 - 4 person/m^2, it will give rise to a dramatic decline in the traveling speed among vehicles, then resulting in severe traffic jam in the internal road network of residential quarters, which may work out anything but alleviating the surrounding traffic pressure.

Therefore, the model will be more effective and authentic when we take the pedestrian factors into account.

4. Empirical Analysis

Choose different types of residential quarters to make the study from the standpoint of

Table 2. Average travel time of each OD.

Starting Point		Route		path length	path time
1	0	0	0	0	0
1	2	0	0	16	0.533333
1	3	4	2	39	1.3
2	4	3	0	29	0.966667
2	4	0	0	9	0.3
2	1	3	4	46	1.533333
2	1	3	5	38	1.266667
2	4	3	5	41	1.366667

floor space, such as Tian Tong Garden and Kang Du Area. In addition, we example Vanke City and a university from the standpoint of geographical location. As follows: (Table 4).

Abstract them into a network diagram, as shown (Figures 2-5).

Use the VISSIM software to make the analogue simulation of the road network shown in Figures 2-5. However, unimpressive road network capacity with transformation was generated under the condition of lacking true statistic date. After the adjustment of some pivotal parameters, the analogue simulation results can be similar closely to the solutions of our model, which can also consider adding the appropriate penalty factors to optimize the analogue simulation model [8]. By means of the calculation of the MATLAB software, the model can reflect the influence of the road traffic after the opening of the residential quarters. For those located in the suburbs, the final effects are not apparent through these patulous operations. Oppositely, when the residential quarters are located in the downtown districts, the eventual results will be more prominent through the opening. What's more, achievements of opening as to residential quarters with small scale may not come into being an attraction significantly.

5. Conclusions

Through model analysis, the development of the road traffic network system after the opening does have the positive function of optimizing the structure of the road network. Vehicles now travel with more and better choices. The shunting effects of the traffic network have coped with the troubles of quite a few paths with conspicuous congestion greatly, and subsequently improve the overall traffic network capacity productively, which also improve citizen's satisfaction degree [9] towards go-out. Cheerfully, the unobstructed traffic network is bound to bring a virtuous cycle, which can improve the operational efficiency of the whole road traffic system, and achieve full and effective use of resource. Finally, the overall economy will also play a leading role of the city's development.

In addition, the location and the acreage of the residential quarters in the existing residential area should also be attached to great importance when connecting with the

Table 3. Pedestrian size statistics in different countries [6].

countries and regions	Female			Male		
	Shoulder width (cm)	Chest Circumference (cm)	Acreage (m^2)	Shoulder width (cm)	Chest Circumference (cm)	Acreage (m^2)
average value	43.17	28.56	0.19	48.17	27.44	0.21

Table 4. Different types of cell.

Name	Vanke City	A university	Tian Tong Garden	Kang Du Area
position	Shanghai	Guangzhou	Beijing	Beijing
acreage	146,938 m^2	53,333 m^2	480,000 m^2	60,000 m^2

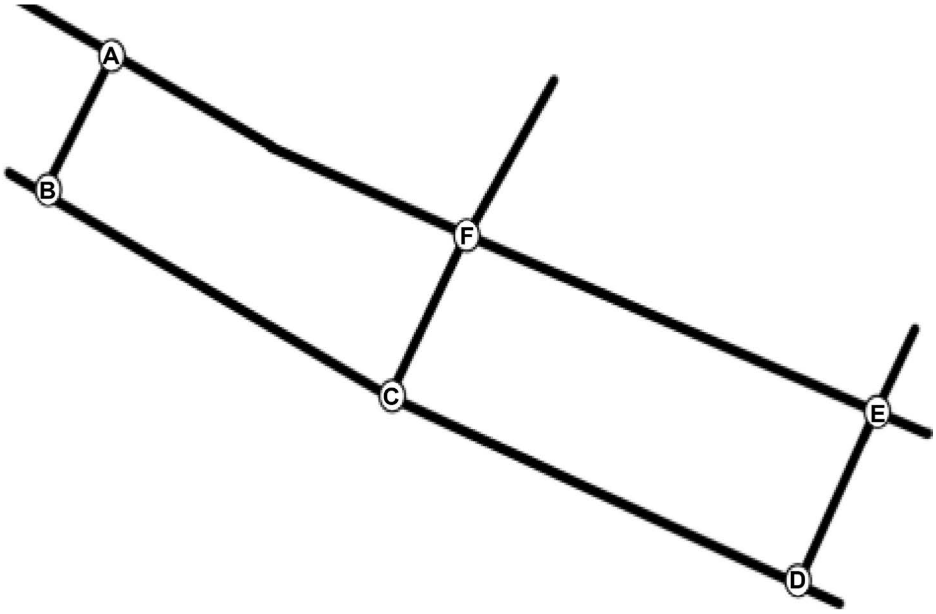

Figure 2. Vanke City road network map.

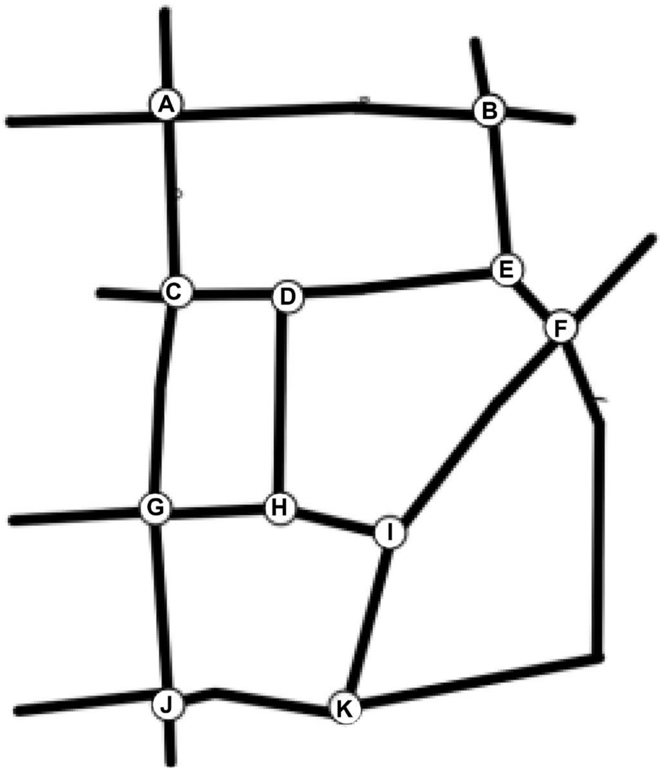

Figure 3. Tian Tong Garden road network map.

obsolete road network for next action as to positive opening. As for those located in the remote suburbs, whether it should be open or not must meet the needs of local economic development, while in the urban parts of the district, it is highly considered to set up the opening operation as soon as possible. And for some residential quarters, in which internal roads are narrow and internal capacity of the area is heavy enough, we can also think of delaying its release time rather than opening it at the moment, in

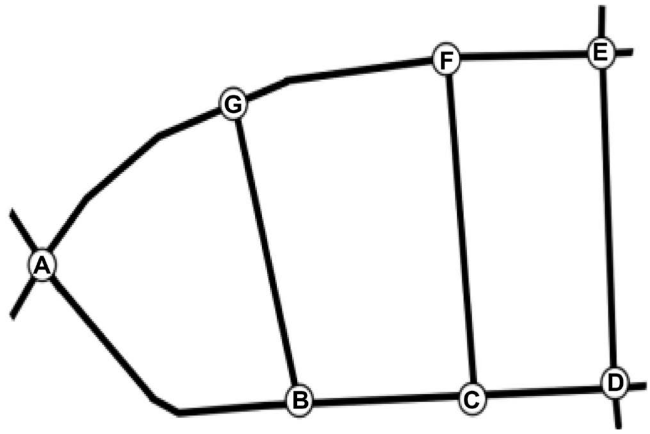

Figure 4. Kang Du road network map.

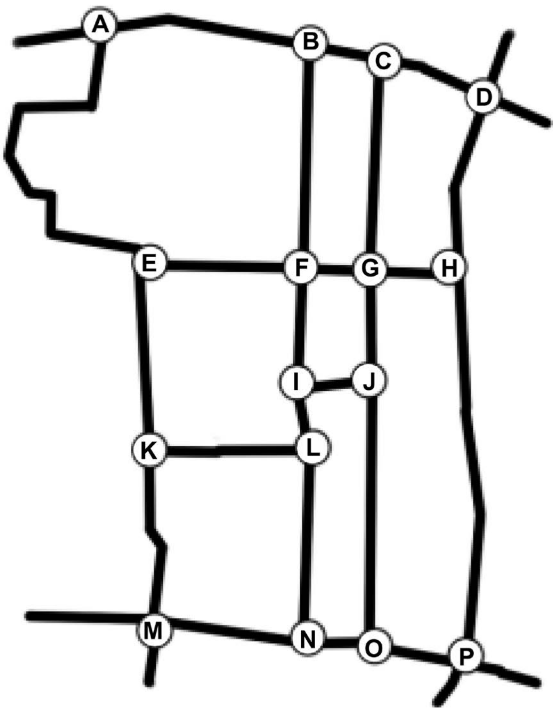

Figure 5. A university road network map.

order to avoid the negative impact of opening up the road traffic network.

However, residents in the open area have been still worrying about whether their legitimate interests can be protected, for example: security issues and environmental governance issues after the open. Although the opening of the road network actually provides a convenient way for residents to travel around, the degree of interference caused by the opening is far greater than the degree of convenience brought through it.

In a word, we should put the mission of opening in the first place and consider citizen's benefit at the same time practicably and effectively. That is to say, residents' rights and interests should be guaranteed genuinely. We can get more supports from the masses about the opening issue rather than counterview and perishing demonstrations only in this way.

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