Characteristics of Strange Hadron Production in Some High Energy Collisions and the Role of Power Laws

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ABSTRACT

Studies on "strange" particle production have always occupied a very important space in the domain of Particle Physics. This was and is so, just because of some conjectures about specially abundant or excess production of "strange" particles, at certain stages and under certain conditions arising out of what goes by the name of "Standard" model in Particle Physics. With the help of Hagedornian power laws we have attempted to understand and interpret here the nature of the p_T -spectra for the strange particle production in a few high energy nuclear collisions, some interesting ratio-behaviors and the characteristics of the nuclear modification factors that are measured in laboratory experiments. After obtaining and analysing the final results we do not confront any peculiarities or oddities or extraneous excesses in the properties of the relevant observables with no left-over problems or puzzles. The model(s) used by us work(s) quite well for explaining the measured data.

Keywords: Hadron-Nucleus Collisions; Inclusive Production; Scaling Phenomena; Power Laws

1. Introduction

Studying the nature of particle production in protonproton collisions is important and interesting in itself, as it might shed light on the basic mechanism for production of particles. Besides, it could also serve as a necessary benchmark for the physics developments in ultra-relativistic heavy ion collisions [1,2]. This is specially important at the large hadron collider (LHC) where the heavy ion programme had started by November 2010 delivering some preliminary results on some aspects of strange hadrons produced as the final product in high energy nuclear collisions and these strange secondaries are supposed to provide valuable insights into the properties of the "controversial" system newly formed. One of the main motivation for measuring strange particles in heavy nucleus-nucleus reactions at LHC is the expectation that their production-rates for participating nucleon should be enhanced with respect to basic nucleon-nucleon interactions. Strangeness enhancement has consistently been proposed as one of the strong diagnostics for a Quark Gluon Plasma (QGP) state [3,4]. The enhancement factor (E) is defined as rapidity-density

of multiplicity (yield) par mean number of nucleon participants $[\langle N_{part} \rangle]$ in heavy ion collisions, divided by the respective value in p + p collisions. The requisite information about $\langle N_{part} \rangle$ etc are to be obtained from Abelev *et al.* [5,6].

As the strange hadrons are not at all present in the initial system (A), the question rises very sharply: how do they make their appearance in the final products. So there must be some specific reflections on the constituent pictures of the particles and specifically the nucleons. Besides, enhancement of strangeness productions was/is one of the powerful diagnostics for the formation of quark gluon plasma (QGP) in relativistic heavy ion collisions and the colliders (RHIC). The observations of an increase of strange baryon production relative to p + p collision in SPS data, confirmed later at the RHIC studies, has brought excitement in this area.Besides, the increase of p/π ratio (B) in such collisions in the non-strange domain had its parallel in the strangeness sector with the observation of slow rise of the Λ/k^0 values.

The organization of this work is as follows. In Section 2 we give an outline of the model to be applied. In the next section (Section 3) we deliver the results by figures and tables with a short discussion on the results obtained.



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In the last chapter, we precisely point out the conclusions to be arrived at.

2. The Background in Some Detail and the Working Formulae

With gradual attainments of larger transverse momenta (p_T) of the secondaries in high- p_T (hard) interactions, the problems of deviations from exponential nature of fits on invariant spectra began to crop up steadily. Gazdzicki and Gorenstein [7] observed rightly that for $p_T > 2$ GeV/c, the data sharply deviate from the exponential nature, for which Darriulat[8] proposed a power law distribution of certain forms for both p_{T} spectra and particle multiplicity. Indeed, for both p_{T} spectra and multiplicity such power law forms have become now the most dominant tools in dealing with the transverse momentum spectra of all hadrons. Gazdzicki and Gorenstein showed that the normalised multiplicities and $(m_T \sim p_T)$ spectra of neutral mesons obey the m_{T} -scaling which has had an approximately power law structure of the form $\sim (m_T)^n$, where m_T is called transverse mass and is defined by $m_T = \sqrt{p_T^2 + m^2}$. This scaling behaviour was analogous to that expected in statistical mechanics, the parameter n plays the role of temperature and any normalization constant to be used resembles the system volume. Thus the basic modification of the statistical approach needed to reproduce the experimental results on some hadron production process in $p(\overline{p}) + p$ interaction in the large $m_T \equiv p_T$ domain is to change the shape of the distribution functions $\exp\left(-\frac{E^*}{T}\right)$ had to be altered to the power law

form as given by $\left(\frac{E^*}{\Lambda}\right)^{-n}$ with some changed parame-

ters, viz, a scale parameter Λ and an exponent n, both are assumed to be common for all hadrons.

Let us now dwell, in brief, on the clues to the possible origin of power laws. One of the basic features of the hadronuclear collisions is: irrespective of the initial state, agitations caused by the impinging projectile (be it a parton or particle/nucleon) generate system effects of producing avalanches of new kind of partons (called gluons) which form an open dissipative system. And these production processes are not at all gradual; rather they are very sudden, drastic and complex. And such complex properties and processes in nature do generally subscribe to the power-law behaviours. In the recent times, it is being propounded consistently that the power law behaviours put into use here are "manifestations of the dynamics of complex systems whose striking feature is of showing universal laws characterized by exponents in scale invariant distributions that happen to be basically independent of the details in the microscopic dynamics" [9]. The avalanches caused by production of excessive number of some new variety of parton called 'gluons' (the process called "gluonisation") give rise to the jettiness of particle production and of cascadisation of the particle production processes leading to the fractality as is shown by Sarcevic [10]. These cascades are self-organizing, self-similar and do just have the fractal behaviour. Driven by the physical impacts of these well-established factors, in the high energy collision processes do crop up the several power-laws.And how such power laws do evolve from exponential origins or roots is now-a-days being taken care of by the induction of Tsallis entropy [11] and a generalisation of Gibbs-Boltzmann statistics for long-range and multifractal processes.

In what follows we are going to choose a specific form of power law which was previously applied by us in the case of hadron-nucleus collisions. With a view to accommodating some observed facts for strange particle production, it is tempting to try to fit the whole distribution for the inclusive p_T -spectra with one single expression in the form of power law as was done by G. Arnison *et al.* [12] and Hagedorn [13].

$$E\frac{\mathrm{d}^{3}\sigma}{\mathrm{d}p^{3}} = const.\frac{d\left(\mathrm{d}N/\mathrm{d}y\right)}{2\pi p_{T}dp_{T}} = A\left(\frac{q}{p_{T}+q}\right)^{n} \tag{1}$$

where the letters and expressions have their contextual significance. This parametrization describe the data well over the entire range of p_T .

Indeed for $p_T \rightarrow 0, \infty$ we have,

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$$\left(\frac{q}{p_T + q}\right)^n \approx \left(1 - \frac{n}{q} p_T\right)$$

$$\approx \begin{cases} \exp\left[\frac{n}{q} p_T\right] & \text{for} \quad p_T \to 0 \\ & \text{and} \\ \left(\frac{q}{p_T}\right)^n & \text{for} \quad p_T \to \infty \end{cases}$$
(2)

Thus along with impressive fit, which now includes the large p_T domain, the estimate of $\langle p_T \rangle$ assumes with the help of expression (1):

$$\langle p_T \rangle = \frac{\int q/(p_T + q)^n p_T^2 dp_T}{\int q/(p_T + q)^n p_T dp_T} = \frac{2q}{n-3}$$
 (3)

So, in clearer terms, let us put the final working formulae here as follows with substitution of p_T (transverse momentum) as x in the power-law model [14,15] respectively

$$f(x) = A(1+x/q)^{-n}$$
 (4)

There is yet another very important observable called nuclear modification factor (NMF), denoted here by R_{CP} , which for production of any hadron is defined by [16]

$$R_{CP}(p_T) = \frac{\left[\left(\frac{d^2 N}{2\pi dp_T dp_T dy} \right)_c / N_{bin} \right]^{Central}}{\left[\left(\frac{d^2 N}{2\pi dp_T dp_T dy} \right)_c / N_{bin} \right]^{Peripheral}}$$
(5)

3. Results

In obtaining the results presented here, no serious statistical calculational procedure was adopted. The graphs are drawn more as fitological-cum-phenomenological exercises with mainly statistical errors in considerations. The experimental data do not provide, in the most cases, any systematic errors. Data points for the heavy, high strangeness-valued particles are too scarce; for which the number of degrees of freedom is too limited for many cases. The quality of fits to the data indicated in the tables by χ^2/ndf terms in the columns is attempted to be kept at a modestly satisfactory value (tending as nearly as possible to unity). And the figures are drawn by Wgnuplot, wherein there are some inbuilt statistical procedures and techniques.

Quite observably, the results are depicted here in graphical plots. And the used values of the corresponding parameters for fits are shown in separate tables. The **Figures 1(a)** and (b) show the production of secondaries k^0, k^+, k^- , Λ and $\overline{\Lambda}$ in proton-proton collisions at $\sqrt{s} = 200$ GeV at the rapidity y < 0.5 (**Table 1**). The figures in **Figures 2(a)** and (b) depict the results for the $\Xi^-, \overline{\Xi}^+, \Lambda, \overline{\Lambda}$ particle production for the same collision at the same energy (**Table 2**). The cases of k^+ and k^- production in gold-gold reaction at the same energy and at different centralities are reproduced by power laws in **Figures 3(a)** and (b) (**Table 3**). In **Figures 4(a)** and (b)



Figure 1. Transverse momentum spectra for production of keon (K^0, K^+, K^-) , lamda (Λ) , lamdabar $(\overline{\Lambda})$, cascade minus (Ξ^-) , cascade plus bar (Ξ^+bar) particles in pp collisions at $\sqrt{s} = 200$ GeV. The experimental data are taken from Ref. [5]. The solid curves are fits for power-law model.

Tab	le 1.	Numerical	values o	f the fit	parameters	of power	law equat	ion for k	keon and	lamda	production	in p-p	collisions	at
$\sqrt{s_N}$	$\overline{v} = 2$	200 GeV, p	$p_T = 0$ to 5	5 GeV/c.										

Sesondaries	Α	q	п	$\frac{\chi^2}{ndf}$
K_s^0	0.563 ± 0.023	3.108 ± 0.025	15.005 ± 0.032	20.707/17
$K^{\scriptscriptstyle +}$	1.067 ± 0.032	1.581 ± 0.055	10.000 ± 0.209	10.943/9
K^{-}	0.066 ± 0.008	2.895 ± 0.026	15.116 ± 0.405	0.584/6
Λ	0.273 ± 0.036	3.092 ± 0.068	15.007 ± 0.074	18.393/15
Лbar	0.029 ± 0.001	3.010 ± 0.068	15.016 ± 0.068	15.673/15



Figure 2. Transverse momentum spectra for production of cascade $(\Xi^-, \Xi^+ bar)$, lamda and lamdabar particles in pp collisions at $\sqrt{s} = 200$ GeV. The experimental data are taken from Ref. [17]. The solid curves are fits for power-law model.



Figure 3. Transverse momentum spectra for production of kaon (K^+, K^-) at different centrality at $\sqrt{s} = 200$ GeV in Au-Au collisions. The experimental data are taken from Ref. [18]. The solid curves are fits for power-law model.

Table 2. Numerical values of the fit parameters of power law equation for lamda and cascade particle production in p-p collisions at $\sqrt{s_{_{NN}}} = 200 \text{ GeV}, p_T = 0 \text{ to 5 GeV/c}.$

Sesondaries	Α	q	п	$\frac{\chi^2}{ndf}$
Λ	1.848 ± 0.094	1.306 ± 0.061	10.417 ± 0.244	26.883/12
Лbar	0.541 ± 0.026	1.910 ± 0.068	12.005 ± 0.061	36.675/13
≘⁻	0.022 ± 0.001	1.804 ± 0.023	10.033 ± 0.119	21.4/8
$\Xi^* bar$	0.063 ± 0.002	2.301 ± 0.015	9.951 ± 0.466	24.246/9



Figure 4. Transverse momentum spectra for production of lamda (Λ) and lamda bar ($\overline{\Lambda}$) particles at different centrality in Au-Au collisions. The experimental data are taken from Ref. [18] The solid curves are fits for power-law model.

Table 3. Numerical values of the fit parameters of Power Law equation for keon production (k^+, k^-) in Au-Au collisions at $\sqrt{s_{_{NN}}} = 200 \text{ GeV}$ at different Centrality, $p_T = 0$ to 2 GeV/c.

Secondaries	Centrality (%)	Α	q	п	$\frac{\chi^2}{ndf}$
	0 - 5%	120.114 ± 8.341	2.508 ± 0.057	10.376 ± 0.172	4.555/14
1.+	20% - 30%	47.371 ± 2.28	1.998 ± 0.029	9.989 ± 0.112	2.477/15
κ	40% - 50%	16.727 ± 2.703	1.843 ± 0.082	10.006 ± 0.111	1.294/12
	60% - 70%	3468.89 ± 253.7	2.030 ± 0.033	11.981 ± 0.154	3.022/13
	0 - 5%	435.675 ± 15.85	2.286 ± 0.076	12.00 ± 0.264	1.032/15
1 -	20% - 30%	68.462 ± 6.505	2.379 ± 0.061	12.003 ± 0.07	0.246/12
ĸ	40% - 50%	25.454 ± 0.901	2.239 ± 0.084	12.052 ± 0.297	1.566/14
	60% - 70%	5.058 ± 0.283	2.016 ± 0.101	11.819 ± 0.382	3.356/14

the production of Λ and $\overline{\Lambda}$ are shown in the same collision at the same energy(**Table 4**). In **Figures 5(a)**, (**b**) and (**c**) the production of cascade, cascade-bar and omega particles production are shown at different centralities and at the same energy (**Table 5**). The cases of production of neutral kaons and lamda particles in Copper-Copper collision at $\sqrt{s} = 200$ GeV are plotted in **Figures 6(a)** and (**b**) with reckoning of the parameters presented in **Tables 6** and 7.

In **Table 8** the values of average transverse momenta $(\langle p_T \rangle)$ for different produced secondaries in protonproton and gold-gold collisions have been computed. All these values tally with the similar ranges arrived at by experimental measurements. This helps us to obtain for us a consistency check-up of the parameter values used for getting fits to the data on $\langle p_T \rangle$ -spectra. In **Figures 7(a)** and **(b)** we see, the lamda-bar to lamda and cascademinus to cascade-plus particle production cross-section ratio as a function of transverse momentum respectively and the ratio gradually fall off with increasing values of p_T . In **Figures 8(a)** and **(b)** the nuclear modification factors (R_{CP}) are plotted against transverse momentum for the production of neutral meson and lamda particles in copper-copper collision. With the increasing p_T , the R_{CP} -values fall off. In addition, the data show the R_{CP} for baryons exhibits a lower fall-off compared with that of mesons in intermediate transverse momentum region. The experimental data show that the baryonmeson difference of R_{CP} disappears at higher p_T .

The data on production of strange particles described

Secondaries	Centrality (%)	Α	q	п	$\frac{\chi^2}{ndf}$
	0 - 5%	63.570 ± 4.841	3.542 ± 0.105	16.124 ± 0.140	10.064/18
	10% - 20%	772.323 ± 48.46	2.000 ± 0.087	13.870 ± 0.259	5.778/14
Λ	20% - 40%	141.42 ± 11.33	2.002 ± 0.023	12.815 ± 0.099	7.512/12
	40% - 60%	34.334 ± 2.317	2.025 ± 0.109	12.820 ± 0.308	7.238/12
	60% - 80%	19.811 ± 0.968	2.000 ± 0.014	11.869 ± 0.235	14.913/12
	0 - 5%	198.233 ± 0.269	2.000 ± 0.014	10.765 ± 0.269	7.588/10
	10% - 20%	29.161 ± 1.727	2.002 ± 0.121	12.003 ± 0.072	17.298/10
Λbar	20% - 40%	122.194 ± 4.406	2.899 ± 0.014	15.001 ± 0.054	3.589/8
	40% - 60%	25.183 ± 0.646	2.500 ± 0.008	12.888 ± 0.028	9.902/12
	60% - 80%	10.994 ± 0.788	2.001 ± 0.022	11.995 ± 0.088	31.900/13

Table 4. Numerical values of the fit parameters in Power Law form for (Λ and $\overline{\Lambda}$) production in Au-Au collisions at $\sqrt{s_{_{NN}}} = 200 \text{ GeV}$ at $p_T = 1$ to 5 GeV/c ranges and for various Centrality-values as given below.

Table 5. Numerical values of the fit parameters in Power Law equation for cascade-minus (Ξ^-), cascade-plus bar (Ξ^+bar) and $\Omega^- + \Omega^+$ particle production in Au-Au collisions at $\sqrt{s_{_{NN}}} = 200$ GeV at different p_T -values = 1 to 5 GeV/c for various Centrality values.

Secondaries	Centrality (%)	A	q	n	$\frac{\chi^2}{ndf}$
	0 - 5%	213.632 ± 16.06	1.344 ± 0.015	10.042 ± 0.271	14.593/8
	10% - 20%	1019.24 ± 81.73	1.001 ± 0.010	10.017 ± 0.058	20.800/6
Ξ⁻	20% - 40%	2978.87 ± 155.4	1.000 ± 0.006	11.003 ± 0.001	6.138/6
	40% - 60%	16.505 ± 0.288	1.500 ± 0.003	10.019 ± 0.055	0.675/7
	60% - 80%	3.065 ± 0.339	1.503 ± 0.027	10.001 ± 0.111	16.968/5
	0 - 5%	1531.25 ± 144	1.001 ± 0.012	9.996 ± 0.070	18.845/7
	10% - 20%	5248.25 ± 468.3	0.801 ± 0.008	10.011 ± 0.053	22.498/6
$\Xi^{*}bar$	20% - 40%	735.176 ± 31.68	0.999 ± 0.005	9.996 ± 0.031	24.258/8
	40% - 60%	233.981 ± 15.39	0.999 ± 0.008	9.997 ± 0.047	16.173/7
	60% - 80%	2.993 ± 0.188	1.496 ± 0.014	10.006 ± 0.057	4.049/4
	0 - 5%	1.248 ± 0.094	3.018 ± 0.055	9.988 ± 0.139	2.073/3
$\Omega^{\scriptscriptstyle -} + \Omega^{\scriptscriptstyle +}$	20% - 40%	2.970 ± 0.229	1.997 ± 0.028	9.978 ± 0.100	1.304/2
	40% - 60%	0.342 ± 0.037	2.001 ± 0.043	9.235 ± 0.131	4.755/3

here pertain, in the main, to the "hard" sector of high energy interactions. And it is well-known that Hagedornian power law forms which have their roots in the physics of quantum chromodynamics (QCD) describe hard particle production in a modestly successful manner for, at least, the light hadrons of which strange K-mesons constitute a part.But some of the strange particles have moderately high masses, for which our objective here was primarily to check whether this generalized power law form could address the issues of invariant p_T spectra and some other related observables in a satisfactory manner for all the strange hadrons. And the outcome is: this study is strongly affirmative by all indications and yardsticks of actual performances.

The data on strange particles are in general quite sparse. The errors in measurements are also in most cases



Figure 5. Transverse momentum spectra for production of cascade minus (Ξ^-) ,cascade plus bar (Ξ^+bar) and omega particles $(\Omega^- + \Omega^+)$ at different centrality in Au-Au collisions at $\sqrt{s} = 200$ GeV. The experimental data are taken from Ref. [19]. The solid curves are fits for power-law model.

Table	6. Numerical	values of	the fit	parameters	of Power	Law	equation	for	keon	production	(k_{s}^{0})	in	Cu-Cu	collisions	; at
$\sqrt{s_{NN}}$	= 200GeV at	different	p_T -val	ues = 1 to 9	GeV/c and	for s	everal Ce	ntral	lity do	omains.					

Secondaries	Centrality (%)	Α	q	п	$\frac{\chi^2}{ndf}$
	0 - 10%	112.079 ± 4.169	2.020 ± 0.035	12.229 ± 0.094	6.675/21
	10% - 20%	67.284 ± 2.154	1.921 ± 0.031	11.712 ± 0.477	6.068/21
7.0	20% - 30%	24.860 ± 0.568	2.879 ± 0.030	14.155 ± 0.262	3.667/21
<i>K</i> _s	30% - 40%	17.099 ± 0.286	2.565 ± 0.070	$13.181 \pm 0.0.245$	6.230/21
	40% - 50%	18.504 ± 0.598	1.985 ± 0.036	11.583 ± 0.476	7.738/21
	50% - 60%	12.159 ± 0.536	2.000 ± 0.006	11.551 ± 0.054	7.842/21

10⁶

10⁴

10²

10⁰

10⁻²

10⁻⁴

10⁻⁶

1

 $d^2N/(2 m_T dp_T dp_T dy)(GeV)^{-2}$



Figure 6. Transverse momentum spectra for production of neutral keons (k_s^0) and lamda (Λ) particles at different centrality in Au-Au collisions at $\sqrt{s} = 200$ GeV. The experimental data are taken from Ref. [18]. The solid curves are fits for power-law model.



Figure 7. Transverse momentum-dependence spectra of Λ bar/ Λ and Ξ^-/Ξ^+bar for pp collision at $\sqrt{s} = 200$ GeV. The data type-points are taken from Ref. [5]. The solid curves or lines are drawn on the basis of Power Law Model.

Table 7. Numerical values of the fit parameters of Power Law equation for lamda particle (Λ) production in Cu-Cu collisions at $\sqrt{s_{_{NN}}}$ = 200 GeV at different Centrality, p_T = 1 to 7 GeV/c.

Secondaries	Centrality (%)	Α	q	п	$\frac{\chi^2}{ndf}$
	0 - 10%	1091.23 ± 47.1	2.000 ± 0.009	14.428 ± 0.166	2.635/12
	10% - 20%	139.73 ± 12.91	2.034 ± 0.103	13.033 ± 0.275	15.067/14
	20% - 30%	105.009 ± 5.974	2.024 ± 0.073	12.996 ± 0.193	16.425/17
Λ	30% - 40%	46.450 ± 1.523	2.026 ± 0.081	12.629 ± 0.216	24.731/17
	40% - 50%	27.007 ± 1.798	2.001 ± 0.018	12.340 ± 0.215	16.803/15
	50% - 60%	24.819 ± 2.077	2.029 ± 0.023	13.042 ± 0.097	17.437/14



Figure 8. Plots of the nuclear modification factor (R_{cp}) versus p_T spectra for Cu-Cu collisions at $\sqrt{s} = 200$ GeV. The experimental data are taken from Ref. [18]. The solid curves indicate the power-law-based description of the data.

Table 8. Calculated values of average transverse momentum $\langle p_T \rangle$ for $y_{cm} < 0.5$ at $\sqrt{s_{NN}} = 200$ GeV.

Nature of Collisions	Secondaries	Value of $\langle p_T \rangle$ in GeV/c
	k_s^0	0.52
	$k^{\scriptscriptstyle +}$	0.45
	k^-	0.48
p-p Collisions	Λ	0.52
	$\overline{\Lambda}$	0.50
	≘⁻	0.51
	$\Xi^{+}bar$	0.86
Au-Au Collisions	$k^{\scriptscriptstyle +}$	0.68
	k^-	0.50
	Λ	0.54
(Control)	$\overline{\Lambda}$	0.52
(Central)	≘⁻	0.38
	$\Xi^{+}bar$	0.28
	$\Omega^{\scriptscriptstyle -} + \Omega^{\scriptscriptstyle +}$	0.86

quite considerable. If these limitations are taken into account, the importance of this comprehensive work, though entirely phenomenological, assumes some degree of importance. Barring these generalized comments on an overall basis, we have some specific observations as well, which are also quite well-merited and are being presented hereafter: 1) The power indices for all the varieties of strange hadrons lie in the most cases in the range of 10 - 12. This is in accord with the prescription on the limit set by Brodsky [20], with the q-values bordering on the values, 2 - 3. 2) The χ^2 /ndf values for production of cascade and omega particles have suffered quantitatively due to very small values of the number of degrees of freedom. 3) Thirdly, the q and n values do not exhibit any clear centrality-dependence or the mass-dependence of the observed heavy secondaries. 4) However, we cannot ascertain at this moment the properties of these parameters with regard to the nature of their energydependence(s), if any. 5) The average momentum values of these measured heavy strange baryons are found to be quantitatively compatible with other non-strange light hadrons, though the expression for the average transverse momentum is not very rigorously derived, for which reliability of expression (3) is certainly limited. 6) Some ratio-values shown by Figures 7(a) and (b) are modestly well-described. 7) However, the values of the nuclear modification factor, denoted by R_{CP} , are not reproduced satisfactorily, especially on the lower-side of the p_T -values. But this is no wonder, as the used power laws are suited to high- p_T values as was remarked above very concretely. 7) Still, with one of the simplest approaches, that we have succeeded in explaining the characteristics of a large bulk of data on these rare hadrons is certainly quite stimulating to and encouraging for us.

4. Concluding Remarks

Let us now sum up;

1) The used power laws explain quite well the mea-

sured data on the observables like, p_T -spectra, some ratio-behaviours and the nuclear modification factors; so none of the questions related to suppression or enhancement is consequential. 2) The centrality-dependences of the p_{T} -spectra of strange hadrons too are well-reproduced. 3) The essential physical content of the power-law models is the modest observance of the p_T -scaling (as is reflected in the $\sim p_T/p_0$ term). And in terms of the functional efficacy, this model seems, so far, to be at par, if not better, with all other numerous existing approaches within the frameworks of either the Hadronic transport models or the statistical models [21-25]. 4) The model obviously bears no relationship with the concept of the quark-gluon plasma (QGP), which is, still, just a conjecture with so far no clear and concrete experimental support. 5) In a conclusive vein, this has to be asserted that we observe nothing too strange about "strangeness" production in high energy interactions.

The latter two points in the preceding paragraph highlight, in the main, the novelty of this study from a global viewpoint against the background of the current trends and streaming ideas in the present day Particle Physics.

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