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Comparison of One-Dimensional Analysis with Experiment for CO_2 Two-Phase Nozzle Flow

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Abstract

The aim of this study is to investigate CO_2 two-phase nozzle flow in terms of both experimental and analytical aspects for the optimum design of two-phase flow nozzle of CO_2 two-phase flow ejector. In the experiment, it is measured that the temperature profile in the stream-wise direction of a divergent-convergent nozzle through which CO_2 in the supercritical pressure condition is blown down into the atmosphere. In the analysis, a one-dimensional model which assumes steady, adiabatic, frictionless, and equilibrium is proposed. In the convergent part of the nozzle the flow is treated as single-phase flow of liquid, whereas in the divergent part the flow is treated as separated two-phase flow with saturated condition. The analytical results indicate that the temperature and the pressure decrease rapidly in the divergent part, and the void fraction increases immediately near the throat. Although this analysis is quite simple, the analytical results can follow the experimental results well within this study.

Keywords

Carbon Dioxide, High-Speed Nozzle Flow, Gas-Liquid Two-Phase Flow with Phase Change, Blow down Test, One-Dimensional Analysis

1. Introduction

In recent years, natural refrigerants are regarded as refrigerants of refrigeration cycles. Because of the highly safety and the small global warming coefficient, carbon dioxide (CO_2) is especially focused. However, wasted available energy which is discarded in a pressure reduction process at an expansion valve in the cycle of using CO_2 is three times as much as that of using R-134a. Therefore, it is important to recover the available energy,

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and a gas-liquid two-phase flow ejector (two-phase flow ejector) is used to recover the energy [1] [2].

A two-phase flow ejector consists of a driving flow nozzle, a suction chamber, a mixing section and a diffuser, respectively. Many researches for single-phase flow ejectors have been carried out. However for two-phase flow ejectors, the mechanisms of the internal flow with phase change have not been sufficiently made clear yet because of the complexly of the flow. This study is carried out on a gas-liquid two-phase flow nozzle (two-phase flow nozzle) of a driving nozzle flow section which is the most important section.

A divergent-convergent nozzle is generally used as a two-phase flow nozzle [3]. In actual CO_2 refrigeration cycle, the typical value of the inlet pressure of the expansion valve is 9 MPa which exceeds the critical pressure of CO_2, 7.38 MPa. Therefore, in a CO_2 two-phase flow nozzle, the pressure of the nozzle inlet and the convergent section pressure are the supercritical pressure. And the flow in the divergent section is the saturated two-phase flow with phase change (boiling). The CO_2 two-phase flow in the nozzle is a complicated high-speed flow with large changes in both temperature and pressure. For the optimum design of the nozzle which has these characteristics, the investigation of the fluid mechanics of the high speed gas-liquid two-phase flow is important.

For the two-phase nozzle flow there are studies conducted by Nakagawa *et al*. [3]-[6]. Nakagawa *et al*. investigated nozzle shapes experimentally [3] [4], and proposed a one-dimensional analytical model for the flow [5] [6]. In their study, they analyzed a shock wave caused by a velocity relaxation and a temperature relaxation with the one-dimensional model. A phenomenon that the flow in the nozzle is not accelerated well owing to the existence of a shock wave has been explained by the analysis. On the other hand, it is necessary to analyze a phenomenon for the case that the flow in the nozzle accelerates and the pressure decreases sufficiently.

The purpose of this study is to conduct experimental and analytical investigations of the flow in the two-phase flow nozzle. In the experiment, in the similar way used by Nakagawa *et al*. [4], the CO_2 flow that goes through the divergent-convergent nozzle into the atmosphere is researched, where the temperature and the pressure distribution in the mainstream direction are investigated. In the analysis, one-dimensional analytical model assumed to be steady, adiabatic process, frictionless and equilibrium is investigated. The model refers to the formularization by Sudo and Katto [7] for the study of the critical flow rate of a compressible gas-liquid two-phase flow. A method to predict various distributions of flow properties along the mainstream is considered by solving fundamental equations, which are obtained from applying conservation of mass, momentum and energy to the flow in the two-phase flow nozzle, simultaneously.

2. Experimental Apparatus and Procedure

Figure 1 is the schematic diagram of experimental apparatus based on Nakagawa *et al*. [4]. The experimental apparatus mainly consists of a high-pressure tank and a test section. The high-pressure tank is filled up with gases in the order of CO_2 and N_2, and high-pressure CO_2 can be obtained. The test section consists of the pressure measuring part at the nozzle inlet and the nozzle.

Figure 2 shows the rectangular nozzle [4] used for the nozzle part, where z is the distance from the nozzle inlet in the mainstream direction. Thermocouples are set on a side wall of the nozzle as shown in **Figure 2**, and the temperature inside the nozzle is measured. The pressure is calculated by using assumption of saturated condition and the measured temperature. CO_2 is emitted through the test section into the atmosphere.

The pressure in the high pressure tank is set four stages from 7.2 MPa to 10.3 MPa, and the CO_2 gas is left until the tank temperature is the same temperature of the room of 300.15 K. At the time of starting the experiment, valve 3 and valve 4 in **Figure 1** are opened to flow CO_2 in the nozzle. The outputs of the thermocouples and the load cell are saved on a computer. The weight of the tank is measured at before and after the test, and the CO_2 mass flow rate is calculated from the duration time of the emission.

3. One-Dimensional Analysis

3.1. Analytical Model

In the CO_2 nozzle flow, the temperature and the pressure distributions change rapidly, and the phase condition changes from supercritical to saturated two-phase condition. To get the most important aspect of the complex flow, in this study, a one-dimensional analytical model is used. For this analysis, the flow is assumed to be steady, adiabatic, no friction and equilibrium.

The flow in the convergent part of the nozzle is treated as a single-phase flow. **Figure 3** shows the single-phase flow model. Where u is the velocity, P is the pressure, T is the temperature, ρ is the density,

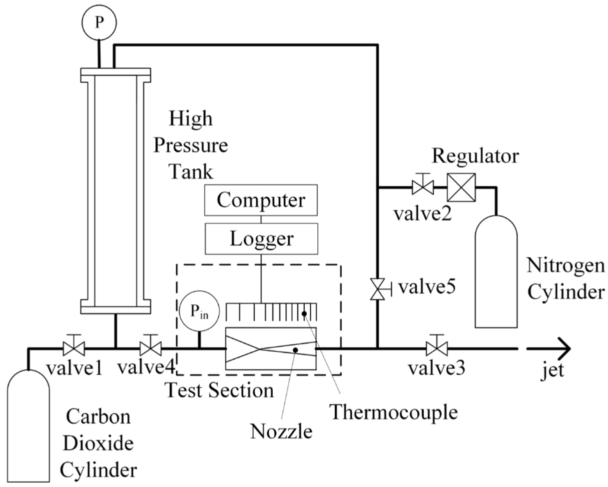

Figure 1. Experimental apparatus.

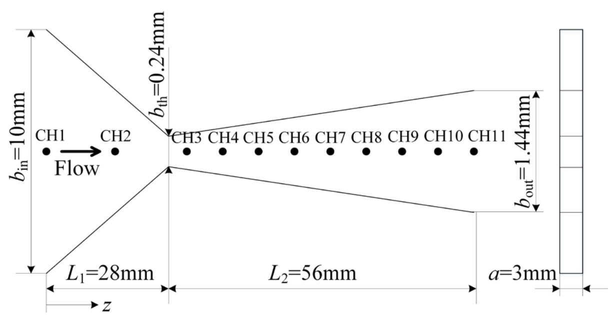

Figure 2. Details of nozzle.

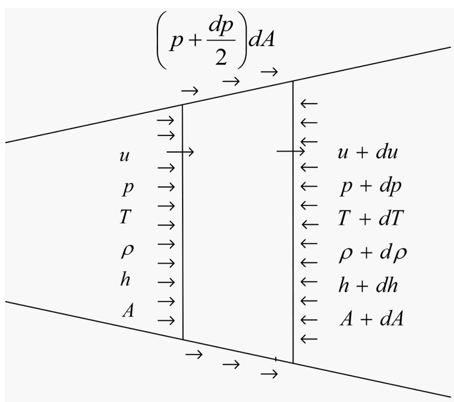

Figure 3. Single-phase flow model for one-dimensional analysis.

h the enthalpy, and A is the area of the cross section. Appling the laws of conservation of mass, momentum and energy to a minute section shown in **Figure 3**, Equations (1) to (3) is obtained,

$$u\left(\frac{\partial \rho}{\partial T}\right)_p \mathrm{d}T + u\left(\frac{\partial \rho}{\partial P}\right)_T \mathrm{d}P + \rho \mathrm{d}u = -\frac{\rho u}{A}\mathrm{d}A \tag{1}$$

$$\frac{1}{\rho u}\mathrm{d}P + \mathrm{d}u = 0 \tag{2}$$

$$\left(\frac{\partial h}{\partial T}\right)_P dT + \left(\frac{\partial h}{\partial P}\right)_T \mathrm{d}P + u\mathrm{d}u = 0 \tag{3}$$

where $\mathrm{d}A$ is positive. To obtain the temperature, the pressure and the velocity variations, $\mathrm{d}T$, $\mathrm{d}P$ and $\mathrm{d}u$, Equations (1) to (3) are solved simultaneously.

The flow in the divergent section is treated as a saturated separated two-phase flow. A model of the separated gas-liquid two-phase flow is shown in **Figure 4**. Appling the laws of conservation of mass, momentum and energy of each phase to a minute section as shown in **Figure 4**, Equations (4) to (7) is obtained,

$$\left\{x\frac{\rho_l u_l}{\rho_g}\frac{\mathrm{d}\rho_g}{\mathrm{d}P} + (1-x)\frac{\rho_g u_g}{\rho_l}\frac{\mathrm{d}\rho_l}{\mathrm{d}P}\right\}\mathrm{d}P + x\frac{\rho_l u_l}{u_g}\mathrm{d}u_g + (1-x)\frac{\rho_g u_g}{u_l}\mathrm{d}u_l + \left(\rho_g u_g - \rho_l u_l\right)\mathrm{d}x = \frac{-\rho_g u_g \rho_l u_l}{\dot{m}}\mathrm{d}A \tag{4}$$

$$\frac{x}{\rho_g u_g}\mathrm{d}P + x\mathrm{d}u_g + \left(u_g - u_i\right)\mathrm{d}x = 0 \tag{5}$$

$$\frac{1-x}{\rho_l u_l}\mathrm{d}P + (1-x)\mathrm{d}u_l + \left(u_i - u_l\right)\mathrm{d}x = 0 \tag{6}$$

$$\left\{x\frac{\mathrm{d}h_g}{\mathrm{d}P} + (1-x)\frac{\mathrm{d}h_l}{\mathrm{d}P}\right\}\mathrm{d}P + xu_g\mathrm{d}u_g + (1-x)u_l\mathrm{d}u_l + \left\{\left(h_g + \frac{1}{2}u_g^2\right) + \left(h_l + \frac{1}{2}u_l^2\right)\right\}\mathrm{d}x = 0 \tag{7}$$

where $\dot{m}$, x and u_i in **Figure 4** are the mass flow rate, the quality and the velocity of the interface of the phases, g and l of subscripts indicate the gas and the liquid phase, and $\mathrm{d}A$ is positive. The pressure, the gas velocity, the liquid velocity and the quality variations of the separated saturated gas-liquid two-phase flow, $\mathrm{d}P$, $\mathrm{d}u_g$, $\mathrm{d}u_l$ and $\mathrm{d}x$, are obtained by solving Equations (4) to (7) simultaneously.

In **Figure 4** and Equations (4) to (7), the phase change model as changing from liquid to gas is applied to the minute mass flow rate, $\mathrm{d}\dot{m} = \dot{m}\mathrm{d}x$, which correspond to the quality variation in minute section, $\mathrm{d}x$.

Simple and typical models of a gas-liquid two-phase flow are a separated flow model neglecting friction between the phases and a homogeneous flow model neglecting velocity difference between the phases. In the divergent section of the two-phase flow nozzle, since it is expected that the distribution of the void fraction changes significantly from 0 to 1, appropriate application of the two models is desirable. However, in this study, the whole flow in the divergent section of the two-phase flow nozzle is simply treated as a separated flow.

3.2. Analytical Object and Methods

The analysis is carried out on the flow in the convergent-divergent nozzle [4] shown in **Figure 2**. The flow in the rectangular nozzle that is 3 mm wide is analyzed in the mainstream direction. The nozzle that the total length

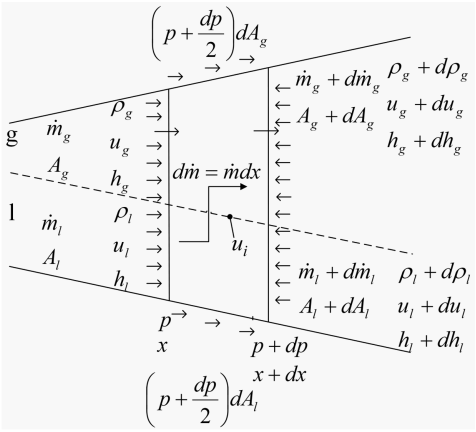

Figure 4. Two-phase flow model for one-dimensional analysis.

is 84 mm is divided into 8400 cells (each section interval is 0.01 mm), and the flow is numerically analyzed by using fundamental equations which is appropriate for each section. There are not much difference between the results of the cases of 8400, 4200 and 16800 cells. Needed CO_2 properties for the calculation process are obtained by PROPATH.

In the convergent section of the nozzle in which flow is treated as the single-phase flow, a flow which reached saturated condition is addressed as saturated liquid. Therefore, excluding $\mathrm{d}T$ in Equations (1) and (3), $\mathrm{d}P$ and $\mathrm{d}u$ is calculated. In the divergent section of the nozzle in which flow is treated as the separated gas-liquid two-phase flow, the velocity at the interface between the phases is the liquid velocity in case of $\mathrm{d}x > 0$, namely $u_i = u_l$. The initial quality is 0.001 at the throat. The experimental results are applied to the inlet boundary conditions of the analysis. Thus, the temperature measured by the thermocouple CH1 shown in **Figure 2** and the measured pressure, P_{in}, shown in **Figure 1** are used as the inlet temperature and the pressure respectively. For the mass flow rate, the value calculated by using the experimental results is used.

4. Results and Discussions

4.1. Experimental Results

The experimental conditions at the nozzle inlet are listed in **Table 1**. The distribution of the temperature measured by the thermocouples in the mainstream direction is shown in **Figure 5**. The CO_2 temperature in the nozzle decreases sharply, and in the case of the tank pressure of 7.2 MPa the difference in temperature between the inlet and the outlet is 80 K. As the tank pressure increases, the temperature at the nozzle outlet also increases. The nozzle outlet temperature for the tank pressure of 10.3 MPa is approximately 17 K larger than that of the tank pressure of 7.2 MPa.

Figure 6 shows the pressure distribution in the mainstream direction in the nozzle obtained by using the assumption of saturated condition and the temperature in **Figure 5**. The CO_2 pressure decreases sharply in the nozzle as well as the temperature. Thus, it could be considered that CO_2 accelerates sufficiently in the nozzle. As the tank pressure increases, the nozzle outlet pressure also increases. Regarding the nozzle outlet pressure, the one for the case of the tank pressure of 10.3 MPa is approximately 0.6 MPa larger than that of 7.2 MPa.

4.2. Comparison of Experiment and Analysis

The comparison of the experimental and the analytical results of the temperature distribution (a) or the pressure distribution (b) in the cases that the tank pressure is set to 7.2, 8.0, 9.2 and 10.3 MPa are shown in **Figures 7-10**.

In the case of the tank pressure of 7.2 MPa, as shown in **Figure 7**, the experimental and the analytical results can be considered almost the same. Since the condition of the nozzle inlet is approximately saturated, the flow in the convergent section of the nozzle is treated as saturated liquid.

In the case of the tank pressure of 8.0 MPa, as shown in **Figure 8**, the temperature distribution of the analysis is almost the same as that of the experiment. In the pressure distribution the analytical results in the convergent section are different from the experimental results, whereas, in the divergent section, the analytical results approximate to the experimental results. In the convergent section, since the critical pressure of CO_2 is 7.38 MPa, it can be considered that the discrepancy is caused by the setting of the tank pressure to the supercritical pressure and the condition is not saturated in the experiment.

Regarding **Figure 9** of the tank pressure of 9.2 MPa, for above reasons, the pressure difference between the experimental and the analytical results in the nozzle convergent section is caused. For the pressure in the divergent section and the temperature distribution of the analysis, the analytical results are rather smaller than the

Table 1. Experimental conditions.

Tank pressure (MPa)	Inlet temperature (K)	Inlet pressure (MPa)	Inlet velocity (m/s)	Mass flow rate (kg/m^3)
7.2	300.0	6.7	2.92	0.06
8.0	300.6	7.6	3.65	0.08
9.2	300.0	8.5	4.34	0.10
10.3	302.2	9.7	5.59	0.13

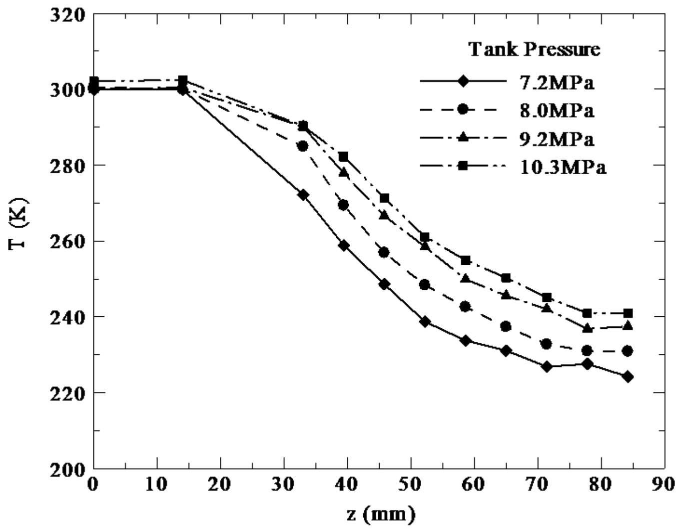

Figure 5. Temperature distributions of experiment.

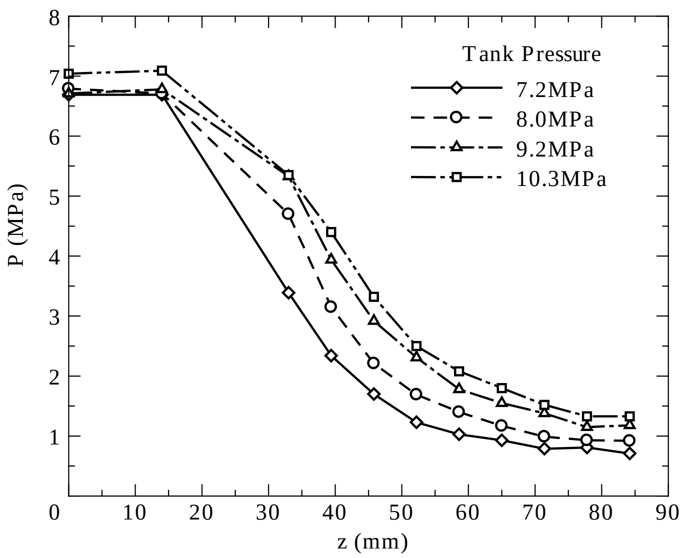

Figure 6. Pressure distributions of experiment.

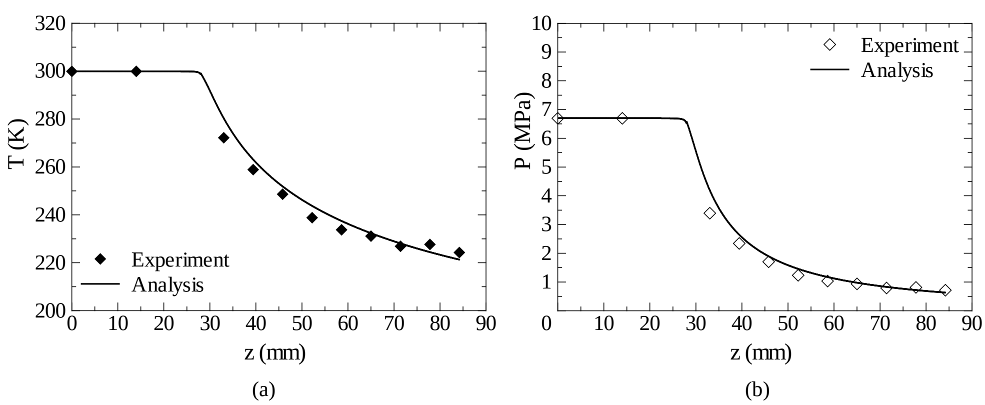

Figure 7. Comparison of analysis with experiment (tank pressure = 7.2 MPa). (a) Temperature; (b) Pressure.

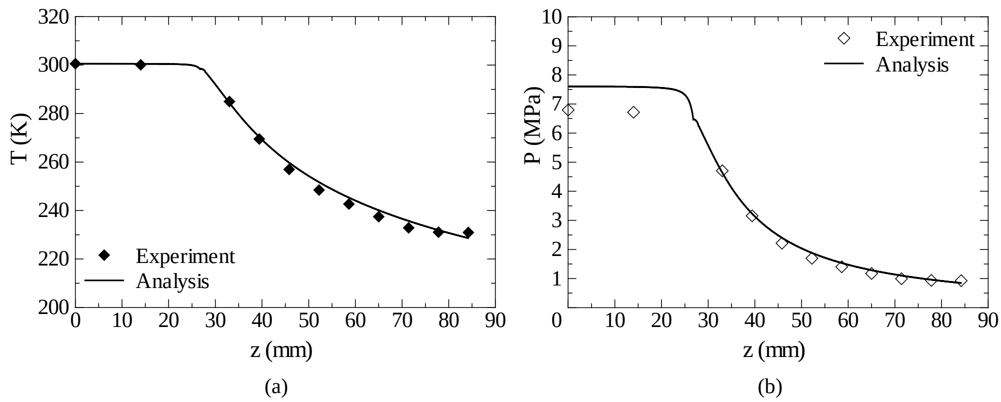

Figure 8. Comparison of analysis with experiment (tank pressure = 8.0 MPa). (a) Temperature; (b) Pressure.

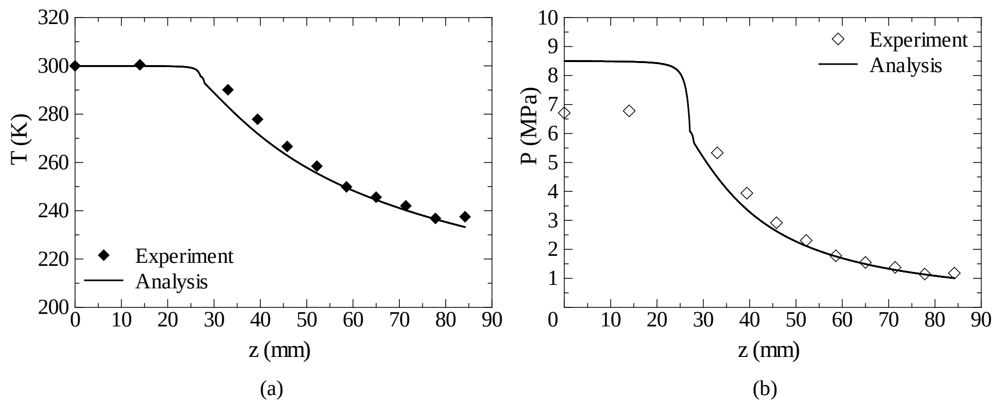

Figure 9. Comparison of analysis with experiment (tank pressure = 9.2 MPa). (a) Temperature; (b) Pressure.

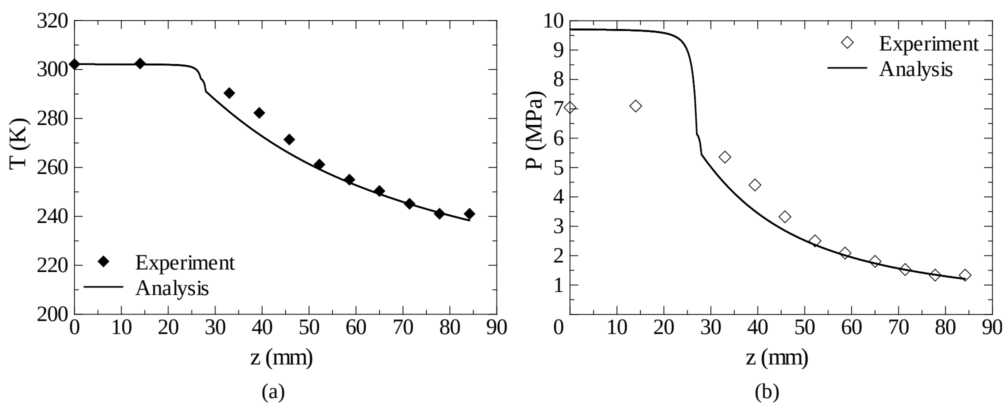

Figure 10. Comparison of analysis with experiment (tank pressure = 10.3 MPa). (a) Temperature; (b) Pressure.

experimental results around the throat. However, in area excepting the near throat, the analytical and the experimental results are approximately the same.

For the pressure in the divergent section and the temperature distribution in the case of the tank pressure of 10.3 MPa shown in **Figure 10**, the differences between the analytical and the experimental results at the near throat is larger than that of 9.2 MPa. Whereas, except the near throat, the analytical and the experimental results are well matched.

4.3. Analytical Results

Figures 11-14 show the analytical flow property distributions of the cases that the tank pressure set to 7.2, 8.0, 9.2 and 10.3 MPa, respectively. In **Figures 11-14**, the analytical results are marked with dotted line for cross sectional area, A/A_{in} (A_{in}: cross sectional area at the nozzle inlet), bold solid line for the temperature, T/T_{in} (T_{in}: the nozzle inlet temperature), small solid line for the pressure, P/P_{in} (P_{in}: the nozzle inlet pressure), bold broken line for the quality, x, small broken line for the void fraction, α, bold chain line for the gas-phase velocity, $u_g/u_{in}/100$ (u_{in}: the nozzle inlet velocity, these values are indicated in **Figures 11-14** respectively), bold two-dot chain line for the liquid-phase velocity, $u_l/u_{in}/100$, small chain line for the gas-phase sonic velocity and small two-dot chain line for $c_g/u_{in}/100$, and the liquid-phase sonic velocity, $c_l/u_{in}/100$. To see these figures clearly, u_g, u_l, c_g and c_l are divided by 100. The single-phase and the two-phase sonic velocity and the void fraction are calculated by following formulas,

$$\alpha = \left(\rho_l u_l x\right) / \left\{\rho_g u_g \left(1-x\right) + \rho_l u_l x\right\} \tag{8}$$

$$c = \sqrt{\left(\mathrm{d}P/\mathrm{d}\rho\right)} \tag{9}$$

$$c_g = \sqrt{\left(\mathrm{d}P/\mathrm{d}\rho_g\right)} \tag{10}$$

$$c_l = \sqrt{\left(\mathrm{d}P/\mathrm{d}\rho_l\right)} \tag{11}$$

where c in the convergent section is marked with the same small chain line as c_g is marked with.

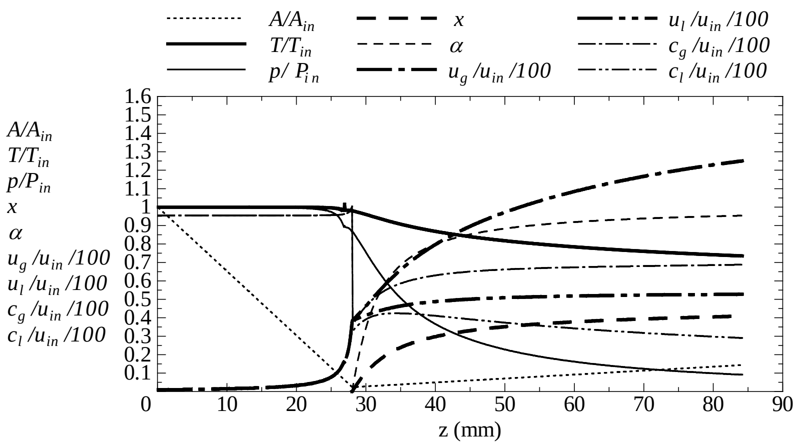

Figure 11. Analytical results (tank pressure = 7.2 MPa).

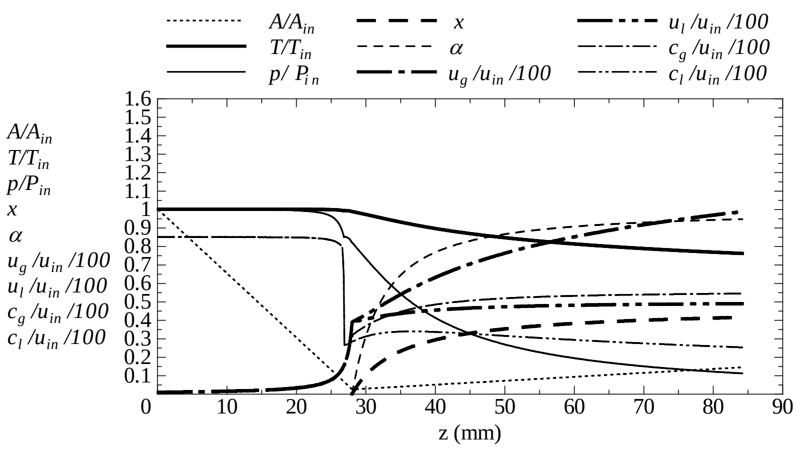

Figure 12. Analytical results (tank pressure = 8.0 MPa).

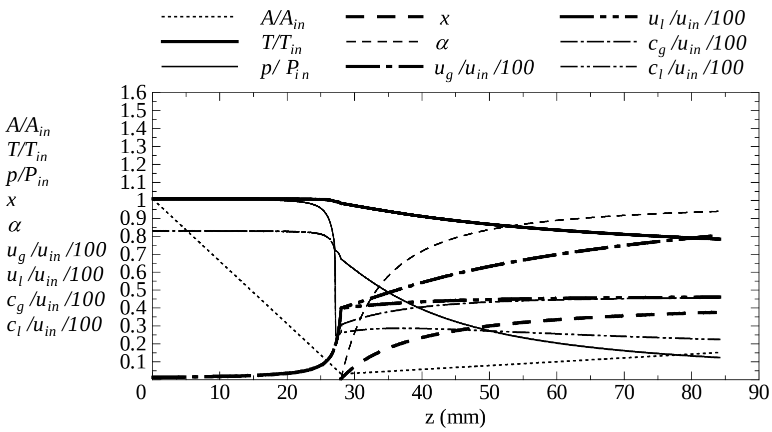

Figure 13. Analytical results (tank pressure = 9.2 MPa).

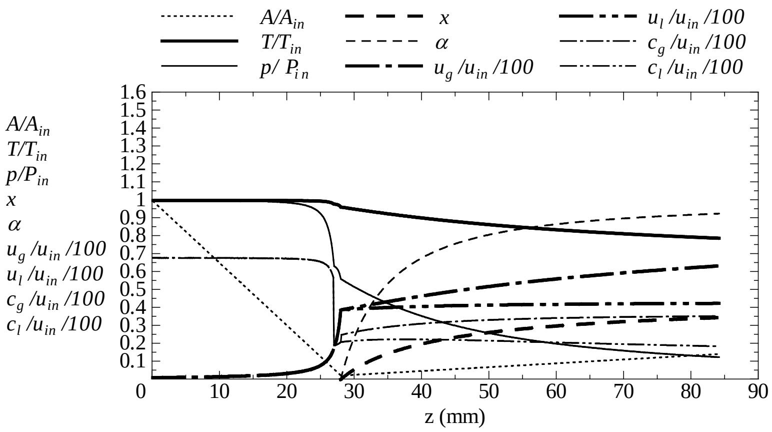

Figure 14. Analytical results (tank pressure = 10.3 MPa).

In the case of the tank pressure of 7.2 MPa, as shown in **Figure 11**, the quality, x, is approximately 0.45, the void fraction, α, is approximately 0.95, the gas-phase velocity, u_g, is 1.3, and the liquid-phase velocity, u_l, is 0.55. Multiplied u_{in} and 100, u_g and u_l are approximately 400 m/s and 160 m/s respectively. In these analytical results, satisfactory accelerating performance of the nozzle is indicated. To elucidate the validity of these analytical results, it is necessary to investigate these flow properties experimentally.

Compared with the sonic velocity defined in this paper, the gas-phase and the liquid-phase velocity is larger. From perspective of the sonic velocity defined in this paper, the flow in the divergent section can be considered as the supersonic flow.

In the case that the tank pressure is set to 8.0 MPa, as shown in **Figure 12**, general trend is the same as the tank pressure of 7.2 MPa. However, the velocity in the case of the tank pressure of 8.0 MPa is smaller than that of the case of 7.2 MPa despite the larger inlet velocity in the case of 8.0 MPa. As shown in **Figure 13** and **Figure 14**, it can be considered that the higher pressure tank is set, the smaller velocity is calculated.

5. Summary

The flow field of the carbon dioxide high-speed two-phase nozzle flow has been investigated by the experiment and the one-dimensional analysis. Obtained results are as follows.

1) For the nozzle form used in this research and the experimental condition, the CO_2 temperature and the pressure have been significantly and monotonously decreased in the nozzle, and satisfactory accelerating performance has been presumed.

2) For the nozzle flow, the one-dimensional analytical model which assumes to be steady, adiabatic, no friction and equilibrium is proposed. In this analytical model, the nozzle convergent section is assumed to be single-

phase flow, and the nozzle divergent section is assumed to be separated gas-liquid two-phase flow.

3) The proposed analytical model is simple, and the assumption that the flow is the separated saturated gas-liquid two-phase flow does not necessarily satisfy the actual flow. However, in this experimental condition, these analytical results satisfactorily predicted these experimental results.

4) By the one-dimensional analysis, the velocity distribution in the mainstream direction in the nozzle is obtained, and the satisfactory accelerating performance has been indicated. Further, it is made clear that from the perspective of the sonic velocity defined in this analysis the flow in the divergent section can be considered as the supersonic flow.

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