

Maintaining an Optimal Flow of Forest Products under a Carbon Market: Approximating a Pareto Set of Optimal Silvicultural Regimes for *Eucalyptus fastigata*

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A competitive co-evolutionary Multi-Objective Genetic Algorithm (cc-MOGA) was used to approximate a Pareto front of efficient silvicultural regimes for *Eucalyptus fastigata*. The three objectives to be maximised included, sawlog, pulpwood and carbon sequestration payment. Three carbon price scenarios (3CPS), i.e. NZ \$25, NZ \$50 and NZ \$100 for a tonne of CO₂ sequestered, were used to assess the impact on silvicultural regimes, against a fourth non-carbon Pareto set of efficient regimes (nonCPS), determined from a cc-MOGA with two objectives, i.e. competing sawlog and pulpwood productions. Carbon prices included in stand valuation were found to influence the silvicultural regimes by increasing the rotation length and lowering the final crop number before clearfell. However, there were no significant changes in the frequency, timing, and intensity of thinning operations amongst all the four Pareto sets of solutions. However, the 3CPS were not significantly different from each other, which meant that these silvicultural regimes were insensitive to the price of carbon. This was because maximising carbon sequestration was directly related to the biological growth rate. As such an optimal mix of frequency, intensity, and timing of thinning maintained maximum growth rate for as long as possible for any one rotation.

Keywords: Optimal Control; Competitive Co-Evolutionary Multi-Objective Genetic Algorithm (cc-MOGA); Pareto Front; Forest Holding Value; Kruskal-Wallis Test; Multiple Comparison Procedure

Introduction

The central focus of our analysis was to approximate a set of optimal silvicultural regimes for a *Eucalyptus fastigata* forest stand under a carbon market. Each estimated regime was expressed as a set of values that included an initial planting stocking, frequency of thinning, timing of thinning, intensity of thinning, final crop number prior to clear-felling, and rotation length. We, therefore, crafted a three-objective optimisation problem, which simultaneously maximised, sawlog, pulpwood and carbon sequestration payment (under three different payment scenarios). This optimisation problem, described later, was based on a two-objective optimisation problem that was successfully solved by simultaneously optimising competing sawlog and pulpwood products (Chikumbo & Nicholas, 2011). The results from the two-objective and three-objective optimisation runs were statistically analysed to decipher the nuances of silvicultural strategies under a carbon market.

Assumptions and Forest Holding Value

To carry out the analysis, we assumed fixed prices for liquid fuels and fossil fuel-based fertilisers such that the carbon price would remain static over the rotation period. This assumption was based on the observation of the EU Emissions Trading Scheme where the carbon price was heavily influenced by fossil fuel prices, which tend to be volatile (White, 2007). The carbon price would in turn influence the forest holding value,

and ultimately impact the stand silvicultural regimes of *E. fastigata*. Forest holding values enable valuation of timber as real property, where timber for immediate harvesting has a liquidation value and the immature resource has a holding value (Mayo & Straka, 2007).

Any forest has an immediate liquidation value if the existing timber is clearfelled and sold along with the land. The forest holding value is the present value of holding the forest until the optimal rotation age (maximum present value) and then selling the timber and land (Klemperer, 1996). The concept is consistent with standard forestry valuation concepts such as land expectation value (Faustmann, 1995; De Jong, Tipper & Montoya-Gómez, 2000). Note that the forest is financially immature for as long as the forest holding value exceeds the forest liquidation value. Therefore, the rotation age should be allowed to increase until the two values are equal (Mayo & Straka, 2007). Thus, the forest holding value provides an ideal financial criterion to evaluate the impact of carbon sequestration payments on the optimal rotation length.

Contribution to Forest Literature

There is an expectation that any forester/land owner wishing to engage in forest-carbon trading, in order to take advantage of a new income stream from carbon sequestration, would want to know the ideal/optimal silvicultural regimes for his/her crop that will not only maximise carbon sequestration (for maximum

pay out), but also maximise production of sawlog and/or pulpwood (De Jong, Tipper & Montoya-Gómez, 2000). However, we do know that many forest analysts have shown that increasing the rotation length would be the sensible thing to do (Appel, 2001; Asante, Armstrong & Adamowicz, 2011; Gutrich & Hossarwarth, 2007). What is scarce in literature is:

- 1) How the frequency, intensity and timing of thinning for a silvicultural regime are affected;
- 2) Whether the longer rotation length is linked to a higher or relatively lower final crop number before clearfell;
- 3) The ability to simultaneously cater for pulpwood and sawlog products under a prevailing carbon market from a single regime instead of different regimes where each specifically caters for a unique product; and
- 4) How the silvicultural regimes are affected by different carbon prices.

Our paper addresses these issues. The specifics of determining the optimal initial planting stocking, optimal rotation length, frequency, timing and intensity of thinning, tree species and site, have been modeled by forest analysts using multi-stage optimisation since the 60s' (Hool, 1965), with mixed successes. The reasons for these mixed successes boiled down to the use of inappropriate growth functions, and an inability to do an exhaustive search for all possible states in a dynamic programming formulation (Chikumbo, 1996; Chen, Rose & Leary, 1980). Bellman (1957) coined this exhaustive search problem, the "curse of dimensionality".

Historical Background of Problem Solving

A pulpwood production was characterised by short rotations and a relatively longer rotation with thinning (i.e., partial harvesting) for a sawlog production regime (Newman & Wear, 1993). To overcome the curse of dimensionality, the determination of optimal silvicultural regimes for separate pulpwood and sawlog production was pursued using a specialised mathematical formulation, i.e. a combined optimal control and optimal parameter selection optimisation (Chikumbo & Mareels, 2003). The growth dynamics of a tree crop stand were described with difference equations in discrete-time and the complete mathematical formulation was as follows:

$$\underset{u,z}{\text{minimise}} \left\{ \begin{array}{l} J_0(\mathbf{u}, z) = \phi_0(\mathbf{x}(T), z) \\ + \sum_{t=0}^T j_0(t, \mathbf{x}(t), \mathbf{u}(t), z) \end{array} \right\} \quad (1)$$

subject to the growth dynamics,

$$\mathbf{x}(t+1) = \mathbf{f}(t, \mathbf{x}(t), \mathbf{u}(t), z), (\text{which is the state variable}), \quad (2)$$

where,

J_0 = the cost functional (i.e. a function of state and control variables or simply the objective function);

ϕ_0 = continuously differentiable function;

j_0 = continuously differentiable function with respect to the state and control variables;

$t = 0, 1, \dots, T - 1$ (for time in years);

T = rotation age; and

z = estimated parameter(s) independent of time,

$$\mathbf{u}(t) = [u_1(t), \dots, u_T(t)], \quad (3)$$

which represented the control vector over the rotation length at one year intervals expressed as the number of trees harvested

per hectare, i.e.

$$sph(t) = sph(t-1) - u_i(t-1), \quad (4)$$

where, $sph(t)$ = the number of standing trees at time, t , and

$$u_i^L(t) \leq u_i(t) \leq u_i^U, \quad (5)$$

where for each u_i , there are Lower and Upper bounds.

The state variable consisted of the mean stand height, which was the mean height in metres per hectare ($mht(t)$), the stand basal area in square metres per hectare ($sba(t)$), and stand volume in cubic metres per hectare ($vol(t)$), i.e.

$$\mathbf{x}(t) = \begin{bmatrix} mht(t) \\ sba(t) \\ vol(t) \end{bmatrix},$$

where,

$$\begin{aligned} mht(t) &= a_1^{mht} mht(t-1) + a_2^{mht} mht(t-2) + b^{mht}, \\ a_1^{mht} &= f(sph(t)), a_2^{mht} = f(sph(t)), \\ b^{mht} &= f(sph(t)), \end{aligned} \quad (6)$$

was a discrete-time dynamical model/difference equation (Ljung, 1987) of mean stand height with parameters, a_1^{mht} , a_2^{mht} , and b^{mht} and that were functions of the number of standing trees, $sph(t)$,

$$\begin{aligned} sba(t) &= a_1^{sba} sba(t-1) + a_2^{sba} sba(t-2) + b^{sba}, \\ a_1^{sba} &= f(sph(t)), a_2^{sba} = f(sph(t)), \\ b^{sba} &= f(sph(t)), \end{aligned} \quad (7)$$

the stand basal area, also a discrete-time dynamical model with parameters a_1^{sba} , a_2^{sba} , and b^{sba} that were functions of the number of standing trees, $sph(t)$,

$$\begin{aligned} vol(t) &= a_1^{vol} vol(t-1) + a_2^{vol} vol(t-2) + b^{vol}, \\ a_1^{vol} &= f(sph(t)), a_2^{vol} = f(sph(t)), \\ b^{vol} &= f(sph(t)), \end{aligned} \quad (8)$$

the stand volume function, a discrete-time dynamical model with parameters, a_1^{vol} , a_2^{vol} , and b^{vol} that were functions of the number of standing trees, $sph(t)$,

$$z = [z_{sph(t_0)}, z_T] \quad (9)$$

the system parameters independent of time,

where,

$z_{sph(t_0)}$ = estimated initial planting stocking; and

z_T = estimated rotation length,

subject to Lower and Upper bounds,

$$z_{sph(t_0)}^L \leq z_{sph(t_0)} \leq z_{sph(t_0)}^U, \text{ and } z_T^L \leq z_T \leq z_T^U \quad (10)$$

The problem (1)-(10) was solved using Pontryagin's Maximum Principle (PMP) (Chikumbo & Mareels, 2003). Only a single objective problem was solved at any one time, i.e. either a value production or a volume production cost functional. It is possible to solve a multi-objective optimal control problem using PMP (Malinowska & Torres, 2007) for a finite number of cost functionals, but this was never meant to be because of one problem. Trying to estimate the optimal rotation length as a system parameter led to ill-conditioning (i.e. an accumulation

of round-off errors from a sequence of matrix inversions that are numerically unstable and that may result in meaningless solutions). We will call this the phase-1 ill-conditioning. The stopgap measure was to solve a series of combined optimal control and a single optimal selection problem (i.e. the initial planting stocking), with a fixed T at increasing intervals, so as to locate the optimal rotation length. Optimal T was the point at which a unit increment in T resulted in “phase-2 ill-conditioning”, caused by growth dynamics that could not perform outside their range. The whole process was time-consuming, giving a clear signal that a better way had to be found.

Research Focus and Problem Solving

A switch to a single objective genetic algorithm eliminated the phase-1 ill-conditioning problem (Chikumbo, 2009a). Chikumbo and Nicholas (2011) demonstrated a two-objective genetic algorithm that simultaneously optimised for a value and volume production for *Eucalyptus fastigata*. **Figure 1** shows a summary of a generic genetic algorithm and how it works.

A genetic algorithm is initialized with a population of randomly generated individuals which is a guided process of “selection”, “crossover/recombination” and “mutation”. Individuals are selected on the basis of their fitness for reproduction. The parent individuals are recombined to produce offspring where only some of them are mutated with a certain probability. The fitness of the offspring is then computed, resulting in the parents being replaced, thus producing a new generation. If the criteria of the objective function(s) are not met, this cycle is performed again until the optimisation criteria are reached (Polheim, 2006).

Therefore, a set of efficient silvicultural regimes that satisfied carbon sequestration payments, sawlog/value production, and pulpwood/volume production at different levels of “trade-offs” was estimated using genetic algorithms, eliminating the need for determining separate regimes for sawlog and pulpwood under a carbon market. Such regimes give the forester/land owner the ability to satisfy the market with all products whilst maximizing carbon sequestration payments, with the flexibility of meeting an increased/decreased supply of any one of the products as dictated by demand.

The critical part of approximating the set of efficient thinning regimes was to find “trade-off” solutions (i.e. non-dominated solutions) where for each solution an improvement in one objective did not lead to worsening in the other (Osborne & Rubenstein, 1994). The set of solutions to the three-objective problem was determined using a competitive co-evolutionary genetic algorithm with five subpopulations of 100 individuals each, computed over 1000 generations. These sub-populations evolved independently for a certain number of generations (isolation time). After the isolation time a number of individuals were distributed between the sub-populations (a process called migration). Each sub-population exerted selective pressure on the other, thereby maintaining diversity a lot longer than each sub-population would do solitarily, thereby guarding against premature convergence. When competition was superimposed between the sub-populations, the ones with higher mean fitness values were allowed to maintain larger sub-population sizes and received more capable individuals, since they had more chances of finding the global optimum (Chikumbo, 2009b; Chikumbo, 2012).

The Fonseca and Fleming ranking scheme (Fonseca & Fleming, 1993) was used to determine the non-dominated solutions, also referred to as the Pareto front. A conflict in the objectives results in a trade-off set (i.e. Pareto), which means that the solutions in the set are optimal in the wider sense that no other solutions in the search space are superior to them when all the objectives are considered.

Fonseca and Fleming called their ranking scheme, Multi-Objective Genetic Algorithm (MOGA) and it involves assigning an individual’s rank (in the objective function space) equal to the number of population individuals that dominated that individual. What this means is that ranking of the individuals prior to selection for recombination is done according to the degree of domination; the more members of the current population that dominate a particular individual, the lower its rank. MOGA, therefore, uses fitness sharing in the objective function space and recombination is also restricted. Reproduction probabilities are determined by means of exponential ranking. Afterwards the fitness values are averaged and shared among individuals having identical ranks (Zitler, Deb, & Thiele, 2000). Finally, stochastic universal sampling, which provides zero bias

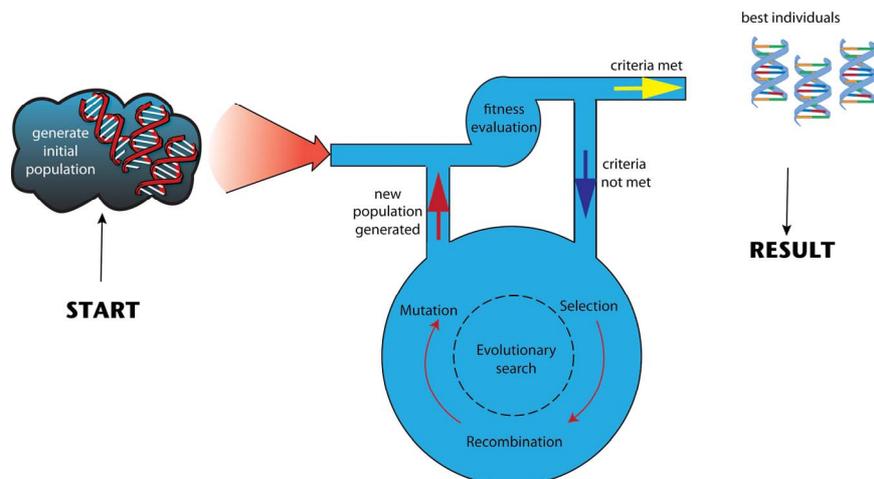


Figure 1. Genetic algorithm search that mimics evolutionary processes to solve optimisation problems (diagram from (Chikumbo, 2012)).

and minimum spread (i.e. the range of possible values for the number of offspring of an individual), is used to fill the sampling pool. The main strength of MOGA is that it is efficient and relatively easy to implement. It has also been successfully implemented in solving optimal control problems with good overall performance (Coello, 1996).

In this paper we follow the formulation by (Chikumbo & Nicholas, 2011), a competitive co-evolutionary Multi-Objective Genetic Algorithm (cc-MOGA), but with an additional objective, i.e. maximisation of carbon sequestration payment. We focus on the three objectives and the forest holding values (held static because of the assumption of fixed prices of liquid fuel and fossil fuel-based fertilisers) on how they are formulated and discuss the results of the three-objective cc-MOGA. The same species, *E. fastigata*, and growth functions by (Chikumbo & Nicholas, 2011) are used in our analysis.

Data

First, *E. fastigata* Deane & Maiden is considered to be the most suitable eucalypt for a wide range of sites in New Zealand because it has a wide site tolerance, performing well from near sea level to altitudes up to 500 m (Chikumbo & Nicholas, 2011). It can be utilised for solid timber as well as providing short-fibre pulp for the production of fine printing paper (Miller, Hay, & Ecroyd, 2000; Haslett, 1988).

The data used for the growth dynamics of *E. fastigata* came from a 1979 Nelder trial (Nelder, 1962) established in Kain-garoo Forest (latitude 38°27.6'S, longitude 176°39.9'E and an altitude of 280 m), near Murupara, New Zealand. A Nelder trial consists of trees planted in a series of concentric circles where the growing space available to each tree is determined by the distance to the nearest eight neighbouring trees.

Further detail on the trial is found in (Chikumbo & Nicholas, 2011) and the summary of the data is shown in **Table 1**. The costs and revenue for silvicultural treatments used in determining forest holding values for the valuation of timber products and carbon sequestered were obtained from (Turner, West, Dungey, Wakelin, Maclaren, Adams & Silcock, 2008).

The Model

There were two main reasons why cc-MOGA was used to solve the three-objective optimisation problem:

1) A competitive co-evolutionary genetic algorithms maintains diversity and controls the selective pressure by balancing

Table 1.

Summary of the 286 data points measured from the Nelder trial (Chikumbo & Nicholas, 2011).

	Minimum	Mean	Maximum
Age (years)	3.4	12.2	28.7
SPH (stems·ha ⁻¹)	89	1181	4356
DBH* (cm)	3.6	20.4	61.7
MD# (cm)	4.2	28.1	62.6
Mean stand height~ (m)	3.6	18.7	42.8
Stand basal area (m ² ·ha ⁻¹)	0.1	28.1	113.6
Stand volume (m ³ ·ha ⁻¹)	0.2	182.1	1075.2
Volume mean annual increment (m ³ ·ha ⁻¹ ·year ⁻¹)	0.1	12.1	40.2

*Diameter at breast height measured at 1.4 m; #Mean of 100 largest diameter trees per hectare; ~Mean height of 100 largest diameter trees per hectare.

exploration and exploitation in the search space, in a way that avoids premature convergence (Chikumbo, 2009b; Menczer, Degeratu, & Street, 2000); and

2) Splitting the population into diverse sub-populations that communicate through migration may result in parallel speed-ups (Menczer, Degeratu, & Street, 2000).

The three objectives or cost functionals were as follows:

$$J_n(u)^1 = \max_{u(t)} \sum_{t=T-(n-1)}^T \frac{u(t)}{sph(t)} * sba(t) * \dots * mht(t) * fhv_{NZZ100}(t) * \frac{vol(t)}{sph(t)}, \quad (11)$$

for value production;

$$J_n(u)^2 = \max_{u(t)} \sum_{t=T-(n-1)}^T \frac{u(t)}{sph(t)} * sba(t) * \dots * mht(t) * fhv_{NZZ100}(t), \quad (12)$$

for volume production; and

$$J_n(u)^3 = \max_{u(t)} \sum_{t=T-(n-1)}^T \frac{(sph(t) - u(t))}{sph(t)} * C_f * \dots * vol(t) * fhv_c(t), \quad (13)$$

for carbon sequestration payments.
subject to the constraints,

$$t \geq T - (n - 1) \quad (14)$$

with upper and lower bounds on the control:

$$0 \leq u(t) \leq 300, \forall t \in [t_1, t_2], \quad (15)$$

$$0 \leq u(t) \leq 200, \forall t \in [t_3, t_4], \quad (16)$$

and

$$0 \leq u(t) \leq 200, \forall t \in [t_5, t_6], \quad (17)$$

where,

$t_1 - t_2 = 5 - 10$ year time window for an initial thinning;

$t_3 - t_4 = 12 - 15$ year time window for a second thinning;

$t_5 - t_6 = 18 - 21$ year time window for a third thinning;

$T =$ rotation length;

$fhv_{NZZ100}(t) =$ the forest holding value at time t for sawlogs;

$fhv_{NZZ20}(t) =$ the forest holding value at time t for pulpwood;

$fhv_c(t) =$ the forest holding value at time t for carbon sequestered; and

$C_f =$ a factor of 0.23 that converts total carbon sequestered (tCO₂ m⁻³) from total stand volume (Meade, Fiuza, & Lu, 2008).

The two-parameter selection constraints were formulated as follows:

$$900 \leq sph(t_0) \leq 2000, \quad (18)$$

for the initial planting stock, and

$$25 \leq T \leq 35, \quad (19)$$

for the rotation length, where t_0 is the initial time.

As for the multi-objective problem with the three cost functionals, $J_n(u)^1$, $J_n(u)^2$, and $J_n(u)^3$, there exists no solution pair (\mathbf{x} , \mathbf{u}) that renders a global minimum value to each of the functionals simultaneously. Rather, there exists a finite set of solutions that represent trade-offs. A key concept in determining this set of solutions is that of Pareto optimality:

Assume two solutions $(\mathbf{x}, \mathbf{u}), (\mathbf{x}', \mathbf{y}') \in \Omega$, where Ω is denoted as the solution space. Then (\mathbf{x}, \mathbf{u}) is said to dominate $(\mathbf{x}', \mathbf{y}')$ (also written $(\mathbf{x}, \mathbf{u}) \succ (\mathbf{x}', \mathbf{y}')$) iff

$$\forall i \in \{1, 2, \dots, n\} : J_i(\mathbf{X}, \mathbf{U}) \geq J_i(\mathbf{X}', \mathbf{U}') \wedge \exists m \in \{1, 2, \dots, n\} : J_m(\mathbf{X}, \mathbf{U}) > J_m(\mathbf{X}', \mathbf{U}'). \quad (20)$$

However, (\mathbf{x}, \mathbf{u}) is said to cover $(\mathbf{x}', \mathbf{u}')$ $((\mathbf{x}, \mathbf{u}) \succ (\mathbf{x}', \mathbf{u}'))$ iff

$$\forall i \in \{1, 2, \dots, n\} : J_i(\mathbf{X}, \mathbf{U}) \leq J_i(\mathbf{X}', \mathbf{U}') \wedge \exists m \in \{1, 2, \dots, n\} : J_m(\mathbf{X}, \mathbf{U}) < J_m(\mathbf{X}', \mathbf{U}'). \quad (21)$$

In this case (\mathbf{x}, \mathbf{u}) is non-dominated by $(\mathbf{x}', \mathbf{y}')$.

Therefore, the set of non-dominated solutions, Ω^N , within the entire search space, Ω , is called the Pareto optimal set.

Forest Holding Value and Carbon Sequestration Accounting

The valuation of timber for a stand defined as $fvh_{NZD100}(t)$, $fvh_{NZD20}(t)$ and $fvh_C(t)$ in the cost functionals (11)-(13), were based on the forest holding value. The holding value is defined as follows:

$$FVH(t) = PVH(t) - PV_{costs} - LOC, \quad (22)$$

where,

$FVH(t)$ = holding value at time, t ;

$PVH(t)$ = Discounted present value of optimal harvest at time, t ;

PV_{costs} = Discounted costs at time t ; and

LOC = Land opportunity cost.

At any point in time carbon sequestered is a function of the change in biomass and the amount of carbon per m^3 of biomass. It is not just the age of the trees *per se* or standing timber volume that is important, but rather the rate of tree growth (van Kooten, Binkley & Delcourt, 1995). As trees grow they sequester carbon, but once carbon has been sequestered no further benefits are forthcoming. In other words the income generated by sequestering carbon at time, t is the value of the extra amount of carbon sequestered between t and $t - 1$ multiplied by the price of carbon, as shown in the following equation:

$$S_s^{t-(t-1)}(t) = Pc * (S(t) - S(t-1)), \quad (23)$$

where,

$S_s^{t-(t-1)}(t)$ = Carbon sequestration payment at time, t ;

Pc = Price for sequestering carbon ($\$/m^3$);

$S(t)$ = Total carbon sequestered at time, t , for a stand; and

$S(t - 1)$ = Total carbon sequestered at time, $(t - 1)$.

The present value of the above equation at the time of planting the crop then becomes:

$$PV_{carb}(t) = \frac{S_s^{t-(t-1)}}{(1+r)^t} \quad (24)$$

where,

$PV_{carb}(t)$ = Discounted carbon sequestration payment at time, t .

Therefore, the forest holding value that includes carbon sequestration payments then takes the following form:

$$FVH(t) = PVH(t) + PV_{carb}(t) - PV_{costs} - LOC \quad (25)$$

An assumption was implicit in the calculation that there was a zero harvesting emissions liability. All it meant was that there was no carbon sequestration payment for the proportion of harvested timber through thinning or clearfelling.

Results and Discussion

The results of the two-objective optimisation problem with the objectives (11) and (12) for value and volume products are shown in **Table 2** with only 6 regimes included in the Pareto set. The other Pareto sets for the three carbon prices scenarios, NZ \$25, NZ \$50, and NZ \$100, are shown in Appendices A (**Table A1**), B (**Table B1**), and C (**Table C1**), because of the larger size of their sets, at 30, 30, and 29 regimes respectively.

All these results in **Tables 1, A1, B1** and **C1** were not averaged, as this would defeat the purpose of finding a Pareto set of solutions in the first place. Therefore, investigating the impact of carbon prices on the silvicultural regimes of *E. fastigata* involved comparisons of the Pareto sets using the Kruskal-Wallis test (Kruskal & Wallis, 1952). This is a non-parametric version of the classical one-way analysis of variance and an extension of the Wilcoxon rank sum test (Gibbons 1985), specifically for testing equality of population medians among groups (which in our case are the four scenarios in **Tables 1, A1, B1** and **C1**). Ordering the data from smallest to largest across all groups and taking the numeric index of this ordering determines the ranks.

Initial Planting Stocking

A p -value of 0.4018 suggests that we accept the null hypothesis that the initial planting stockings from the different groups have the same mean. The box plots in **Figure 2** confirm this with the overlap of the medians at one standard deviation of the group means. The Wilcoxon rank sum test, plotted in **Figure 3**, shows more clearly the degree of overlap and no statistical difference between the initial planting stockings of the different groups. We conclude here that different carbon prices do not seem to have an impact on the initial planting stocking of *E. fastigata*.

Table 2.

Thinning regimes (showing number of trees thinned at T1, T2 and T3) for *E. fastigata* derived from a two-objective optimisation problem with objectives, sawlog (value) and pulpwood (volume) productions (Chikumbo & Nicholas, 2011).

REGIME	Age at T1 (yrs)	Age at T2 (yrs)	Age at T3 (yrs)	Clear felling age (yrs)	Init. stocking (stems·ha ⁻¹)	T1 (stems·ha ⁻¹)	T2 (stems·ha ⁻¹)	T3 (stems·ha ⁻¹)
1	5	13	20	35	988	287	153	196
2	5	15	18	35	995	274	168	186
3	5	15	20	35	936	259	196	170
4	5	14	19	35	939	240	190	176
5	5	15	18	35	937	234	185	197
6	5	12	18	35	903	228	193	187

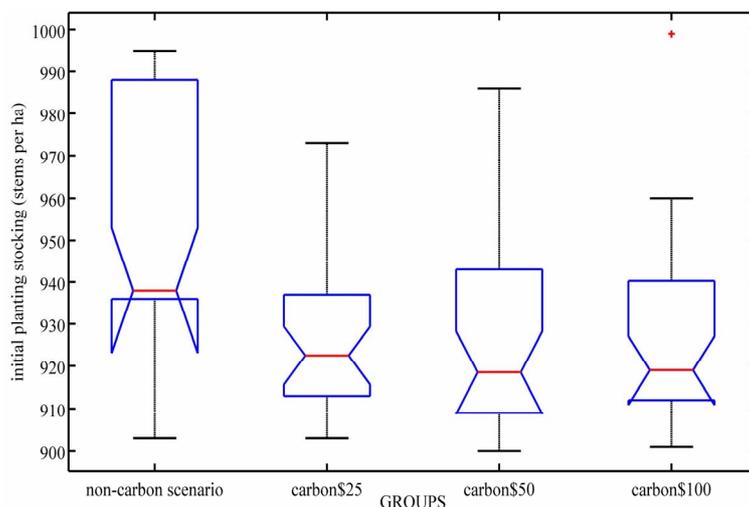


Figure 2. Box plots of the four scenarios where, the central mark in each box is the median; the edges of the box are the 25th and 75th percentiles; the notches in the boxes represent one standard deviation of the mean; the ends of the whiskers are the minimum and maximum; and outliers are plotted individually with the “+” sign.

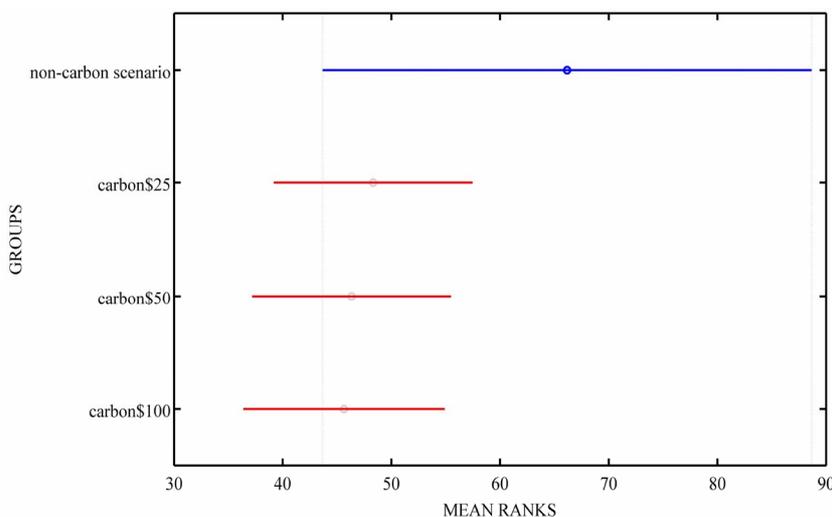


Figure 3. Wilcoxon rank sum test for the initial planting stockings of the different groups/scenarios.

Final Crop Number

A p -value of 0.0014 suggested that there were differences between the groups as shown in the box plots in **Figure 4**. The non-carbon scenario showed a higher number of retained crops before final harvesting, than all the carbon scenarios, which showed an overlap of the one standard deviation of their final crop numbers. The mean ranking in **Figure 5** showed more clearly the degree of overlap among the three carbon scenarios.

Rotation Length

There was also a difference in the rotation length medians amongst the different groups with a p -value of 0.003. The non-carbon scenario showed no variation in the rotation length, which remained at 35 years for all the individual regimes in the Pareto set, whereas the other three carbon scenarios had higher medians, which overlapped at one standard deviation of the

medians. The summary is shown in **Figure 6**. The summary of the mean ranks of the rotation lengths in **Figure 7** confirmed the differences between the non-carbon scenario with a lower rotation length, and the three carbon scenarios with higher ranges of rotation lengths. This observation agrees with conventional wisdom that rotation lengths will be longer in order to sequester more carbon. What is interesting with our results here is that the rotation lengths are insensitive to the variations in the carbon price.

We assert that this insensitivity to the price of carbon is because our optimisation model is maxing out the sequestration of carbon as much as the equations for the growth dynamics of *E. fastigata* will allow. This is good news for the forester, in that for any forest signed up to an emissions trading scheme, there is only one set of optimal regimes to consider that will simultaneously satisfy a sawlog and pulpwood market, regardless of fluctuations in the carbon price.

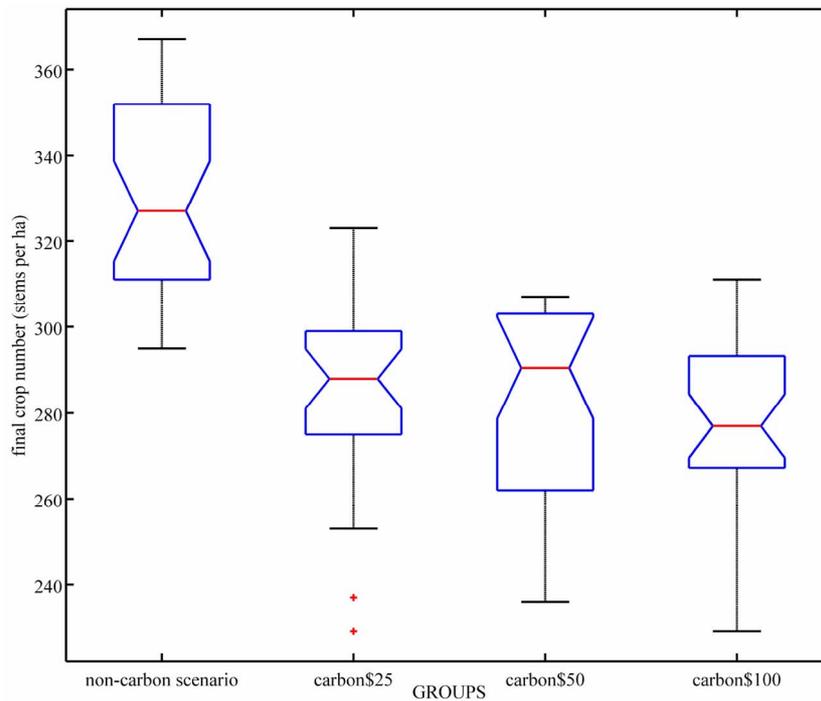


Figure 4.
Box plots of the four scenarios.

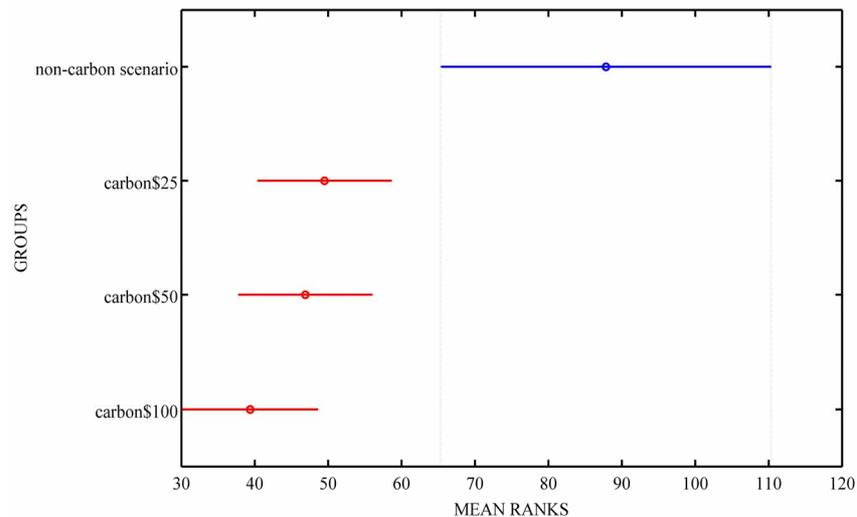


Figure 5.
Wilcoxon rank sum test for the final crop numbers of the different groups/scenarios.

Frequency, Timing and Intensity of Thinning

The frequency, timing and intensity of thinnings did not show any statistical differences for all the four scenarios. Carbon prices do not seem to influence the thinning strategy where it is already optimized for value and volume productions. This might be explained by the fact that though stand volume is reduced through thinning, the sudden availability of more nutrients, light and moisture to the residual trees boosts their growth. It is this growth that will guarantee more sequestration and possibly more payment. Given that the final crop numbers of all the carbon scenarios were lower than those of the non-

carbon scenario, and that the initial planting stockings of all the four scenarios were statistically the same, it therefore stands to confirm this assertion. Also with less number of trees as a final crop, it is possible to keep the trees a little longer than one would normally do in a non-carbon market environment because this may guarantee more growth until full-site occupancy is reached. This might encourage fertilisation following a late-age thinning, in order to boost growth and subsequently sequester more carbon. Implications of late-age fertilisation following a thinning may also mean a premium sawlog/veneer product at the end of the rotation.

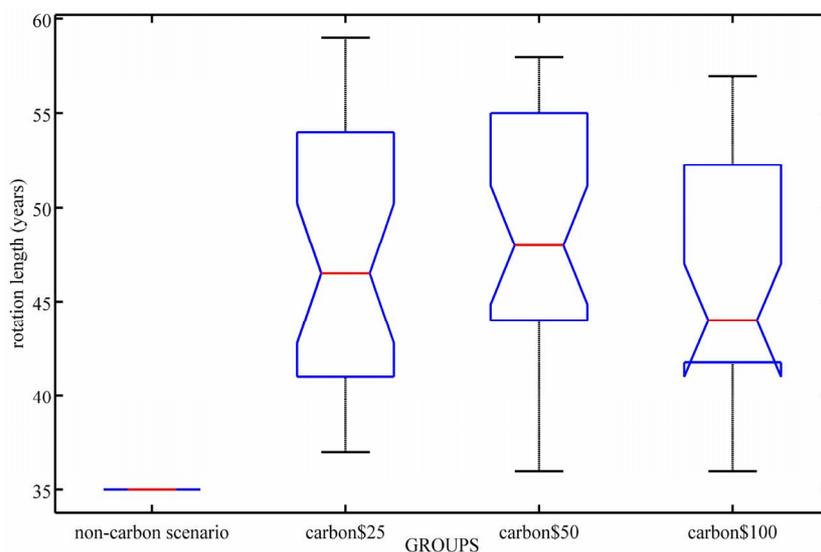


Figure 6.
Box plots of the four scenarios.

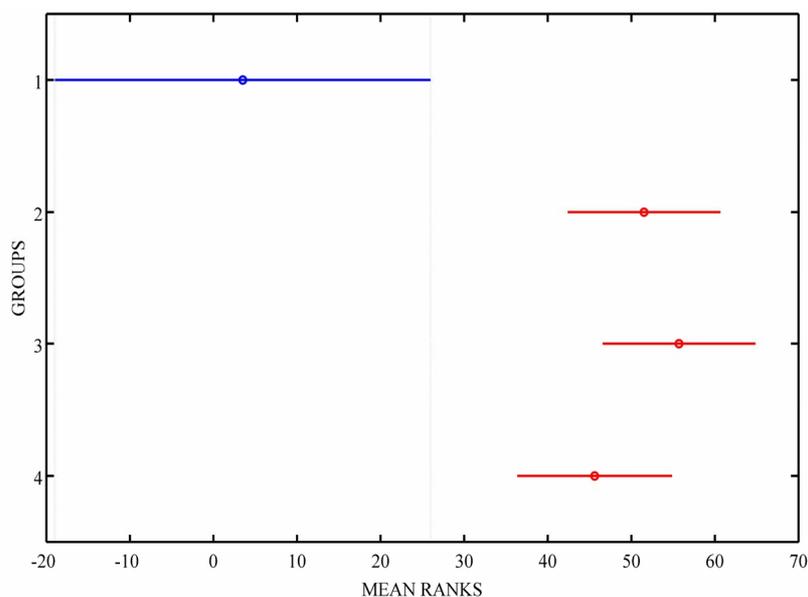


Figure 7.
Wilcoxon rank sum test for the rotation lengths of the different groups/scenarios.

Further Discussion

These results may have been different if the carbon scenarios were matched with forecasted fluctuations in the prices of liquid fuel and fossil fuel-based fertilisers, as this would have meant changes in the forest holding values. It is difficult to imagine how that would have impacted our results. It may well be that we need to take this research further by developing:

- 1) A dynamic stumpage model; and
- 2) A fossil fuel-based fertiliser price model, under a carbon trading scheme, and revisit our analysis. Both (1) and (2) will expose the forest holding value to a more realistic output, and maybe provide us with further insight into the impact of carbon prices on the silvicultural regimes of *E. fastigata*.

It is important to note that although our results are based on a stand level, they are Pareto sets which can still be used at a forest-estate level, where one has hundreds or thousands of stands at different age classes. With each stand with a Pareto set of possible silvicultural regimes, it is possible to optimise at an estate level, assigning the appropriate regime to each stand, which may mean simultaneously meeting harvesting commitments and optimally sequestering carbon under prevailing market constraints. In other words the Pareto optimality at a stand level gives flexibility at an estate level planning. We have not touched on environmental constraints because of the focus of this paper. However, environmental issues are best dealt with at a forest estate level both temporally and spatially, given their long-term gestation period.

Conclusion

The consequences on the silvicultural regimes for *Eucalyptus fastigata*, when the crop is simultaneously managed for, carbon sequestration, sawlog and pulpwood were, decreased final crop numbers, and increased rotation lengths. It will be well worth it to investigate fertilisation following late-age thinning in order to boost growth. This would mean sequestering more carbon and guarantee of a premium sawlog/veneer product at the end of the rotation. The regimes under a carbon market were also found to be insensitive to fluctuations in the price of carbon. Our findings were based on a fixed stumpage used in the calculation of forest holding values.

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Appendix A: NZ \$25 Carbon Price Scenario Silvicultural Regimes

Table A1.

Pareto optimal set for the NZ \$25 carbon price scenario.

REGIME	Age at 1 st thinning (years)	Age at 2 nd thinning (years)	Age at 3 rd thinning (years)	Clear felling age (years)	Initial planting density (stems·ha ⁻¹)	Number thinned at 1 st thinning (stems·ha ⁻¹)	Number thinned at 2 nd thinning (stems·ha ⁻¹)	Number thinned at 3 rd thinning (stems·ha ⁻¹)
1	6	14	21	46	918	297	174	194
2	8	13	20	43	930	279	188	198
3	8	15	19	59	924	266	193	188
4	8	13	20	39	913	274	182	198
5	8	13	18	39	906	298	178	193
6	6	15	20	57	973	287	199	178
7	10	14	19	41	913	287	198	199
8	6	14	21	37	937	267	168	197
9	7	15	21	39	968	270	195	180
10	7	12	19	39	960	296	187	178
11	6	14	21	54	937	278	162	195
12	5	15	19	47	937	291	188	174
13	5	12	21	46	934	286	194	168
14	6	14	18	59	925	298	184	150
15	5	14	20	52	915	273	182	170
16	5	13	19	47	935	273	200	187
17	7	12	21	46	946	292	189	165
18	6	15	21	57	912	283	193	137
19	5	14	20	52	907	254	191	192
20	6	12	21	51	921	265	173	189
21	5	13	19	44	939	261	199	200
22	5	15	21	57	905	294	125	196
23	5	15	19	58	903	264	182	187
24	7	14	19	46	938	274	195	178
25	7	14	21	48	927	236	183	189
26	7	15	20	39	913	295	177	155
27	7	12	19	42	921	264	181	198
28	7	15	20	39	913	295	177	155
29	7	15	20	59	916	231	194	186
30	7	13	20	48	908	283	133	193

Appendix B: NZ \$50 Carbon Price Scenario Silvicultural Regimes**Table B1.**

Pareto optimal set for the NZ \$50 carbon price scenario.

REGIME	Age at 1 st thinning (years)	Age at 2 nd thinning (years)	Age at 3 rd thinning (years)	Clear felling age (years)	Initial planting density (stems·ha ⁻¹)	Number thinned at 1 st thinning (stems·ha ⁻¹)	Number thinned at 2 nd thinning (stems·ha ⁻¹)	Number thinned at 3 rd thinning (stems·ha ⁻¹)
1	6	13	19	53	912	283	182	192
2	5	15	20	44	917	299	200	164
3	7	12	20	47	954	299	198	197
4	7	14	19	48	920	299	189	191
5	7	12	19	58	911	260	197	197
6	7	13	21	58	900	278	195	191
7	7	13	20	46	909	262	198	185
8	6	15	21	55	970	292	186	185
9	6	15	21	55	970	292	186	185
10	6	14	20	38	944	256	197	187
11	7	12	18	51	941	276	196	178
12	6	15	20	46	937	273	166	195
13	5	15	19	53	946	292	150	198
14	5	13	20	52	938	294	166	185
15	5	14	21	56	933	293	192	156
16	5	14	20	48	986	296	200	196
17	5	12	21	52	943	287	198	167
18	5	15	18	47	916	289	194	158
19	5	13	19	56	909	298	175	174
20	5	15	19	44	964	282	194	200
21	5	13	19	40	916	281	176	198
22	6	12	19	46	931	281	172	183
23	6	14	19	44	931	258	192	176
24	6	15	20	36	901	297	167	164
25	6	13	19	48	912	292	169	185
26	5	15	20	57	901	275	146	190
27	5	14	19	38	901	231	180	186
28	6	15	19	51	900	283	127	189
29	5	15	20	57	907	225	183	192
30	6	15	21	38	911	245	186	197

Appendix C: NZ \$100 Carbon Price Scenario Silvicultural Regimes**Table C1.**
Pareto optimal set for the NZ \$100 carbon price scenario.

REGIME	Age at 1 st thinning (years)	Age at 2 nd thinning (years)	Age at 3 rd thinning (years)	Clear felling age (years)	Initial planting density (stems·ha ⁻¹)	Number thinned at 1 st thinning (stems·ha ⁻¹)	Number thinned at 2 nd thinning (stems·ha ⁻¹)	Number thinned at 3 rd thinning (stems·ha ⁻¹)
1	6	14	18	40	952	300	200	195
2	6	13	21	42	915	294	198	194
3	7	14	18	49	921	289	198	182
4	7	14	19	43	907	272	195	200
5	8	15	19	46	941	285	200	182
6	7	14	21	51	903	254	189	191
7	8	15	19	44	912	278	179	190
8	8	15	19	44	912	278	179	190
9	6	15	19	47	960	291	200	182
10	7	15	19	40	999	300	200	197
11	5	12	21	50	947	288	193	197
12	5	14	18	53	942	296	196	189
13	5	14	20	56	940	273	184	190
14	6	12	19	36	932	283	183	189
15	5	14	21	36	930	269	176	191
16	5	12	20	42	924	282	177	180
17	6	13	21	37	915	298	165	184
18	6	15	20	52	919	278	173	176
19	7	15	18	50	958	289	181	194
20	5	12	19	44	913	243	185	199
21	6	12	18	56	902	294	174	152
22	5	12	19	57	919	296	177	177
23	6	15	21	53	922	222	198	191
24	6	13	21	41	901	287	147	194
25	5	13	20	38	908	237	169	200
26	5	13	21	42	918	231	197	193
27	6	14	19	43	915	243	198	195
28	6	15	19	53	912	260	182	167
29	5	15	18	53	903	257	193	180