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Mathematical Modeling of Crown Forest Fire Spread

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Mathematical model of forest fire was based on an analysis of known experimental data and using concept and methods from reactive media mechanics. In this paper the assignment and theoretical investigations of the problems of crown forest fire spread in windy condition were carried out. In this context, a study—mathematical modeling—of the conditions of forest fire spreading that would make it possible to obtain a detailed picture of the change in the temperature and component concentration fields with time, and determine as well as the limiting condition of fire propagation in forest with fire break.

Keywords: Forest Fire; Mathematical Model; Turbulence; Ignition; Fire Spread; Control Volume; Discrete Analogue

Introduction

A great deal of work has been done on the theoretical problem of crown forest fire initiation. Crown fires are initiated by convective and radiative heat transfer from surface fires. However, convection is the main heat transfer mechanism (Van Wagner, 1977). The theory proposed by Van Wagner (1977) depends on three simple crown properties: crown base height, bulk density of forest combustible materials and moisture content of forest fuel. Also, crown fire initiation and hazard have been studied and modeled in details later (Alexander, 1998); Van Wagner (1989); Xanthopoulos (1990); Rothermel (1991); Cruz and others (2002); Albini and others (1195); Scott and Reinhardt (2001). The more complete discussion of the problem of crown forest fires is provided by coworkers at Tomsk University (Grishin, 1997); Grishin et al. (1998); Perminov (1995, 1998). In particular, a mathematical model of forest fires was obtained by Grishin (1997) based on an analysis of known and original experimental data; Konev (1977), and using concepts and methods from reactive media mechanics. The physical two-phase models used by Morvan & Dupuy (2001, 2004) may be considered as a continuation and extension of the formulation proposed in (Grishin 1997). This study gives a two dimensional averaged mathematical setting and method of numerical solution of a problem of a forest fire spread. The boundary-value problem is solved numerically using the method of splitting according to physical processes. It was based on numerical solution of two dimensional Reynolds equations for the description of turbulent flow taking into account for diffusion equations chemical components and equations of energy conservation for gaseous and condensed phases, volume of fraction of condensed phase (dry organic substance, moisture, condensed pyrolysis products, mineral part of forest fuel).

Mathematical Model

It is assumed that the forest during a forest fire can be modeled as 1) a multi-phase, multistoried, spatially heterogeneous medium; 2) in the fire zone the forest is a porous-dispersed, two-temperature, single-velocity, reactive medium; 3) the forest canopy is supposed to be non-deformed medium (trunks, large branches, small twigs and needles), which affects only the magnitude of the force of resistance in the equation of conservation of momentum in the gas phase, i.e., the medium is assumed to be quasi-solid (almost non-deformable during wind gusts); 4) let there be a so-called "ventilated" forest massif, in which the volume of fractions of condensed forest fuel phases, consisting of dry organic matter, water in liquid state, solid pyrolysis products, and ash, can be neglected compared to the volume fraction of gas phase (components of air and gaseous pyrolysis products); 5) the flow has a developed turbulent nature and molecular transfer is neglected; 6) gaseous phase density doesn't depend on the pressure because of the low velocities of the flow in compareson with the velocity of the sound. Let the point x_1, x_2, $x_3 = 0$ is situated at the centre of the surface forest fire source at the height of the roughness level, axis $0x_1$ directed parallel to the Earth's surface to the right in the direction of the unperturbed wind speed, axis $0x_2$ directed perpendicular to $0x_1$ and axis $0x_3$ directed upward (**Figure 1**).

Because of the horizontal sizes of forest massif more than height of forest—h, system of equations of general mathematiccal model of forest fire (Grishin, 1997) was integrated between the limits from height of the roughness level—0 to h. Besides, suppose that

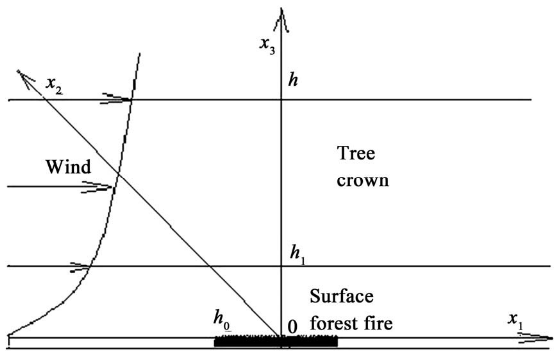


Figure 1.
Basic scheme of forest fire initiation and propagation.

$$\int_0^h \phi \, dx_3 = \overline{\varphi} h$$

$\ddot{\phi}$ —average value of ϕ. The problem formulated above is reduced to a solution of the following system of equation:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j}\left(\rho v_j\right) = Q - \left(\dot{m}^- - \dot{m}^+\right)/h, \quad j = 1, 2, 3; \qquad (1)$$

$$\rho \frac{dv_i}{dt} = -\frac{\partial p}{\partial x_j} + \frac{\partial}{\partial x_j}\left(-\rho \overline{v_i' v_j'}\right) - \rho s c_d v_i |\mathbf{v}| - \rho g_i - Q v_i + \left(\tau_i^- - \tau_i^+\right)/h, \quad i = 1, 2, 3; \qquad (2)$$

$$\rho c_p \frac{dT}{dt} = \frac{\partial}{\partial x_j}\left(-\rho c_p \overline{v_j' T'}\right) + q_5 R_5 - \alpha_v \left(T - T_s\right) + -\left(q_T^- - q_T^+\right)/h; \qquad (3)$$

$$\rho \frac{dc_\alpha}{dt} = \frac{\partial}{\partial x_j}\left(-\rho \overline{v_j' c_\alpha'}\right) + R_{5\alpha} - Q c_\alpha + (J_\alpha^- - J_\alpha^+)/h, \quad \alpha = \overline{1,5}; \qquad (4)$$

$$\frac{\partial}{\partial x_j}\left(\frac{c}{3k} \times \frac{\partial U_R}{\partial x_j}\right) - k\left(cU_R - 4\sigma T_S^4\right) + \left(q_R^- - q_R^+\right)/h = 0; \qquad (5)$$

$$\sum_{i=1}^4 \rho_i c_{pi} \phi_i \frac{\partial T_S}{\partial t} = q_3 R_3 - q_2 R_2 + k\left(cU_R - 4\sigma T_S^4\right) + \alpha_V \left(T - T_S\right); \qquad (6)$$

$$\rho_1 \frac{\partial \varphi_1}{\partial t} = -R_1, \; \rho_2 \frac{\partial \varphi_2}{\partial t} = -R_2, \quad \rho_3 \frac{\partial \varphi_3}{\partial t} = a_C R_1 - \frac{M_C}{M_1} R_3, \; \rho_4 \frac{\partial \varphi_4}{\partial t} = 0 \qquad (7)$$

$$\sum_{a=1}^5 c_a = 1, \; p_e = \rho RT \sum_{a=1}^5 \frac{c_a}{M_a}, \; \mathbf{v} = \left(v_1, v_2, v_3\right), \; \mathbf{g} = \left(0, 0, g\right)$$

The system of Equations (1)-(7) must be solved taking into account the initial and boundary conditions:

$$t = 1: v_1 = 0, v_2 = 0, v_3 = 0, T = T_e, c_a = c_{ae}, T_s = T_e, \varphi_1 = \varphi_{ie};$$

$$x_1 = -x_{1e}: v_1 = V_e, v_2 = 0, \frac{\partial v_3}{\partial x_1} = 0, T = T_e, c_a = c_{ae}, \quad -\frac{c}{3k} \times \frac{\partial U_R}{\partial x_1} + cU_R/2 = 0; \qquad (8)$$

$$x_1 = x_{1e}: \frac{\partial v_1}{\partial x_1} = 0, \frac{\partial v_2}{\partial x_1} = 0, \frac{\partial v_3}{\partial x_1} = 0, \frac{\partial c_\alpha}{\partial x_1} = 0, \frac{\partial T}{\partial x_1} = 0, \quad \frac{c}{3k} \times \frac{\partial U_R}{\partial x_1} + \frac{c}{2} U_R = 0; \qquad (9)$$

$$x_2 = x_{20}: \frac{\partial v_1}{\partial x_2} = 0, \frac{\partial v_2}{\partial x_2} = 0, \frac{\partial v_3}{\partial x_2} = 0, \frac{\partial c_\alpha}{\partial x_2} = 0, \frac{\partial T}{\partial x_2} = 0, \quad -\frac{c}{3k} \times \frac{\partial U_R}{\partial x_2} + \frac{c}{2} U_R = 0; \qquad (10)$$

$$x_2 = x_{2e}: \frac{\partial v_1}{\partial x_2} = 0, \frac{\partial v_2}{\partial x_2} = 0, \frac{\partial v_3}{\partial x_2} = 0, \frac{\partial c_\alpha}{\partial x_2} = 0, \frac{\partial T}{\partial x_2} = 0, \quad \frac{c}{3k} \times \frac{\partial U_R}{\partial x_2} + \frac{c}{2} U_R = 0; \qquad (11)$$

$$x_3 = 0: v_1 = 0, v_2 = 0, \frac{\partial c_\alpha}{\partial x_3} = 0, -\frac{c}{3k} \frac{\partial U_R}{\partial x_3} + \frac{c}{2} U_R = 0, \quad v_3 = v_{30}, T = T_g, |x_1| \le \Delta, |x_2| \le \Delta \quad v_3 = 0, \; T = T_e, \; |x_1| > \Delta, |x_2| > \Delta; \qquad (12)$$

$$x_3 = x_{3e}: \frac{\partial v_1}{\partial x_3} = 0, \frac{\partial v_2}{\partial x_3} = 0, \frac{\partial v_3}{\partial x_3} = 0, \frac{\partial c_\alpha}{\partial x_3} = 0, \frac{\partial T}{\partial x_3} = 0, \quad \frac{c}{3k} \times \frac{\partial U_R}{\partial x_3} + \frac{c}{2} U_R = 0. \qquad (13)$$

Here and above $\frac{d}{dt}$ is the symbol of the total (substantial) derivative; αv is the coefficient of phase exchange; ρ—density of gas-dispersed phase, t is time; v_i—the velocity components; T, T_S—temperatures of gas and solid phases, U_R—density of radiation energy, k—coefficient of radiation attenuation, P—pressure; c_p—constant pressure specific heat of the gas phase, c_{pi}, ρ_i, φ_i—specific heat, density and volume of fraction of condensed phase (1—dry organic substance, 2—moisture, 3—condensed pyrolysis products, 4—mineral part of forest fuel), R_i—the mass rates of chemical reactions, q_i—hermal effects of chemical reactions; k_g, k_S—radiation absorption coefficients for gas and condensed phases; T_e—the ambient temperature; c_α—mass concentrations of α-component of gas-dispersed medium, index α = 1, 2, 3, where 1 corresponds to the density of oxygen, 2—to carbon monoxide CO, 3—to carbon dioxide and inert components of air, 4, 5—soot and ash; R—universal gas constant; M_α, M_C, and M molecular mass of α-components of the gas phase, carbon and air mixture; g is the gravity acceleration; c_d is an empirical coefficient of the resistance of the vegetation, s is the specific surface of the forest fuel in the given forest stratum. In system of Equations (1)-(7) are introduced the next designations:

$$\dot{m} = \rho v_3, \; \tau_i = -\rho \overline{v_i' v_3'}, \; J_\alpha = -\rho \overline{v_3' c_\alpha'}, \; J_T = -\rho \overline{v_3' T'}$$

Upper indexes "+" and "–" designate values of functions at $x_3 = h$ and $x_3 = 0$ correspondingly. It is assumed that heat and mass exchange of fire front and boundary layer of atmosphere are governed by Newton law and written using the formulas:

$$\left(q_T^- - q_T^+\right)/h = -\alpha\left(T - T_e\right)/h,$$

$$\left(J_\alpha^- - J_\alpha^+\right)/h = -\alpha\left(c - c_{\alpha e}\right)/hc_p.$$

To define source terms which characterize inflow (outflow of mass) in a volume unit of the gas-dispersed phase, the following formulae were used for the rate of formulation of the gas-dispersed mixture $\dot{m}$, outflow of oxygen R_{51}, changing carbon monoxide R_{52}.

$$Q = \left(1 - \alpha_c\right) R_1 + R_2 + \frac{M_c}{M_1} R_3, \; R_{51} = -R_3 - \frac{M_1}{2M_2} R_5,$$

$$R_{52} = \nu_g \left(1 - \alpha_c\right) R_1 - R_5, \; R_{53} = 0.$$

Here ν_g—mass fraction of gas combustible products of pyrolysis, α_4 and α_5—empirical constants. Reaction rates of these various contributions (pyrolysis, evaporation, combustion of coke and volatile combustible products of pyrolysis) are approximated by Arrhenius laws whose parameters (pre-exponential constant k_i and activation energy E_i) are evaluated using data for mathematical models (Grishin, 1997; Perminov, 1995).

$$R_1 = k_1 \rho_1 \varphi_1 \exp\left(-\frac{E_1}{RT_s}\right), \; R_2 = k_2 \rho_2 \varphi_2 T_s^{-0.5} \exp\left(-\frac{E_2}{RT_s}\right),$$

$$R_3 = k_3 \rho \varphi_3 s_\sigma c_1 \exp\left(-\frac{E_3}{RT_s}\right),$$

$$R_5 = k_5 M_2 \left(\frac{c_1 M}{M_1}\right)^{0.25} \frac{c_2 M}{M_2} T^{-2.25} \exp\left(-\frac{E_5}{RT}\right).$$

The initial values for volume of fractions of condensed phases are determined using the expressions:

$$\varphi_{1e} = \frac{d\left(1-\nu_z\right)}{\rho_1}, \varphi_{2e} = \frac{Wd}{\rho_2}, \varphi_{3e}\frac{a_c\varphi_{1e}\rho_1}{\rho_3}$$

where d—bulk density for surface layer, ν_z—coefficient of ashes of forest fuel, W—forest fuel moisture content. It is supposed that the optical properties of a medium are independent of radiation wavelength (the assumption that the medium is "grey"), and the so-called diffusion approximation for radiation flux density were used for a mathematical description of radiation transport during forest fires.

To close the system (1)-(7), the components of the tensor of turbulent stresses, and the turbulent heat and mass fluxes are determined using the local-equilibrium model of turbulence (Grishin, 1997). The system of Equations (1)-(7) contains terms associated with turbulent diffusion, thermal conduction, and convection, and needs to be closed. The components of the tensor of turbulent stresses $\rho\overline{v'w'}$, as well as the turbulent fluxes of heat and mass $\rho\overline{v'_j c_p T'}$, $\rho\overline{v'_j c'_a}$ are written in terms of the gradients of the average flow properties using the formulas:

$$-\rho\overline{v'_i v'_j} = \mu_t\left(\frac{\partial v_i}{\partial x_j}+\frac{\partial v_j}{\partial x_i}\right)-\frac{2}{3}K\delta_{ij},$$

$$-\rho\overline{v_j c_p T'} = \lambda_t \times \frac{\partial T}{\partial x_j}, -\rho\overline{v_j c'_a} = \rho D_t \times \frac{\partial c_a}{\partial x_j}$$

$$\lambda_t = \mu_t c_p / \mathrm{Pr}_t, \rho D_t = \mu_t / Sc_t, \mu_t = c_\mu \rho K^2/\varepsilon,$$

where μ_t, λ_t, D_t are the coefficients of turbulent viscosity, thermal conductivity, and diffusion, respectively; Pr_t, Sc_t are the turbulent Prandtl and Schmidt numbers, which were assumed to be equal to 1. In dimensional form, the coefficient of dynamic turbulent viscosity is determined using local equilibrium model of turbulence (Grishin, 1997). The system of Equations (1)-(7) must be solved taking into account the initial and boundary conditions. The thermodynamic, thermophysical and structural characteristics correspond to the forest fuels in the canopy of a different type of forest; such as, pine forest (Grishin, 1997).

Numerical Methods and Results

The boundary-value problem (1)-(13) is solved numerically using the method of splitting according to physical processes (Perminov, 1995). In the first stage, the hydrodynamic pattern of flow and distribution of scalar functions was calculated. The system of ordinary differential equations of chemical kinetics obtained as a result of splitting was then integrated. A discrete analog was obtained by means of the control volume method using the SIMPLE like algorithm (Patankar, 1981). The accuracy of the program was checked by the method of inserted analytical solutions. Analytical expressions for the unknown functions were substituted in (1)-(7) and the closure of the equations were calculated. This was then treated as the source in each equation. Next, with the aid of the algorithm described above, the values of the functions used were inferred with an accuracy of not less than 1%. The effect of the dimensions of the control volumes on the solution was studied by diminishing them. The time step was selected automatically. Fields of temperature, velocity, component mass fractions, and volume fractions of phases were obtained numerically. The distribution of basic functions shows that the process of crown forest fire initiation goes through the next stages. The first stage is related to increasing maximum temperature in the fire source. At this process stage the fire source a thermal wind is formed a zone of heated forest fire pyrolysis products which are mixed with air, float up and penetrate into the crowns of trees. As a result, forest fuels in the tree crowns are heated, moisture evaporates and gaseous and dispersed pyrolysis products are generated. Ignition of gaseous pyrolysis products of the ground cover occurs at the next stage, and that of gaseous pyrolysis products in the forest canopy occurs at the last stage. As a result of heating of forest fuel elements of crown, moisture evaporates, and pyrolysis occurs accompanied by the release of gaseous products, which then ignite and burn away in the forest canopy. At the moment of ignition the gas combustible products of pyrolysis burns away, and the concentration of oxygen is rapidly reduced. The temperatures of both phases reach a maximum value at the point of ignition. The ignition processes is of a gas-phase nature. Note also that the transfer of energy from the fire source takes place due to radiation; the value of radiation heat flux density is small compared to that of the convective heat flux. At $V_e \neq 0$, the wind field in the forest canopy interacts with the gas-jet obstacle that forms from the forest fire source and from the ignited forest canopy and burn away in the forest canopy. **Figures 2-5** present the distribution of temperature $\overline{T}\left(\overline{T} = T/T_e, T_e = 300\ \mathrm{K}\right)$ (1 - 2, 2 - 2.6, 3 - 3, 4 - 3.5, 5 - 4) for gas phase, oxygen $\overline{c}_1$ (1 - 0.1, 2 - 0.5, 3 - 0.6, 4 - 0.7, 5 - 0.8, 6 - 0.9), volatile combustible products of pyrolysis $\overline{c}_2$ concentrations (1 - 1, 2 - 0.1, 3 - 0.05, 4 - 0.01) $\left(\overline{c}_\alpha = c_\alpha/c_{1e}, c_{1e} = 0.23\right)$, temperature of condensed phase $\overline{T}_S\left(\overline{T}_S = T_S/T_e, T_e = 300\ \mathrm{K}\right)$ (1 - 2, 2 - 2.6, 3 - 3, 4 - 3.5, 5 - 4) for wind velocity Ve = 10 m/s at h = 10 m: 1) t = 3 sec., 2) t = 10 sec, 3) t = 18 sec., 4) t = 24 sec.

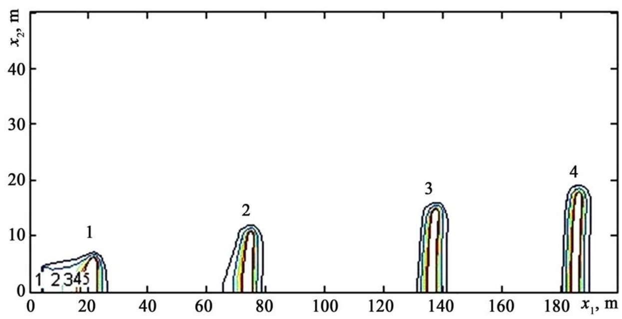


Figure 2.
Field of isotherms of the forest fire spread (gas phase).

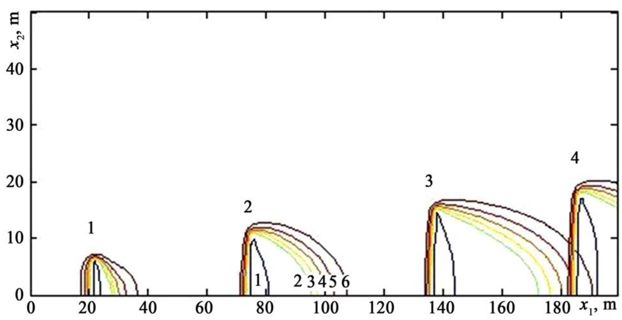


Figure 3.
The distribution of oxygen $\overline{c}_1$.

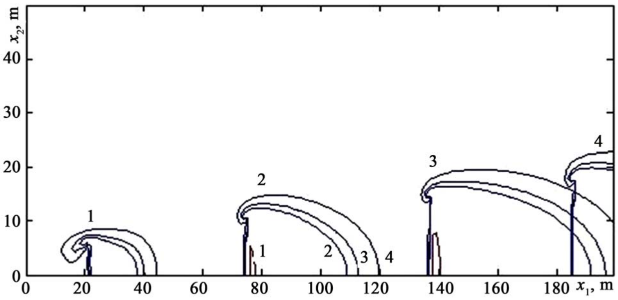


Figure 4.
The distribution of $\bar{c}_2$.

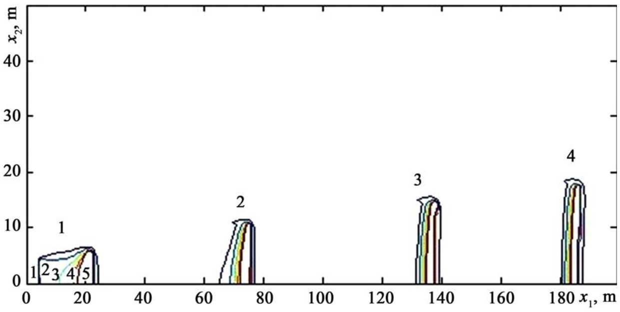


Figure 5.
Field of isotherms of the forest fire spread (solid phase).

We can note that the isotherms and lines of equal levels are moved in the forest canopy and deformed by the action of wind. Similarly, the fields of component concentrations are deformed. It is concluded that the forest fire begins to spread. Mathematical model and the result of the calculation give an opportunity to consider forest fire spread for different wind velocity.

Figures 6(a)-(d) present the distribution of temperature for gas phase, concentration of oxygen and volatile combustible products of pyrolysis c_2 concentrations and temperature of condensed phase for wind velocity Ve = 5 m/s at h = 10 m: 1) t = 3 sec., 2) t = 10 sec., 3) t = 18 sec., 4) t = 20 sec., 5) t = 31 sec., 6) t = 40 sec. The results reported in **Figure 6** show the decrease of the wind induces a decrease of the rate of fire spread.

One of the objectives of this paper could be to develop modeling means to reduce forest fire hazard in forest or near towns. In this paper it presents numerical results to study forest fire propagation through firebreak. This problem was considered by Zverev (1985) in one dimensional mathematical model approach.

Figures 7 and **8** (**Figure 8(b)** is a continuation of **Figure 8(a)**) present the forest fire front movement using distributions of temperature at different instants of time for two sizes of firebreaks (4.5 and 4 meters). The fire break is situated in the middle of domain (x_1 =100 m). In the first case the fire could not spread through this fire break.

If the fire break reduces to 4 meters (**Figure 8**) the fire continue to spread but the isotherm (isotherm 5) of forest fire is decreased after overcoming of fire break. In the **Figure 9**. The

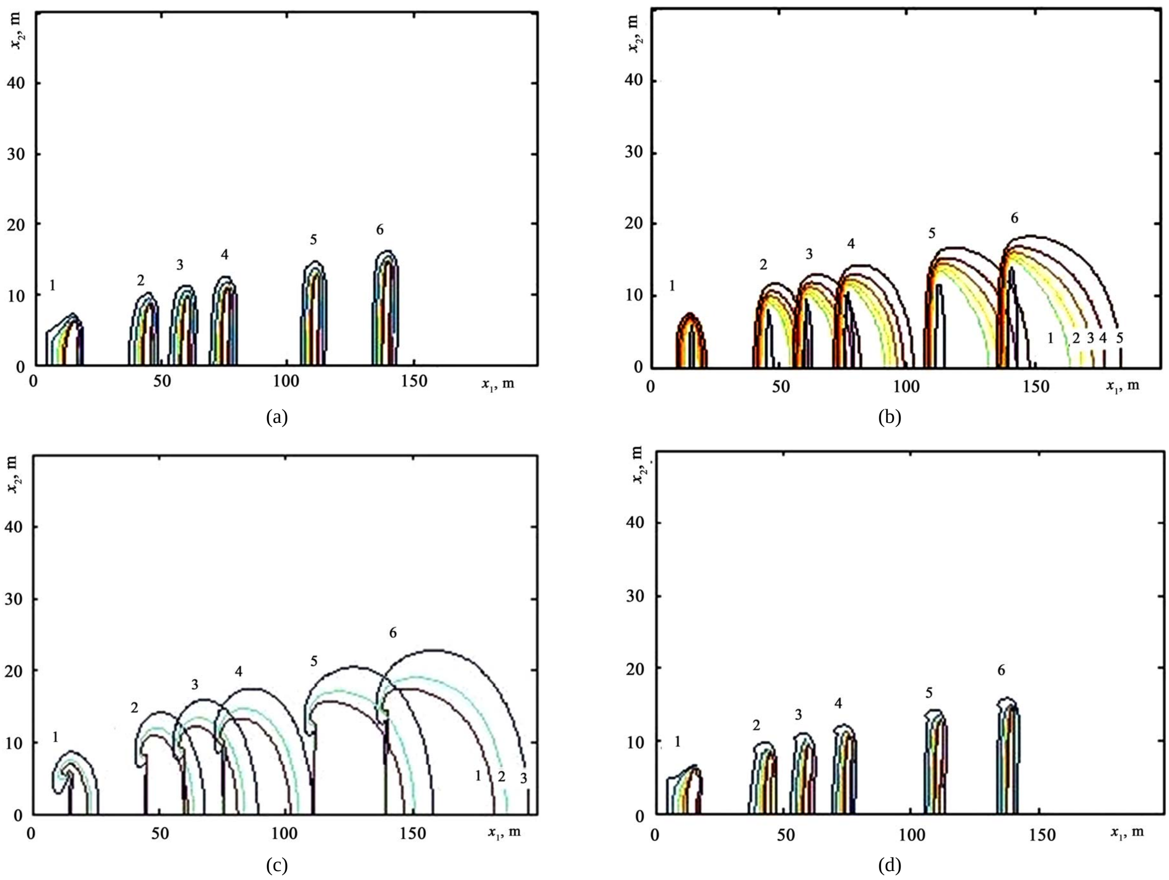


Figure 6.
Fields of isotherms of gas (a) and solid phase (d), isolines of oxygen (b) and gas products of pyrolysis (c).

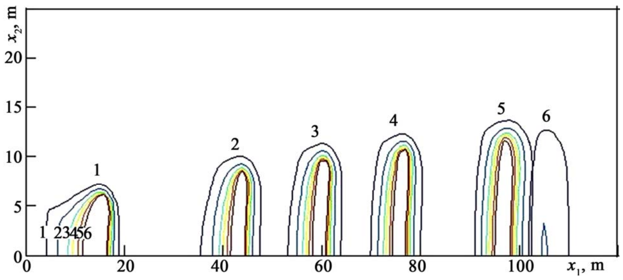
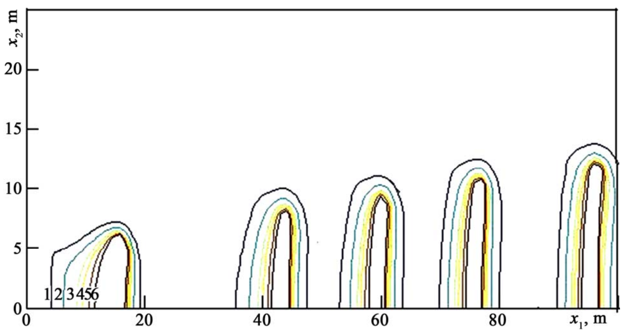
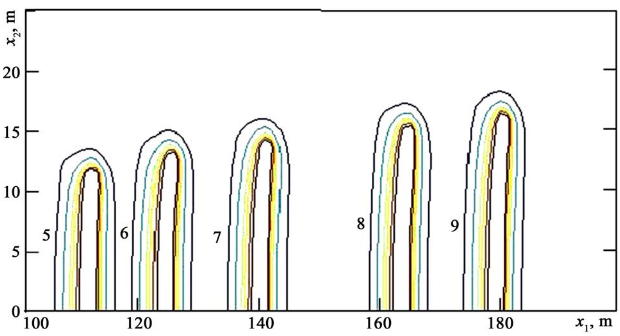


Figure 7.
Field of isotherms. Fire break equals 4.5 m.



(a)



(b)

Figure 8.
Field of isotherms. Fire break equals 4 m.

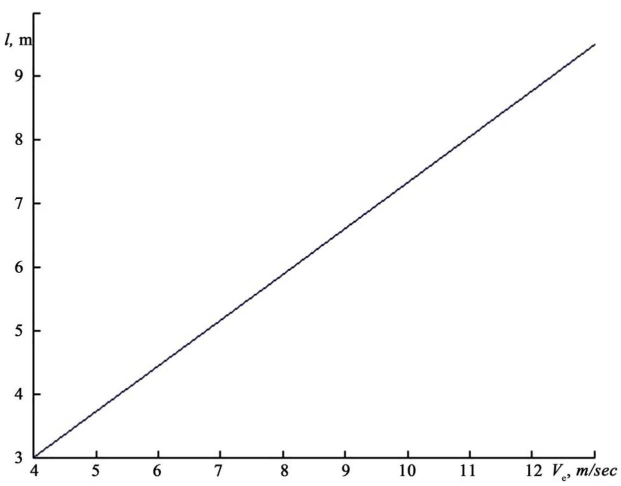


Figure 9.
The influence of wind velocity at the size of fire break.

dependence of critical fire break value for different wind velocities is presented. Of course the size of safe distance depends not only of wind velocity, but type and quality of forest combustible materials, its moisture, height of trees and others conditions. This model allows to study an influence all these main factors.

Conclusion

The results of calculation give an opportunity to evaluate critical condition of the forest fire spread, which allows applying the given model for preventing fires. It overestimates the velocity of crown forest fire spread that depends on crown properties: bulk density, moisture content of forest fuel and etc. The model proposed there gives a detailed picture of the change in the temperature and component concentration fields with time, and determine as well as the influence of different conditions on the crown forest fire initiation. The results obtained agree with the laws of physics and experimental data (Konev, 1977; Grishin, 1997). From an analysis of calculations and experimental data it was found that for the cases in question the minimum total incendiary heat pulse is 2600 kJ/m^2 (Grishin, 1997). Calculations demonstrated that the value of the radiant heat flux for both problems is considerably less than the convective one, therefore radiation has a weak effect on local and integral characteristics of the problem discoursed above.

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