Scientific Research Publishing

Open Journal of Earthquake Research, 2018, 7, 28-38
http://www.scirp.org/journal/ojer
ISSN Online: 2169-9631
ISSN Print: 2169-9623

On the Dynamical Analysis in Aftershock Networks

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How to cite this paper: Baek, W.-H., Kim, K., Chang, K.-H., Seo, S.-K., Lee, J.-H. and Lee, D.-I. (2018) On the Dynamical Analysis in Aftershock Networks. *Open Journal of Earthquake Research*, **7**, 28-38.
https://doi.org/10.4236/ojer.2018.71002

Received: December 17, 2017
Accepted: February 8, 2018
Published: February 11, 2018

Abstract

We investigate the dynamical behavior of aftershocks in earthquake networks, and the earthquake network calculated from a time series is constructed by contemplating cell resolution and temporal causality. We attempt to connect an earthquake network using relationship between one main earthquake and its aftershocks from seismic data of California. We mainly examine some topological properties of the earthquake such as the degree distribution, the characteristic path length, the clustering coefficient, and the global efficiency. Our result cannot presently determine the universal scaling exponents in statistical quantities, but the topological properties may be inferred to advance and improve by implementing the method and its technique of networks. Particularly, it may be dealt with a network issue of convenience and of importance in the case how large networks construct in time to proceed on earthquake systems.

Keywords

Earthquake Network, Aftershock, Characteristic Path Length, Clustering Coefficient, Global Efficiency

1. Introduction

Scientists have treated emerging problems in order to cover the basic concept and the important principle or clearly describe the scientific phenomenon. In several drifts of the complex system, they have pursued and settled many scientific phenomena [1] [2] [3] [4] [5] such as agent-based model, economic network, social interaction, neural and cerebral networks, world wild web, and ecosystem, which we have been able to recognize exactly in the past. Furthermore, the research of complex systems has recently been applied to the method and its technique of studying the various financial time series [6] [7], wavelet transform approaches [8] [9], transport networks [10] [11], social network [12] [13], multilayer networks [14] [15] and earthquake phenomena [16].

Particularly, the remarkable potential to calculate and analyze the dynamical behavior of complex systems has gradually been an increasing trend in new fields of research in the natural, engineering, medical, social sciences over the last two decades. In the network theory, small-world and scale-free network models [17] [18] have been studied widely in various applications of these scientific fields and played a crucial role in understanding complex phenomena [19] [20] [21]. Researchers have as yet concentrated on statistical and stochastic problems that the degree distribution for scale-free networks follows a power law, while that for random networks decays faster than exponentially. In complex seismic time series [22] [23] [24] [25] [26], the shallow earthquake has constructed and analyzed the form of distributions in the relevant region that leads to many aftershocks [27] [28] [29]. The Gutenberg-Richter law [24] has used to measure the number of aftershocks, and the slip-size of faults or the seismic moment has followed a power law. The Omori-Utsu law [25] has analyzed the calculation of earthquakes by a theoretical formula, and the frequency of aftershocks has decayed in a power law. Nanjo *et al.* [30] have investigated the seismicity modeled by the Gutenberg-Richter law and the Omori-Utsu law in M6.5 quake of Kumamoto 2016. They speculated the reduction of the occurrence of larger shocks causing the notable increase in the b value and the large p value.

Abe and Suzuki have analyzed the spatio-temporal properties of seismicity from the viewpoint of the Tsallis entropy under appropriate constraints [31]. They have argued the spatial distance and the time interval between two successive earthquakes in the characteristic of the nonextensive statistical mechanics [29]. The correlation function has particularly been a main issue in theoretical and numerical investigations of aftershock phenomena. Several theoretical formulae have been used to carry out the calculation of earthquakes. It has been suggested to construct complex network from seismic data by Abe and Suzuki [16]. Baek *et al.* have studied the earthquake network by considering the cell resolution and the temporal causality based on earthquake activity data for the Korean Peninsula [32]. They mainly estimated and analyzed several global network metrics. Min *et al.* have performed the numerical computations for network metrics from seismic time series data taken in Japan [33]. They have investigated the topological robustness of the earthquake network against the spatial shift and the scale after constructing the earthquake network in a cubic cell.

The aftershocks represent many smaller earthquakes that it occurs after a large earthquake. Our method is to construct the network between one main earthquake and its aftershocks, different from the Abe and Suzuki method [16]. It is as yet an open problem that the statistical quantities in earthquake networks find the universal property and the regular feature. In the future, we think that this is an important problem coming to a settlement. In this paper, we study and analyze the topological property and its feature in the aftershock network. After constructing an aftershock network, we mainly examine some topological properties such as the mean degree, the degree distribution, the characteristic path length, the clustering coefficient, and the global efficiency from seismic data of California. Section 2 describes the theoretical method of complex networks. In section 3, we treat the numerical calculation and its result, and our case is particularly compared to both the Abe and Suzuki network and the random network in earthquake networks. Our result is summarized in section 4.

2. Theoretical Method

In this section, we mainly consider the theoretical background of the several global network metrics. First of all, there are some important ingredients of complex networks, and these are different from ingredients of random networks. The mean degree $\langle k \rangle$ is defined as

$$\langle k \rangle = \frac{1}{N}\sum_{m=1}^{N} k_m , \tag{1}$$

where k_m is the degree of a node m, and N is the number of total degrees. The random network is that it constructed by randomizing the earthquake network under fixed links and nodes. From our method for constructing network, a newly creating node of the growing earthquake network is linked with preferential attachment probability. A network generated with this rule characterized by power-law connectivity distribution. The degree distribution that is the probability distribution function as a function of degrees k is represented in terms of

$$P(k) \sim k^{-\gamma} , \tag{2}$$

where γ is the degree exponent. Theoretically, the scaling exponent γ of the scale free network [18] [33] is in the range between 2 to 3. The clustering coefficient C_i for a node i is defined as the fraction of links that exists among its nearest-neighbor nodes to the maximum number of possible links among them. The clustering coefficient of a node with degrees k follows the scaling law

$$C(k) \sim k^{-\beta} , \tag{3}$$

where the scaling exponent β is a hierarchy coefficient. The network with the mentioned feature of Equations (2) and (3) is called the scale free network.

The characteristic path length is defined the statistical quantity that the sum of all the shortest path length between two nodes is divided by all links of nodes. We introduce the characteristic path length L given as

$$L = \frac{2}{N(N-1)}\sum_{m=1}^{N-1}\sum_{m=i+1}^{N} L_{mn} , \tag{4}$$

where L_{mn} is the shortest path length between nodes m and n [34]. We consider that the diameter of the network is the largest of all the shortest path lengths. If the averaged shortest path length is proportional to log$N$, it can be ascertained that the property of a network is satisfied the small-worldness. The average clustering coefficient C is calculated as $C = 1/N \sum_{m=1}^{N} C_m$. Here, the clustering coefficient C_m for a node m is defined as the fraction of links that exist among its nearest neighbor nodes to the maximum number of possible links among them. The global clustering coefficient C_g is defined as the transitivity ratio that is the fraction of the closed triangles over the whole triangles.

The global efficiency is defined as the average of inverses of the global distance for all nodes [35]. We calculate the global efficiency as

$$E_g = \frac{2}{N(N-1)} \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \frac{1}{L_{ij}}. \quad (5)$$

When we construct a network for the neighbors of node m, the local efficiency E_{lc} can be calculated to be the average value of the efficiencies of node as $E_{lc} = 1/N \sum_{m=1}^{N} E_m$, where E_m is the subgraph efficiency of the neighbors of the m-th node.

From Equations (1)-(5), we mainly calculate and analyze the topological measures in the complex network in next section. These methods and techniques are able to be treated in the study of the diverse earthquake models. Further result of other phase metrics will appear in a future publication.

3. Numerical Calculations and Results

The aftershocks have a trend occurring during definitive time intervals after one main earthquake occurs. If an aftershock is larger than the main shock, then the aftershock is considered as the role of the main shock and the previous main shock is designated as a foreshock. Aftershocks are formed as the crust around the displaced fault plane adjusts to the effects of the main shock. Hence, we suggest our method to construct complex network using the property of aftershocks. An earthquake network is constructed by segmenting the whole region into three-dimensional cubic cells and making a link between consecutive events. Each cubic cell is regarded as node of a network, and the network constructed in that manner is basically directed, but we transform it into an undirected one because we focus on the topology of the network.

Our network is introduced the method constructing in aftershock, while Abe-Suzuki network constructs in earthquakes for all the time series data consecutive earthquake events. Our method constructing the network is compared to that of Abe-Suzuki. Our procedure is as follows: 1) We segment the whole region into cells, each of which has the same size. 2) If the magnitude of second earthquake is smaller than the first one, we link two earthquakes. 3) If the magnitude of third earthquake is smaller than the second one, we also make a link between first earthquake and third one. In this manner, smaller earthquakes as the role of aftershock are linked with a main shock. Otherwise, if the magnitude of third earthquake is bigger than that of second earthquake, the third one becomes a main shock. 4) If two consecutive events belong to the same cubic cell, then their link is disregarded. 5) The number of links, if two directed links form between two cubic cells. 6) Hence, we regard the links made by all events belonging to the cubic cell with others in another cubic cell as links of a network, by considering each cubic cell as a node. Next, the method of Abe-Suzuki is as follows: 1) We segment the whole region into N-by-N-by-L cubic cells, each of which has the same size. 2) We link two earthquakes occurring consecutively. 3) If two consecutive events belong to the same cubic cell, their link is disregarded. 4) If two directed links form between two cubic cells, the number of links is counted as one. 5) By considering each cubic cell as a node, we regard the links made by all events belonging to the cubic cell with others in another cubic cell as links of a network.

We construct and analyze seismic data collected from California of USA. The data sources are USGS [36] that the time intervals are between 20th May 2001 and 19th May 2010. The region covered is 32˚N - 37˚N latitude and 115˚W - 120˚W longitude to the depth of 797 km on California. The maximal magnitude is 7.2, and the data for the total numbers of events is 147,193. We configure two different earthquake networks, that is, the OAS and the ASN. We have formed 11 networks with various cubic cell scales from (1˚/10) × (1˚/10) × 10 km^3 to ((1˚/20) × (1˚/20) × 10 km^3.

In **Figure 1**, the number of links versus the number of nodes plots in the OAS and the ASN. Due to the different method to construct the network, our aftershock networks have smaller links than the ASN, compared to **Table 1**. **Figure 2**

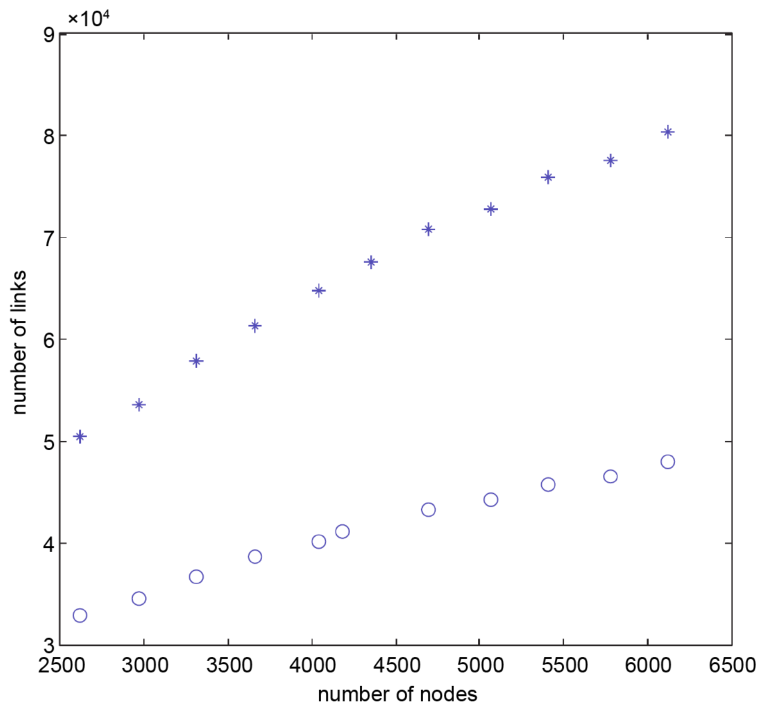

Figure 1. The number of links versus the number of nodes in our aftershock network (OAS, circle) and the Abe-Suzuki network (ASN, cross).

Table 1. Numerical computation of statistical quantities performed from seismic time series data taken in Japan. The statistical quantities *N*, *k*, <*k*>, *L*, C_l , and E_g denote the number of nodes, the number of links, mean degree, characteristic path length, average clustering coefficient, and global efficiency, respectively. We summarize the values of these statistical quantities at the 11 cell widths from 1˚/10 degree and 1˚/20 degree, and the depth is 10 km. Here, the two networks are our aftershock network (OAS) and Abe-Suzuki network (ASN).

cell width	*N*	*k*		<*k*>		*L*		C_l		E_g	
		OAN	ASN	OAN	ASN	OAN	ASN	OAN	ASN	OAN	ASN
1˚/10	2,616	32863	50465	25.12	38.58	2.66	2.49	0.41	0.59	0.37	0.71
1˚/11	2,963	34501	53604	23.29	36.18	2.70	2.52	0.38	0.57	0.36	0.69
1˚/12	3,309	36735	57832	22.20	34.95	2.76	2.56	0.35	0.53	0.35	0.66
1˚/13	3,660	38557	61386	21.07	33.54	2.79	2.59	0.33	0.51	0.34	0.64
1˚/14	4,041	40156	64690	19.87	32.02	2.84	2.63	0.30	0.49	0.33	0.62
˚/15	4,350	41154	67578	19.66	31.07	2.86	2.66	0.29	0.47	0.33	0.60
1˚/16	4,691	43245	70811	18.44	30.19	2.91	2.68	0.27	0.45	0.33	0.57
1˚/17	5,063	44186	72824	17.45	28.77	2.94	2.70	0.25	0.44	0.32	0.56
1˚/18	5,408	45764	75902	16.92	28.07	2.99	2.74	0.23	0.42	0.31	0.54
1˚/19	5,783	46499	77580	16.08	26.83	3.01	2.76	0.21	0.39	0.31	0.51
1˚/20	6,127	47955	80398	15.65	26.24	3.04	2.79	0.20	0.38	0.30	0.49

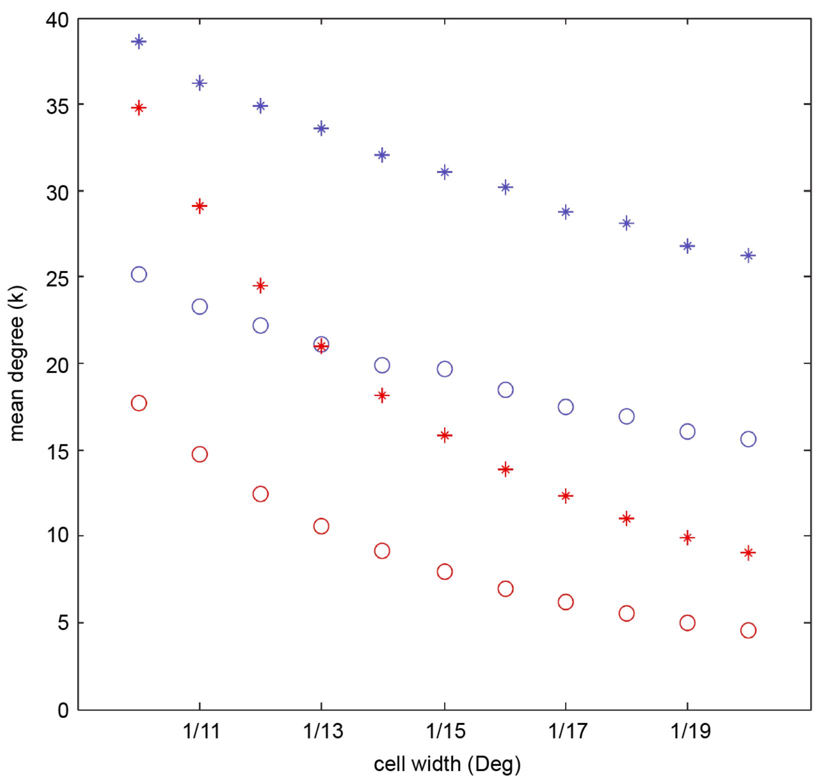

Figure 2. Mean degree versus the cell width in the OAS (blue circle) and the ASN (blue cross). The red circle (red cross) denotes our (Abe-Suzuki) random network.

plots the mean degree as a function of the cell width in our aftershock network and the ASN. We show that the mean degree of the earthquake network becomes relatively smaller than that of random network as the cell width approaches to small values.

In **Figure 3**, the characteristic path length plots in the OAS and the ASN. We also show that the average shortest path length of the earthquake network is relatively smaller than that of random network, as the cell width approaches to smaller value. **Figure 4** shows the average clustering coefficient as a function of the cell width, and the global efficiency is plotted as a function of the cell width in **Figure 5**. Since the random network constructs from mixed or shuffled time series data, it is certain that the values of the global efficiency and the average clustering coefficient are different from those of the regular network. In the OAS and the ASN, both the global efficiency and clustering coefficient are larger than those of random networks.

We find that the scaling exponent in the degree distribution [32] has 1.53 and 1.60 (1.37 and 1.36) in the case of (1˚/20) × (1˚/20) × 10 km^3 and (1˚/10) × (1˚/10) × 10 km^3 in the OAS (the ASN), respectively. **Table 1** summarizes the numerical computation of statistical quantities in various scales performed from seismic time series data taken in California of USA. We compare the OAS to the ASN, and these are the values of these statistical metrics at the 11 cells from 1˚/20 to 1˚/10 and 10 km of the depth.

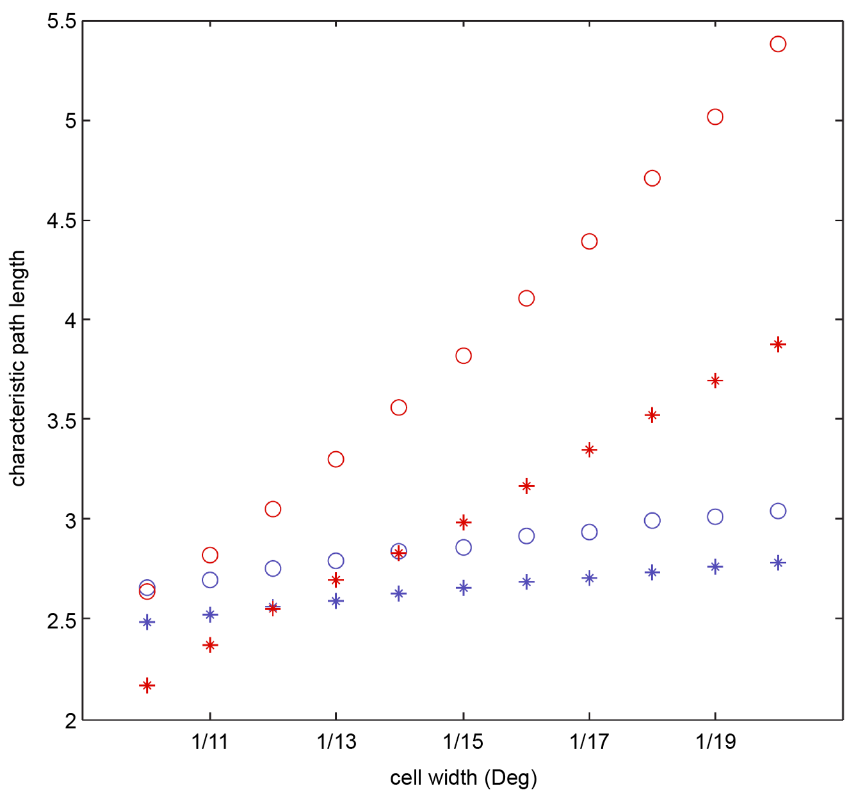

Figure 3. Characteristic path length in our network (blue circle) and Abe-Suzuki network (blue cross). The red circle (red cross) denotes our (Abe-Suzuki) random network.

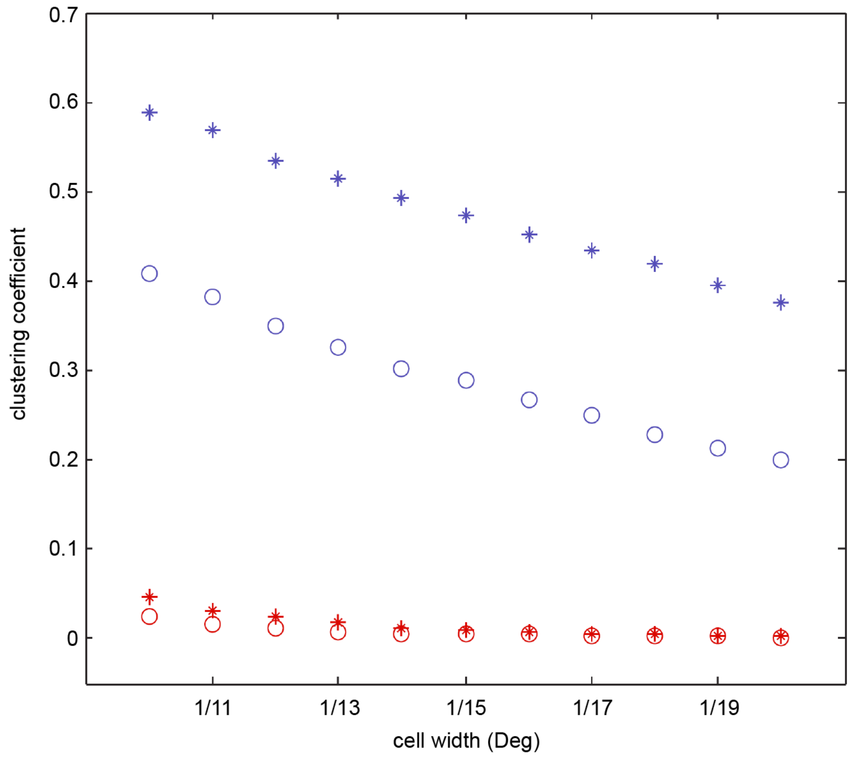

Figure 4. Average clustering coefficient in the OAS (blue circle) and the ASN (blue cross). The red circle (red cross) denotes our (Abe-Suzuki) random network.

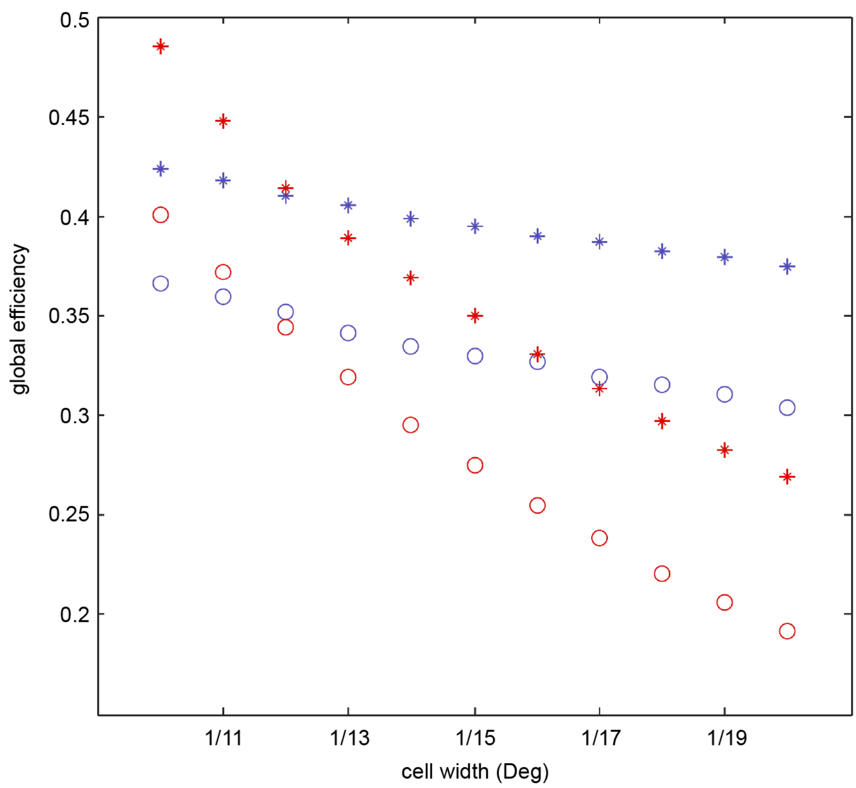

Figure 5. Global efficiency versus the cell width in the OAS (blue circle) and the ASN (blue cross). The red circle (red cross) denotes our (Abe-Suzuki) random network.

4. Summary

In this paper, we have calculated and analyzed the fundamental network metrics such as the mean degree, degree distribution, characteristic path length, average clustering coefficient, and global efficiency from seismic data of California. We have compared the OAS and the ASN to their random networks. Through other works, the seismicity has taken the features of complex network for the average clustering coefficient. Min *et al.* have found the values of average clustering coefficient between 0.85 and 0.90 in the cell widths between 60 km and 100 km [33]. We show that the average clustering coefficient in our case has smaller values than that of ASN in the cell widths between 1˚/20 and 1˚/10.

We have novelly treated the network of aftershocks in the field of complex networks. In the future, we think that this method will extend and measure its topological metrics to other earthquake networks how to show a universal property in networks of other regions. We conclude from the results of the calculation that our aftershock network is a scale free network and has the hierarchical structure. Particularly, our method is able to perceive one way to construct the aftershock network, significantly different from the constructing method of the ASN.

The results of this investigation may provide useful and effective information for prediction of scaling behaviors under the impacts of earthquake network changes in other earthquake regions. Our findings support that a recent network approach to earthquake analysis is very useful and reliable in three-dimensional cells. In order to argue our suggestion, a further work about the calculation of network constructions in other nations is needed. It is anticipated that the formalism of our analysis can be extended to both discrimination and the characterization of various aftershocks and earthquakes.

Acknowledgements

This work was supported by a Research Grant of Pukyong National University (2017).

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