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Graphs with *k*-Role Assignments

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Abstract

For a given graph *G*, a *k*-role assignment of *G* is a surjective function $r:V(G)\to\{1,2,\cdots,k\}$ such that $r(x)=r(y)\Rightarrow r(N(x))=r(N(y))$, where $N(x)$ and $N(y)$ are the neighborhoods of *x* and *y*, respectively. Furthermore, as we limit the number of different roles in the neighborhood of an individual, we call *r* a restricted size *k*-role assignment. When the hausdorff distance between the sets of roles assigned to their neighbors is at most 1, we call *r* a *k*-threshold close role assignment. In this paper we study the graphs that have *k*-role assignments, restricted size *k*-role assignments and *k*-threshold close role assignments, respectively. By the end we discuss the maximal and minimal graphs which have *k*-role assignments.

Keywords

Neighborhood, *k*-Role Assignment, Restricted Size *k*-Role Assignment, *k*-Threshold Close Role Assignment

1. Introduction and Preliminary

Role assignments, introduced by Everett and Borgatti [1], who called them role colorings, formalize the idea, arising in the theory of social networks, that individuals of the same social role will relate in the same way to individuals playing counterpart roles.

Let *G* be a graph with vertices representing individuals and edges representing relationships. For any vertex $v\in V(G)$, the *neighborhood* $N_G(v)=N(v)$ of vertex *v* is the set of all vertices adjacent to *v* in *G*. Let $r:V(G)\to Z$, where *Z* is the set of positive integers. For $S\subseteq V(G)$, denote $r(S)=\{r(x):x\in S\}$ and $G[S]$ be the subgraph of *G* induced by set *S*. For any $i\in Z$, $r^{-1}(i)=\{v\in V(G):r(v)=i\}$. The function *r* is a *role assignment* if

$$r(x)=r(y)\Rightarrow r(N(x))=r(N(y)). \quad (1)$$

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In other words, in a role assignment, if two individuals have the same role, they are related to individuals with the same sets of roles. A *k-role assignment* for graph G is a role assignment r so that $r(V(G)) = \{1, 2, \cdots, k\}$. We say that G is *k-role assignable* if it has a k-role assignment.

Pekeč and Roberts [2] proposed a modification of the role assignment model, which would limit the number of different roles in the neighborhood of an individual. This limit could be different for different individuals or could be uniform for the network. For a group of nonnegative integers $s_1, s_2, \cdots, s_k$, we define a *restricted size k-role assignment r* of a graph G to be a k-role assignment in which $|r(N(v))| \le s_i$ must hold for all $v \in r^{-1}(i)$. When $s_1 = s_2 = \cdots = s_k = s$, we call the restricted size k-role assignment a *restricted s-size k-role assignment*, *i.e.*, a k-role assignment in which $|r(N(v))| \le s$ must hold for all $v \in V(G)$.

If S and T are two sets of real numbers, let the *distance* $d(S,T)$ be defined by $d(S,T) = \min\{|s-t| : s \in S, t \in T\}$, with $d(S,\varnothing) = \infty$ if $S \ne \varnothing$ and 0 otherwise. For a given graph G, we say the function r from $V(G)$ into the set of positive integers is a *threshold role assignment* (Roberts [3]) if

$$r(x) = r(y) \Rightarrow d\left(r(N(x)), r(N(y))\right) \le 1. \tag{2}$$

If $r(V(G)) = \{1, 2, \cdots, k\}$, we say r is a *k-threshold role assignment*.

Roberts [3] also introduced an alternative definition of distance, the *Hausdorff distance* d_h, $d_h(S,T) = \max\{\max_{x\in S} d(x,T), \max_{x\in T} d(x,S)\}$, By convention $d_h(S,\varnothing) = \infty$ if $S \ne \varnothing$ and 0 otherwise. For a given graph G, we say the function r from $V(G)$ into the set of positive integers is a *threshold close role assignment* (Roberts [3]) if

$$r(x) = r(y) \Rightarrow d_h\left(r(N(x)), r(N(y))\right) \le 1. \tag{3}$$

If $r(V(G)) = \{1, 2, \cdots, k\}$, we say r is a *k-threshold close role assignment*. If such an assignment exists, we say G is *k-threshold close role assignable*.

A graph with $n \ge 3$ vertices is *role primitive* if it has no k-role assignment for $2 \le k \le n-1$. Roberts and Sheng [4] studied it for a special class graphs called indifference graphs.

A good survey about role assignment is [5]. Recently, many new papers related to role assignment appeared, see [6] [7] [8]. All the graphs in the paper are simple graphs. In Section 2 we give a characterization of graphs that are k-role assignable. In Section 3 we study the restricted k-role assignable graphs and characterize the graphs that are restricted size k-role assignable or restricted s-size k-role assignable. In Section 4 we discuss the graphs that are k-threshold close role assignable. In Section 5 we discuss the maximal and minimal k-role assignable graphs.

2. *k*-Role Assignable Graphs

The assignment $r(x) \equiv 1$ for any vertex $x \in V(G)$ is a role assignment if and only if the graph has no isolated vertices or all isolated vertices, so this describes the graphs which are 1-role assignable. The assignment where $r(x)$ is different for each vertex $x \in V(G)$ is always a role assignment, so every graph of *n* vertices is *n*-role assignable. The other cases are not straightforward.

The 2-role assignable graphs are the first interesting class of graphs. Roberts and Sheng [9] showed that the problem of determining if *G* has a 2-role assignment is NP-complete. Sheng [10] characterized 2-role assignable indifference graphs and extended some results about indifference graphs to the broader class of triangulated graphs. For the general case, we have the following results.

Theorem 1 *Let G be a graph and S a proper subset of* $V(G)$. *If S satisfies the following properties:*

- $|S| = k-1$,
- $G-S$ has no isolated vertices or all isolated vertices, and
- for any $v \in S$, $N(v) \cap (V(G) \setminus S) = \varnothing$ or $V(G) \setminus S$.

Then *G* is *k*-role assignable.

Proof. We give an assignment *r* for *G* by assigning each role of $\{1,2,\cdots,k-1\}$ to one vertex of *S* and *k* to all vertices of $G-S$. The following is to verify that *r* is a *k*-role assignment of *G*. In fact, we just need to check the vertices with role *k*. By property (3), we may assume that $S = S_1 \cup S_2$ and $S_1 \cap S_2 = \varnothing$, where $S_1 = \{v \in S : N(v) \cap (V(G) \setminus S) = V(G) \setminus S\}$ and $S_2 = \{v \in S : N(v) \cap (V(G) \setminus S) = \varnothing\}$. For any vertex $v \in V(G) \setminus S$, since $N(v) = S_1 \cup N_{G-S}(v)$, then

$$r(N(v)) = \begin{cases} r(S_1) \cup \{k\}, & \text{if } G-S \text{ has no isolated vertices,} \\ r(S_1), & \text{if all vertices of } G-S \text{ are isolated vertices.} \end{cases}$$

So *r* is a *k*-role assignment of *G* in each case. The proof is complete.

Theorem 2 *A graph G with at least k vertices has a k-role assignment if and only if* $V(G)$ *can be partitioned into k nonempty sets* $V_1, V_2, \cdots, V_k$, *and the following two properties are satisfied.*

- $G[V_i]$ has no isolated vertices or all isolated vertices, where $i = 1,2,\cdots,k$, and
- for each $i \in \{1,2,\cdots,k\}$, there exist k_i sets $V_{i_1}, V_{i_2}, \cdots, V_{i_{k_i}} \in \{V_1, V_2, \cdots, V_k\}$, where $0 \le k_i \le k$, such that for any $x \in V_i$, $N(x) \cap V_{i_j} \ne \varnothing$ when $j = 1,2,\cdots,k_i$, and $N(x) \cap V_j = \varnothing$ when $j \ne i_1, i_2, \cdots, i_{k_i}$.

Proof. Let *r* be a *k*-role assignment of graph *G* and $r^{-1}(i) = V_i$, in which $i = 1,2,\cdots,k$. Then it is obvious that $V(G)$ is partitioned into *k* nonempty sets $V_1, V_2, \cdots, V_k$. For any $x \in V_i$, if $i \in r(N(x))$, then $G[V_i]$ has no isolated vertices; if $i \notin r(N(x))$, then all vertices of $G[V_i]$ are isolated vertices. For any $x \in V_i$, let $k_i = |r(N(x))|$, then there exist k_i sets $V_{i_1}, V_{i_2}, \cdots, V_{i_{k_i}} \in \{V_1, V_2, \cdots, V_k\}$, where $0 \le k_i \le k$, such that for any $x \in V_i$, $N(x) \cap V_{i_j} \ne \varnothing$ when $j = 1,2,\cdots,k_i$, and $N(x) \cap V_j = \varnothing$ when $j \ne i_1, i_2, \cdots, i_{k_i}$.

On the converse case we suppose $V(G)$ can be partitioned into *k* nonempty sets $V_1, V_2, \cdots, V_k$, and *G* satisfies the two properties mentioned in the theorem. Let $r(V_i) = i$ for $i = 1,2,\cdots,k$. Then it is easy to check that *r* is a *k*-role assignment of graph *G*.

Roberts and Sheng [9] did some work on 2-role assignable graphs. For $k=2$, Sheng [10] have studied the indifference graphs and triangulated graphs. For $k=n-1$, we also have the following result.

Theorem 3 *A graph G with n vertices has a $(n-1)$-role assignment if and only if there exist $x,y\in V(G)$ such that $N(x)\setminus\{y\}=N(y)\setminus\{x\}$.*

Proof. Let *r* be a $(n-1)$-role assignment of *G*. Since $|V(G)|=n$, there must be two vertices *x* and *y* with the same role, *i.e.*, $r(x)=r(y)=k$, where $1\le k\le n-1$, and each of the other vertices have unique roles. Because $r(N(x))=r(N(y))$, if *x* is not adjacent to *y*, then $N(x)=N(y)$, and $N(x)\setminus\{y\}=N(y)\setminus\{x\}$ is obvious; if *x* is adjacent to *y*, then $N(x)\setminus\{y\}=N(y)\setminus\{x\}$.

If there are two vertices *x* and *y* satisfying $N(x)\setminus\{y\}=N(y)\setminus\{x\}$, then let $r(x)=r(y)=1$, and the *r* values of the other $n-2$ vertices are $2,3,\cdots,n-1$ respectively. It is easy to see that *r* is a $(n-1)$-role assignment of graph *G*.

3. Restricted *k*-Role Assignable Graphs

In this section we study the restricted size *k*-role assignment and the restricted *s*-size *k*-role assignment of a graph *G*. It is easy to see that any *k*-role assignment of graph *G* is a restricted *k*-size *k*-role assignment. So we may assume in the following that $0\le s\le k$ and $0\le s_i\le k$, in which $i=1,2,\cdots,k$.

Pekeč and Roberts [2] just discussed the restricted *s*-size *k*-role assignment for the special case when $s=1$, *i.e.*, when everyone in the neighborhood of an individual should be assigned the same role. They showed that *G* has a restricted 1-size *k*-role assignment if and only if $k\le c(G)+b(G)$, where $c(G)$ is the number of connected components of *G* and $b(G)$ is the number of connected components of *G* that are bipartite but not isolated vertices. In particular, a connected graph *G* on at least two vertices has a restricted 1-size 2-role assignment if and only if *G* is bipartite. For the restricted size *k*-role assignment, we have the following result.

Theorem 4 *For k nonnegative integers $s_1,s_2,\cdots,s_k$, a graph G with at least k vertices has a restricted size k-role assignment if and only if $V(G)$ can be partitioned into k nonempty sets $V_1,V_2,\cdots,V_k$, and the following two properties are satisfied.*

- $G[V_i]$ has no isolated vertices or all isolated vertices, where $i=1,2,\cdots,k$, and
- for each $i\in\{1,2,\cdots,k\}$, there exist t_i sets $V_{i_1},V_{i_2},\cdots,V_{i_{t_i}}\in\{V_1,V_2,\cdots,V_k\}$, where $0\le k_i\le s_i$, such that for any $x\in V_i$, $N(x)\cap V_{i_j}\ne\varnothing$ when $j=1,2,\cdots,t_i$, and $N(x)\cap V_j=\varnothing$ when $j\ne i_1,i_2,\cdots,i_{t_i}$.

Proof. For *k* nonnegative integers $s_1,s_2,\cdots,s_k$, let *r* be a restricted size *k*-role assignment of graph *G* and $r^{-1}(i)=V_i$. Then we have $|r(N(v))|\le s_i$ for all $v\in V_i$, in which $i=1,2,\cdots,k$. The following is similar to the proof of Theorem 2.

When $s_1=s_2=\cdots=s_k=s$, we have the following corollary for the restricted *s*-size *k*-role assignment.

Corollary 1 *A graph G with at least k vertices has a restricted s-size k-role* *assignment if and only if* $V(G)$ *can be partitioned into* k *nonempty sets* $V_1, V_2, \cdots, V_k$ *, and the following two properties are satisfied.*

- $G[V_i]$ has no isolated vertices or all isolated vertices, where $i = 1, 2, \cdots, k$, and
- for each $i \in \{1, 2, \cdots, k\}$, there exist t_i sets $V_{i_1}, V_{i_2}, \cdots, V_{i_{t_i}} \in \{V_1, V_2, \cdots, V_k\}$, where $0 \le t_i \le s$, such that for any $x \in V_i$, $N(x) \cap V_{i_j} \ne \varnothing$ when $j = 1, 2, \cdots, t_i$, and $N(x) \cap V_j = \varnothing$ when $j \ne i_1, i_2, \cdots, i_{t_i}$.

4. *k*-Threshold Close Role Assignable Graphs

Roberts [3] showed that every graph G has a k-threshold role assignment for every $2 \le k \le |V(G)|$. Roberts [3] [11] showed that every graph with k or more vertices is k-threshold close role assignable for the cases $k = 2, 3, 4$ and 5.

Let $[1, d+1]$ denote the set of all positive integers less or equal to $d+1$ and the diameter of a graph G be defined as

$$\operatorname{diam} G = \max\{d(x, y) : x, y \in V(G)\}. \tag{4}$$

Then we have the following result.

Theorem 5 *Given a connected graph G. If* $\operatorname{diam} G = d$ *, then G is k-threshold close role assignable for any* $k \in [1, d+1]$.

Proof. Without loss of generality, we may assume that there are two vertices $x, y \in V(G)$ such that $\operatorname{diam} G = d(x, y) = d$, *i.e.*, x and y are the endpoints of some path of *G*. We partition $V(G)$ into $d+1$ nonempty sets $V_1, V_2, \cdots, V_{d+1}$ by the following way: Let

$$V_1 = \{x\} \text{ and } V_i = \{v : d(x, v) = i - 1\}, \text{ where } i = 2, 3, \cdots, d+1. \tag{5}$$

Then it is easy to see that $y \in V_{d+1}$ and the following properties are valid.

- $\bigcup_{i=1}^{d+1} V_i = V(G)$,
- $V_i \cap V_j = \varnothing$, when $i \ne j$, and
- for any $v \in V_i, N(v) \cap V_{i-1} \ne \varnothing$ and $N(v) \cap V_j = \varnothing$, where $j = 1, 2, \cdots, i-2, i+2, i+3, \cdots, d, d+1$.

Note that $N(v) \cap V_{i+1}$ may be null or not.

Now we construct other $d+1$ nonempty sets $V_1^*, V_2^*, \cdots, V_{d+1}^*$ from $V_1, V_2, \cdots, V_{d+1}$. Let

$$V_i^* = (V_i \setminus H_i) \cup H_{i+1} \text{ and } V_{d+1}^* = V_{d+1}, i = 1, 2, \cdots, d, \tag{6}$$

in which $H_1 = \varnothing$, $H_{d+1} = \varnothing$ and
$H_i = \{v : v \text{ is an isolated vertex of } G[V_i] \text{ and } N(v) \cap V_{i+1} = \varnothing\}$, $i = 2, 3, \cdots, d$.

Then $\bigcup_{i=1}^{d+1} V_i^* = V(G)$, $V_i^* \cap V_j^* = \varnothing$ when $i \ne j$, and for any vertex $v \in V_i^*$, the neighborhood $N(v)$ must be one of the following four cases.

- $N(v) \cap V_i^* = \varnothing,\ N(v) \cap V_{i+1}^* \ne \varnothing,\ N(v) \cap V_{i-1}^* \ne \varnothing$,
- $N(v) \cap V_i^* \ne \varnothing,\ N(v) \cap V_{i-1}^* = \varnothing,\ N(v) \cap V_{i+1}^* = \varnothing$,
- $N(v) \cap V_i^* \ne \varnothing,\ N(v) \cap V_{i-1}^* \ne \varnothing,\ N(v) \cap V_{i+1}^* = \varnothing$,
- $N(v) \cap V_i^* \ne \varnothing,\ N(v) \cap V_{i-1}^* \ne \varnothing,\ N(v) \cap V_{i+1}^* \ne \varnothing$.

Then, for any $k \in [1, d+1]$, let $r(V_i^*) = i$ for $i = 1, 2, \cdots, k$, and $r(V_j^*) = k$ for $j = k+1, k+2, \cdots, d+1$. Then $r(N(v)) = \{k\}$ for any vertex $v \in \bigcup_{i=k+1}^{d+1} V_i^*$. For any vertex $v \in V_k^*$, $r(N(v))$ must be $\{k\}$ or $\{k-1, k\}$. For any vertex $v \in V_i^*$ where $i = 1, 2, \cdots, k-1$, since $N(v)$ is in one of the four cases above, $r(N(v))$ must be one of the following four sets: $\{i-1, i+1\}$, $\{i\}$, $\{i-1, i\}$ and $\{i-1, i, i+1\}$. Thus it is easy to see that $d_h(r(N(u)), r(N(v))) \le 1$ for any two vertices u and v with same role, *i.e.*, r is a k-threshold close role assignment of G. The proof is complete.

Corollary 2 *Any graph that has no isolated vertices or all isolated vertices is k-threshold close role assignable for any* $k \in [1, d+\omega]$, *where* $d = d_1 + d_2 + \cdots + d_\omega$, ω *is the number of components of G and* d_i *is the diameter of each component of G,* $i = 1, 2, \cdots, \omega$.

Proof. When all vertices of G are isolated vertices, the result is obvious. Now we assume that G has no isolated vertices and that $G_1, G_2, \cdots, G_\omega$ are the connected components of G. For each G_i, we define V_{ij}^* as V_j^* for G in Theorem 5, where $j \in \{1, 2, \cdots, d_{i+1}\}$. Furthermore, we order them as follows:

$$V_{11}^*, V_{12}^*, \cdots, V_{1(d_1+1)}^*, V_{21}^*, V_{22}^*, \cdots, V_{2(d_2+1)}^*, V_{31}^*, V_{32}^*, \cdots, V_{\omega(d_\omega+1)}^*.$$

For any $k \in [1, d+\omega]$, we assign all the vertices of the ith set in above sequence with role i, where $1 \le i \le k$, and assign all vertices of the other sets with role k. Then the result is valid by the similar discussion in Theorem 5.

The following three results are immediate from Theorem 5.

Corollary 3 *Any path with n vertices is k-threshold close role assignable for any* $k \in [1, n]$.

Corollary 4 *Any tree whose longest path has t vertices is k-threshold close role assignable for any* $k \in [1, t]$.

Corollary 5 *Any cycle with n vertices is k-threshold close role assignable for any* $k \in \left[1, \left\lceil \frac{n+1}{2} \right\rceil\right]$.

Remark. Any cycle C with n vertices is k-threshold close role assignable for any $k \in [1, n]$. In fact, let

$$v_1, v_3, v_5,, v_{n-1}, v_n, v_{n-2}, \cdots, v_4, v_2, (n \text{ even})$$

or

$$v_1, v_3, v_5, \cdots, v_{n-2}, v_n, v_{n-1}, \cdots, v_4, v_2. (n \text{ odd})$$

be the n vertices of C in clockwise order. For any $k \in [1, n]$, let $r(v_i) = i$ for any $i = 1, 2, \cdots, k$, $r(v_{k+1}) = k-1$ and $r(v_j) = k$ for $j = k+2, k+3, \cdots, n$. Then it is easy to see that r is a k-threshold close role assignment of C.

5. Maximal and Minimal *k*-Role Assignable Graphs

Suppose graph G with n vertices is k-role assignable. We may add (or delete) edges on G to get a complete (or empty) graph G'. It is easy to see that G' is k-role assignable too. So we can maximize or minimize the graph G and keep $r(N(x))$ fixedness for any $x \in V(G)$ such that the resulting graph is still *k*-role assignable.

For a given *k*-role assignment *r* of graph *G*, the maximal (minimal, respectively) *k*-role assignable graph respect to *r*, denoted by G_r^+ (G_r^-, respectively), is the graph that will get by the following ways.

If $V_i = r^{-1}(i)$, where $i = 1, 2, \cdots, k$, we may use the following ways to get G_r^+.

• If $G[V_i]$ has no isolated vertices, then add some edges to join each pair of vertices unconnected in V_i, where $i = 1, 2, \cdots, k$.

• If $xy \in E(G)$, where $x \in V_i$ and $y \in V_j$, $i, j \in \{1, 2, \cdots, k\}$, then add some edges on graph *G* such that each vertex of V_i joins with every vertex of V_j.

We may also use the following ways to get G_r^-.

• If $G[V_i]$ has no isolated vertices, then delete some edges of $G[V_i]$ at mostly and keep $G[V_i]$ without any isolated vertex, where $i = 1, 2, \cdots, k$.

• If $xy \in E(G)$, where $x \in V_i$ and $y \in V_j$, $i, j \in \{1, 2, \cdots, k\}$, then delete some edges between V_i and V_j at mostly but keeping $N(x) \cap V_j \neq \varnothing$ for any $x \in V_i$ and $N(y) \cap V_i \neq \varnothing$ for any $y \in V_j$.

By the ways of constructing G_r^+ and G_r^-, the following theorem is obvious.

Theorem 6 *If a graph G is k-role assignable, then both* G_r^+ *and* G_r^- *are all k-role assignable. Furthermore,* G_r^+ *(*G_r^-*, respectively) is the maximal (minimal, respectively) k-role assignable graph such that* $r(N(x))$ *keeps fixedness for any* $x \in V(G)$*, where r is some k-role assignment of graph G.*

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