

# The Spread of Infectious Disease on Network Using Neutrosophic Algebraic Structure

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## Abstract

Network theory and its associated techniques has tremendous impact in various discipline and research, from computer, engineering, architecture, humanities, social science to system biology. However in recent years epidemiology can be said to utilizes these potentials of network theory more than any other discipline. Graph which has been considered as the processor in network theory has a close relationship with epidemiology that dated as far back as early 1900 [1]. This is because the earliest models of infectious disease transfer were in a form of compartment which defines a graph even though adequate knowledge of mathematical computation and mechanistic behavior is scarce. This paper introduces a new type of disease propagation on network utilizing the potentials of neutrosophic algebraic group structures and graph theory.

## Keywords

Neutrosophic Group  $N(G)$ , Infectious Disease, Graph Representation of  $N(G)$ , Cayley Graphs

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## 1. Introduction

Mathematical models is being used in studying the dynamics, transfer and control of infectious disease. Most of the epidemiological models are continuous time models which uses ordinary differential equation [2] and assumed that the population is fixed and well mixed and the transmission is evaluated over period of time. However the environment in which the population lives is often heterogeneous, therefore there is need to distinguish between the locations. There is closed relationship between epidemiology and graph theory that dated as far back as early 1900. This is because compartments, groupings and household

models are created and used to study dynamics of epidemics even though mathematical and mechanistic knowledge is scarce. More recently graphs and trees are being used to study the dynamics of infectious disease. In this type, nodes represents infected agents and arc represents transmission from one agent to another. During the review of the literature we found out that most of the authors used compartment-based modelling in the transmission of the pathogens. In order to demonstrate how the population moves through, during the course of the epidemic processes, different compartment were formulated. Common among them are SIR, SIS and SEIR. In most of these models, two basic assumptions were often made so as to simplify the modelling process: 1) uniform mixing assumption and 2) homogeneity assumption, both of which may not yield a good result. We also found out that considerable importance is placed on analytical approaches and statistical methods which are both rapidly expanding fields. We note that less attention is paid to the algebraic structures, therefore we intend to explore a novel approach with the algebraic graph theory and the neutrosophic algebraic structure first introduce by [3]. This is a new branch of philosophy which actually studies the origin, nature, and scope of neutralities. This theory is being applied in many fields in order to solve problems related to indeterminacy.

In this paper we represent each stage of the viral transmission with element of  $Z_7$ , where each stage is accorded with a unique element in  $Z_7$ . Therefore we are able to distinguish between those who are unable to catch the infection with those susceptible. Moreover we noticed that the infected population were counted as one entity in all of those models, so that transmission from infected class to the next compartment raises a lot of questions. Here these shortcomings were fully taken care of by splitting the infected class into two categories, *i.e.*, Infected vs transmitter. We are focusing on the neutrosophic graphs created by [4], because of the indeterminacy in some nodes and edges which distinguishes it from the classical graph theory. So we can label each point of the viral transmission stages with the notation  $a + bI$  where  $a, b \in \mathbb{R}, \mathbb{Z}$ , or  $\mathbb{C}$ . Here  $N(G) = \langle \{Z_7 \cup I\}, '+' \text{ modulo } 7 \rangle$  is use and we found that this neutrosophic group is also a cyclic group modulo 7. But since  $N(G)$  is a union of elements of  $Z_7$  and the indeterminacy " $I$ " then there is no single generator for the group, instead a disjoint class of generators with a maximum of five (5) elements each. So what we are present here is a transformation of the representation of different stages of the viral transmission into neutrosophic group structure thereby constructing neutrosophic graphs from this group.

In many disease transmission models it is unfeasible to exactly retrieve the complete network structure but with neutrosophic integer modulo this shortcoming is overcome because we enlarge the graph so as to accommodate more nodes.

## 2. Basic Definition

Feinleib defines epidemiology as "the study of the distribution and determinants of health related states or events in specified populations, and the application of

this study to control of health problems” [5].

Epidemiology is defined by Kramer as “it is a scientific method which can be applied to a broad range of health and medical problems, from infectious diseases to health care” [6].

When referring to disease spread various terminologies are used, such as:

1) Disease Outbreak: a disease outbreak occurs when there is a sudden increase in the expected number of affected people.

2) Epidemic: during a disease outbreak, when the incidence rate increases above the previous occurrences of the outbreak it is known as epidemic.

3) Pandemic: epidemic affecting a large population is known as pandemic.

4) Endemic: if the epidemic remains within a particular community, it is called endemic [7].

Stages of Infection:

The infection has different stages in a person and the duration and strength vary according to a person’s immunity, age, health, previous exposure to the infection etc. A person who is not infected and has a chance of getting infected is said to be in susceptible stage. A person’s immunity system may destroy the infectious organism. It may continue to stay inside a person’s body depending upon his characteristics. Different stages of infection are as follows:

- Latent Period

The latent period is the period between the start of infection to the infectious period.

- Incubation Period

It is the time period between the infectious period and the onset of symptoms.

- Symptomatic Period

Symptomatic period starts after the incubation period when the symptoms show up till the infected person stops infecting other.

The stages of infection summarized by Nelson as shown in **Figure 1** which is grouped into Susceptible, infected, infective and removed. It shows the level of agent in host from the start of the infection and increase instantaneously over time until it reaches its pick and then dies out momentarily.

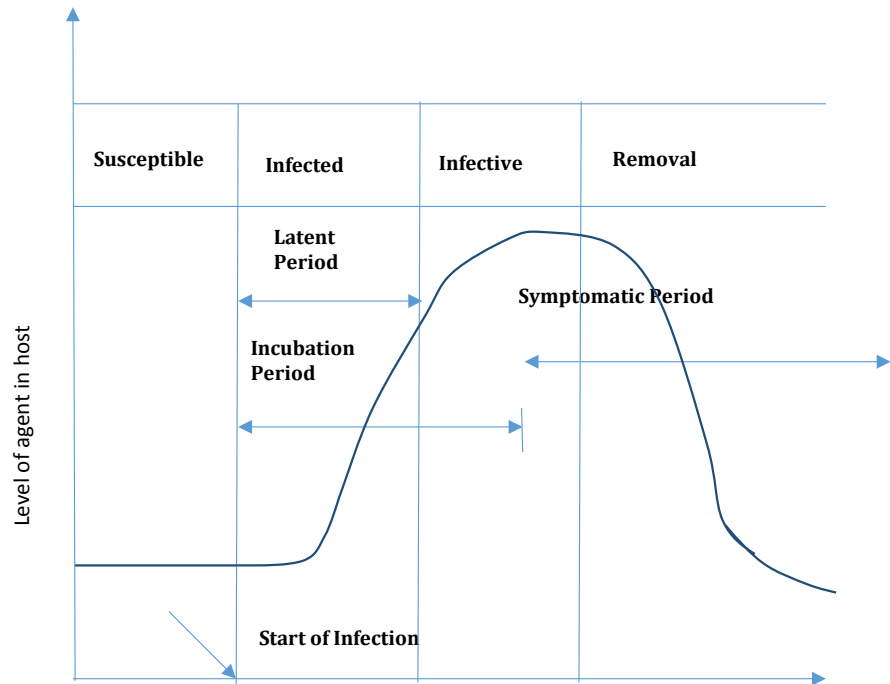
Although Nelson was able to distinguish between infected and infective period that is a person might be infected but yet not infectious until later in life, what is lacking in his model is the category of infected individuals who will never be infectious but rather will remain in the infected class until been recovered. Our proposed model is set to capture this targeted population together with the category of those not susceptible members of the population whom might have significant effects in determining the epidemic threshold and other epidemiological concepts.

Neutrosophic group:

Let  $(G, *)$  be any group, the neutrosophic group is generated by  $I$  and  $G$ , under  $*$  denoted by  $N(G) = \{ \langle G \cup I \rangle, * \}$  [9].

Where “ $I$ ” is the indeterminate and,

$$I^2 = I, I + I + I \cdots n \text{ times} = nI - I = 0, 0I = 0.$$



**Figure 1.** Stages of infection [8].

It is however to note that:

- 1)  $N(G)$  in general is not a group by itself.
- 2)  $N(G)$  always contains a group [10].

Form this definition, however by defining

$\langle \langle Z_7 \cup I \rangle, '+' \text{ modulo } 7 \rangle = \{a + bI/a, b(b \in Z_7)\}$ , then  $N(G)$  can be made a group.

Graph:

We know that a graph  $G$  consists of two things:

- 1) A set  $V$  of elements called nodes (or points or vertices).
- 2) A set  $E$  of edges such that each edge in  $E$  is identified with a unique (unordered) pair  $\{u, v\}$  of nodes in  $V$ , denoted by  $e = \{u, v\}$ . So  $G = (V, E)$ . So from the fundamental concepts on graph theory and the knowledge of neutrosophic group we can construct a neutrosophic graph which will serve as our viral transmission model.

From the review of literature we discovered that the first representation of groups by graphs is the Cayley graphs. These graphs were introduced by Arthur Cayley in 1878 and it shows pictorial representations of finite groups [11].

Cayley graph:

Giving a group  $A$  and a set of generators for  $A$ . The Cayley digraph  $G = (V, E)$  denoted by  $\langle A, S \rangle$  is constructed as follows:

- 1) The elements of the group  $A$  forms the vertices  $V$  of the digraph  $G$ .
- 2) The edge  $(a, b)$  is in  $E$  if and only if  $ag = b$  for some generators  $g$  in  $S$ .

If  $S = S \cup S'$ , then  $G$  is Cayley graph [12] [13].

$G$  graph:

For a group  $G$  with generating set  $S = \{s_1, s_2, \dots, s_k\}$ , the  $G$ -graph of  $G$ , de-

noted by  $\Gamma(G, S)$  is the graph whose vertices are distinct cosets of  $\langle s_i \rangle$  in  $G$ . Two distinct vertices are joined by an edge when the set intersection of the cosets is nonempty [14].

Combining the above two definitions, Cayley graph and  $G$ -graph it is evidently clear that:

The generators of a cyclic group plays a vital role in constructing a graph from group and there is a relationship between a generator and its generating set which can be considered to be an arc, whether directed or undirected.

### 3. Methodology

1) Assumptions:

- a) The total population is taken as a constant.
- b) An initial infected class is introduced to the total population.
- c) The dead or recovered people in the community belongs to the same class.
- d) The population's contact is heterogeneous.

2) Variables Declaration:

$S^*$  = Not Susceptible, represents those members of the population who are negated from the disease or whose probability of being contracted with the disease is zero.

$S$  = Susceptible, represents those members of the population who stand the risk of being infected.

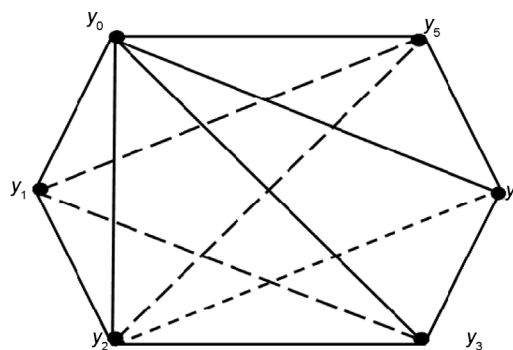
$L$  = Incubation, represents those members of the population who are infected and the body's defence mechanisms are unable to destroy the virus.

$E$  = Exposed, This represents the period when the symptoms start to shows up.

$I$  = Infected, This represents a fraction of the population who are infected and remain in the infected class but potentially cannot transmit the disease. From **Figure 2** this indicates that the curve increase in the infected class and then drops without touching the infective class. Epidemiologically this means it remains stagnant in the infected class, until being removed or return to susceptible.

$T$  = Transmitter, This class of the population are infected and are capable of transmitting the infection.

$R$  = Recovered, These are fraction of the population, that are immune, vaccinated, dead, isolated.



**Figure 2.** Neutrosophic viral transmission graph.

$G =$  Represents the total population. In this case  $S, L, E, I, T, R, S^*$  are subset of  $G$ .

Hence  $(S, L, E, I, T, R, S^*) \subset G$ .

Let us use the integer modulo arithmetic  $Z_n$  to represents the subsets of  $G$ , that is

$$S^* = 0, S = 1, L = 2, E = 3, I = 4, T = 5, R = 6.$$

Observed that  $G = \{0,1,2,3,4,5,6\}$ . Consider the elements of  $G$  to be integer modulo 7 hence  $(Z_7 +)$  is a group.

Lets define  $N(G) = \{\langle Z_7 \cup I \rangle, '+' \text{ modulo } 7\}$  is a neutrosophic group which is in fact a group. For  $N(G) = \{a + bI/a, b \in (Z_7)\}$  is a group under “+” modulo 7. Thus this neutrosophic group is also a group.

Hence

$$N(G) = \left\{ \begin{array}{l} 0, 1, 2, 3, 4, 5, 6, I, 2I, 3I, 4I, 5I, 6I, 1+I, 1+2I, 1+3I, 1+4I, 1+5I, \\ 1+6I, 2+I, 2+2I, 2+3I, 2+4I, 2+5I, 2+6I, 3+I, 3+2I, 3+3I, \\ 3+4I, 3+5I, 3+6I, 4+I, 4+2I, 4+3I, 4+4I, 4+5I, 4+6I, 5+I \\ 5+2I, 5+3I, 5+4I, 5+5I, 5+6I, 6+I, 6+2I, 6+3I, 6+4I, \\ 6+5I, 6+6I. \end{array} \right\}$$

Here “ $\mathcal{P}$ ” cannot stand for the group identity it is only an indeterminate and hence suitable for the nature of our disease propagation model.

Theorems:

Hence the following theorems were proposed based on the group  $N(G)$ .

Theorem (1) Every stage of viral transmission is allocated a unique representation in  $N(G)$ .

Proof: Suppose we have two representation for  $S$  i.e.  $s, s' \in S$ .

If  $e \in N(G)$  is the identity element and be such that has no effect on the elements of  $N(G)$ , then  $e * s = s * e = s$  since  $e$  is the identity element.

Also  $s' * e = e * s' = s'$  since  $e$  is the identity element.

Hence  $s = s'$  and the result follows.

Theorem (2) The operation  $*$  on  $N(G)$ , is well define.

Let  $\underline{a}, \underline{b} \in N(G)$ . By the definition of  $*$  on  $N(G)$ , we would have  $a * b = \overline{a + b}$ . Now suppose  $a_1 \in a$  and  $b_1 \in b$ , be chosen as class representative of  $a$  and  $b$  respectively by defining a suitable homomorphism on  $N(G)$ , in this case we would have  $a + b = a_1 + b_1 = \overline{a_1 + b_1}$ . Thus we can show that  $\overline{a_1 + b_1} = \overline{a + b}$ .

Now  $a_1 \in a \Rightarrow a_1 \equiv a \pmod{n} \Rightarrow a_1 = a + k_n$  for some  $k \in Z$ , and  $b_1 \in b \Rightarrow b_1 \equiv b \pmod{n} \Rightarrow b_1 = b + h_n$  for some  $h \in Z$ , thus  $a_1 + b_1 = a + b + (k + h)n \Rightarrow a_1 + b_1 \equiv a + b \pmod{n}$ , this implies that  $a * b = \overline{a + b}$  and the operation  $*$  on  $N(G)$ , is well define.

Theorem (3) The order of the neutrosophic group  $\{\langle Z_n \cup I \rangle, '+' \text{ modulo } n\}$  denoted by  $|N(G)|$  is  $n^2$ .

Proof: Since  $Z_n$  has  $n$  elements, and the indeterminate “ $\mathcal{P}$ ” is such that:  $I + I + \dots n$  times is  $nI$  i.e.  $|nI|$  is  $n$ . Therefore  $\langle Z_n \cup I \rangle$  is a combinations of the elements of  $Z_n$  and  $nI$  that is  $n \times n = n^2$  hence the proof.

### 3) Building graphs from neutrosophic group structure.

Kandasamy W and Smarandache F in 2015 defines neutrosophic graphs as follows: "If the edge values are from the set  $R \cup I$  or  $Q \cup I$  or  $Z_n \cup I$  or  $Z \cup I$  or  $C \cup I$  they are term as neutrosophic graph" [4]. In our own case the edge are from  $\{\langle Z_7 \cup I \rangle, '+' \text{ modulo } 7\}$  and the elements of  $(Z_7)$  where carefully allocated with a unique stages of the viral transmission. Now lets define a neutrosophic graph generated from neutrosophic group.

Definition: Henceforth we defined neutrosophic graph  $\mathcal{O}(V, E)$  generated from  $N(G)$  as follows:

1. Given an element  $a \in N(G)$  and generates a set  $B$  of elements in  $N(G)$  then element  $a$  and set  $B$  connects or has a relation. Hence For any  $b \in B, (a, b)$  is an edge.
2. The elements of  $N(G)$  forms the vertices  $V$  of the neutrosophic graph.

As mentioned above and confirmed by Dinnen and Breto the generator(s) of a group  $G$  is very vital in the construction of a neutrosophic graph from group.

### 4) Finding set of generators

Since the neutrosophic group  $N(G)$ , is generated by  $Z_7 \cup I$ , the generators are precisely the class of union of generators which partitioned  $N(G)$ .

Hence in  $Z_7$  the generators are  $(1, 2, 3, 4, 5, 6)$ . Moreover

$I$  is a generator of  $2I, 3I, 4I, 5I, 6I$ ,

$1+I$  is a generator of  $2+2I, 3+3I, 4+4I, 5+5I, 6+6I$ ,

$1+2I$  is a generator of  $2+4I, 3+6I, 4+I, 5+3I, 6+5I$ ,

$1+3I$  is a generator of  $2+6I, 3+2I, 4+5I, 5+I, 6+4I$ ,

$1+4I$  is a generator of  $2+I, 3+5I, 4+2I, 5+6I, 6+3I$ ,

$1+5I$  is a generator of  $2+3I, 3+I, 4+6I, 5+4I, 6+2I$ ,

$1+6I$  is a generator of  $2+5I, 3+4I, 4+3I, 5+2I, 6+I$ ,

$2+I$  is a generator of  $4+2I, 6+3I, 1+4I, 3+5I, 5+6I$ ,

$2+2I$  is a generator of  $4+4I, 6+6I, 1+I, 3+3I, 5+5I$ ,

$2+3I$  is a generator of  $4+6I, 6+2I, 1+5I, 3+I+4I$ ,

$2+4I$  is a generator of  $4+I, 6+5I, 1+2I, 3+6I, 5+3I$ ,

$2+5I$  is a generator of  $4+3I, 6+I, 1+6I, 3+4I, 5+2I$ ,

$2+6I$  is a generator of  $4+5I, 6+4I, 1+3I, 3+2I, 5+I$ ,

$3+I$  is a generator of  $6+2I+3I, 5+4I, 1+5I, 4+6I$ ,

$3+2I$  is a generator of  $6+4I, 2+6I, 5+I, 1+3I, 4+5I$ ,

$3+3I$  is a generator of  $6+6I, 2+2I, 5+5I, 1+I, 4+4I$ ,

$3+4I$  is a generator of  $6+I, 2+5I, 5+2I, 1+6I, 4+3I$ ,

$3+5I$  is a generator of  $6+3I, 2+I, 5+6I, 1+4I, 4+2I$ ,

$3+6I$  is a generator of  $6+5I, 2+5I, 5+3I, 1+2I, 4+I$ ,

$4+I$  is a generator of  $1+2I, 5+3I, 2+4I, 6+5I, 3+6I$ ,

$4+2I$  is a generator of  $1+4I, 5+6I, 2+I, 6+3I, 3+5I$ ,

$4+3I$  is a generator of  $1+6I, 5+2I, 2+5I, 6+I, 3+4I$ ,

$4+4I$  is a generator of  $1+I, 5+5I, 2+2I, 6+6I, 3+3I$ ,

$4+5I$  is a generator of  $1+3I, 5+I, 2+6I, 6+4I, 3+2I$ ,

$4+6I$  is a generator of  $1+5I, 5+4I, 2+3I, 5+2I, 3+I$ ,

$5 + I$  is a generator of  $3 + 2I, 1 + 3I, 6 + 4I, 4 + 5I, 2 + 6I,$   
 $5 + 2I$  is a generator of  $3 + 4I, 1 + 6I, 6 + I, 4 + 3I, 2 + 5I,$   
 $5 + 3I$  is a generator of  $3 + 6I, 1 + 2I, 6 + 5I, 4 + I, 2 + 4I,$   
 $5 + 4I$  is a generator of  $3 + I, 1 + 5I, 6 + 2I, 4 + 6I, 2 + 3I,$   
 $5 + 5I$  is a generator of  $3 + 3I, 1 + I, 6 + 6I, 4 + 4I, 2 + 2I,$   
 $5 + 6I$  is a generator of  $3 + 5I, 1 + 4I, 6 + 3I, 4 + 2I, 2 + I,$   
 $6 + I$  is a generator of  $5 + 2I, 4 + 3I, 3 + 4I, 2 + 5I, 1 + 6I,$   
 $6 + 2I$  is a generator of  $5 + 4I, 4 + 6I, 3 + I, 2 + 3I, 1 + 5I,$   
 $6 + 3I$  is a generator of  $5 + 6I, 4 + 2I, 3 + 5I, 2 + I, 1 + 4I,$   
 $6 + 4I$  is a generator of  $5 + I, 4 + 5I, 3 + 2I, 2 + 6I, 1 + 3I,$   
 $6 + 5I$  is a generator of  $5 + 3I, 4 + I, 3 + 6I, 2 + 4I, 1 + 2I,$   
 $6 + 6I$  is a generator of  $5 + 5I, 4 + 4I, 3 + 3I, 2 + 2I, 1 + I,$

#### 4. Discussion

From the result so far generated above we can see that every element  $a + bI$  in  $N(G)$  partitioned  $N(G)$  into subset which are themselves groups each when the identity  $0 + 0I$  is embedded in it. Therefore a neutrosophic group  $N(G)$  is cyclic if for any  $a + bI \in (Z_n \cup I)$ , there exist an integer  $r \in Z_n$  such that  $r(a + bI) = c + dI$  for some  $c + dI \in N(G)$ . Then  $a + bI$  is a generator of  $c + dI$ .

Observed that the principal generators denoted by  $\{\langle p_g \rangle\}$  are those point in  $N(G)$  whose generating cycles are distinct. Also the sub generators are those who are either subset of  $\{\langle p_g \rangle\}$  or parts of its generating cycles have been generated by  $\{\langle p_g \rangle\}$ .

Corollary If  $\{\langle Z_n \cup I \rangle, '+' \text{ modulo } n\}$  is a cyclic neutrosophic group and  $\langle a \rangle$  is a generator, then it generates only  $n - 2$  elements.

Proof: The generalization of  $N(G)$  as a group is false, but  $N(G)$  always contained a group.

Let  $Z_n$  be the cyclic group in  $N(G)$  by the theorem “let  $G = \langle Z_n \rangle$  be a finite cyclic group of order  $n$ . The generators of  $G$  are the elements  $ra$  where the  $\text{gcd}(r, n) = 1$  coprime or relatively prime”.

Now  $r$  is taking from  $\langle Z_7 \rangle$  where  $r = (0, 1, 2, 3, 4, 5, 6)$ ,  $\text{gcd}(0, 7) = 1$  is impossible,  $\text{gcd}(1, 7) = 1$  Obvious.

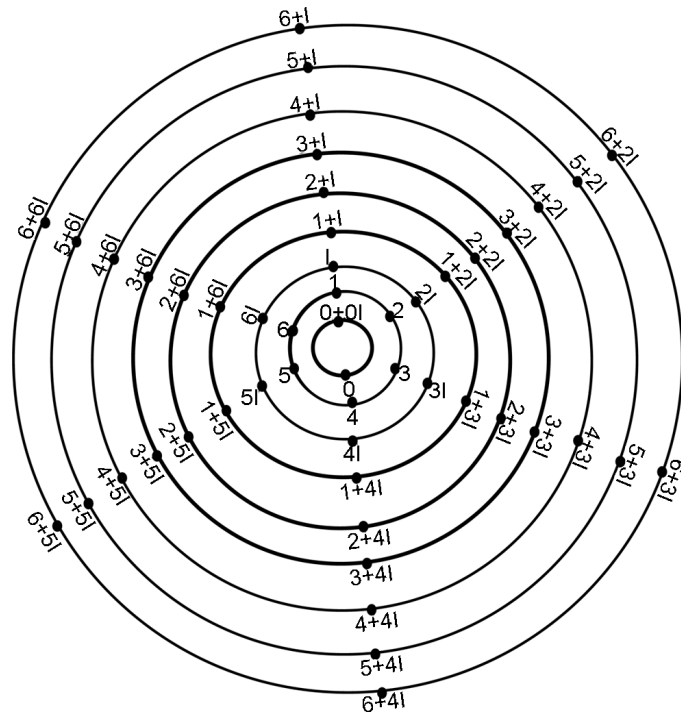
Hence each generator, generate only  $n - 2$  elements in  $N(G)$ .

##### Results

Hence given that  $y_0 = \langle 1 + I \rangle$ , is a generator with its generating sets as  $y_1 = 2 + 2I, y_2 = 3 + 3I, y_3 = 4 + 4I, y_4 = 5 + 5I, y_5 = 6 + 6I$ , then the nodes are  $V = (y_0, y_1, y_2, y_3, y_4, y_5)$ . The dotted lines shows neutrosophic edges, indicating that the connections between those distinct nodes are indeterminate as shown in **Figure 2**.

Since  $y_0$  is the generator, then connections to any of its generating sets is guarantee and the tendency of contracting is high, meaning it has a direct communication to those neighbors. But  $y_1$  to  $y_3, y_2$  to  $y_5, y_4$  to  $y_2$  all has indeterminate connections, this indicates that the tendencies of contraction is vague.





**Figure 3.** Cyclic neutrosophic viral graph.

The figure above is a cyclic representation of the neutrosophic group  $(G)$ . It depicts the connections between the nodes in order of their generation. Thus eight (8) cycles were formed with six (6) elements each together with the terminating cycle for the null neutrosophic or identity as shown in **Figure 3**.

This is indicating that circulation of viral infection within the same class is faster than its interconnectivity. And from the large circle we can see that, connection from node  $6+I$  to node  $6+2I$  indicates the reversal since the graph is undirected, which shows involution. Moreover the distance between these three nodes  $6+6I$ ,  $6+I$ ,  $6+2I$  remains equal when considering the shortest possible routes.

### 5. Conclusions

In this paper we represented an entire epidemic population with integer modulo 7, *i.e.*,  $(Z_7)$  and realize that  $(Z_7, +)$  is a group. Then we transform this group  $(Z_7, +)$  into a neutrosophic group  $N(G)$ , by defining  $N(G) = \{(Z_7 \cup I), '+ \text{ modulo } 7\} = \{a+bI/a, b(Z_7)\}$ . We found that this is a special type of neutrosophic group which is also a group and in particular a cyclic group and having a set of generators each, which partitioned  $N(G)$  into classes of subset which are themselves groups when the identity  $0+0I$  is embedded to each subset.

The results obtained is being converted into neutrosophic graph  $\emptyset(V, E)$  with elements of  $N(G)$  forming the vertices  $V$  and  $E$  is an arc determined by defining a suitable homeomorphisms between the groups  $(Z_7, +)$  and  $N(G)$  as shown above.

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