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Computation of Topological Indices of Dutch Windmill Graph

M. R. Rajesh Kanna[1], R. Pradeep Kumar[2], R. Jagadeesh[3]

[1]Post Graduate Department of Mathematics, Maharani's Science College for Women, Mysore, India
[2]Department of Mathematics, The National Institute of Engineering, Mysore, India
[3]Research and Development Centre, Bharathiar University, Coimbatore, India
Email: mr.rajeshkanna@gmail.com, pradeepr.mysore@gmail.com, jagadeeshr1978@gmail.com

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Abstract

In this paper, we compute Atom-bond connectivity index, Fourth atom-bond connectivity index, Sum connectivity index, Randic connectivity index, Geometric-arithmetic connectivity index and Fifth geometric-arithmetic connectivity index of Dutch windmill graph.

Keywords

ABC **Index,** $\boldsymbol{ABC_4}$ **Index, Sum Connectivity Index, Randic Connectivity Index,** ***GA*** **Index,** $\boldsymbol{GA_5}$ **Index**

1. Introduction

The Dutch windmill graph is denoted by $D_n^{(m)}$ and it is the graph obtained by taking *m* copies of the cycle C_n with a vertex in common. The Dutch windmill graph is also called as friendship graph if $n=3$. *i.e.*, friendship graph is the graph obtained by taking *m* copies of the cycle C_3 with a vertex in common. Dutch windmill graph $D_n^{(m)}$ contains $(n-1)m+1$ vertices and *mn* edges as shown in the **Figures 1-3**.

All graphs considered in this paper are finite, connected, loop less and without multiple edges. Let $G=(V,E)$ be a graph with *n* vertices and *m* edges. The degree of a vertex $u \in V(G)$ is denoted by d_u and is the number of vertices that are adjacent to *u*. The edge connecting the vertices *u* and *v* is denoted by *uv*. Using these terminologies, certain topological indices are defined in the following manner.

Topological indices are numerical parameters of a graph which characterize its topology and are usually graph invariants.

The atom-bond connectivity index, *ABC* index was one of the degree-based molecular descripters, which was introduced by Estrada *et al*. [1] in late 1990's. Some upper bounds for the atom-bond connectivity index of

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Figure 1. $D_3^{(4)}$.

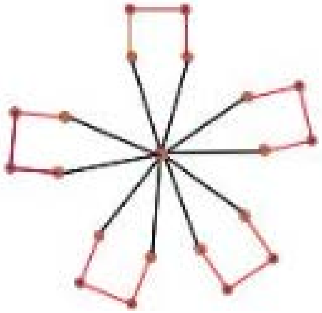

Figure 2. $D_5^{(5)}$.

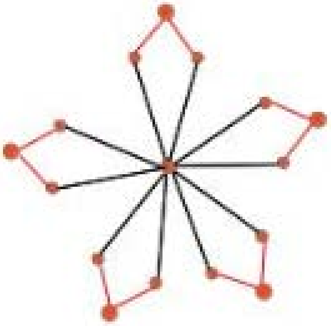

Figure 3. $D_4^{(5)}$.

graphs can be found in [2], The atom-bond connectivity index of chemical bicyclic graphs and connected graphs can be seen in [3] [4]. For further results on *ABC* index of trees, see the papers [5]-[8] and the references cited there in.

Definition 1.1. *Let* $G=(V,E)$ *be a molecular graph and* d_u *is the degree of the vertex u, then ABC index of G is defined as,* $ABC(G)=\sum_{uv\in E}\sqrt{\frac{d_u+d_v-2}{d_u d_v}}$.

The fourth atom bond connectivity index, $ABC_4(G)$ index was introduced by M. Ghorbani *et al*. [9] in 2010. Further studies on $ABC_4(G)$ index can be found in [10] [11].

Definition 1.2. *Let G be a graph, then its fourth ABC index is defined as,* $ABC_4(G)=\sum_{uv\in E(G)}\sqrt{\frac{S_u+S_v-2}{S_u S_v}}$, *where* S_u *is sum of the degrees of all neighbours of vertex u in G. In other words,* $S_u=\sum_{uv\in E(G)} d_v$, *Similarly for* S_v.

The first and oldest degree based topological index was Randic index [12] denoted by $\chi(G)$ and was introduced by Milan Randic in 1975.

Definition 1.3. *For the graph G Randic index is defined as,* $\chi(G)=\sum_{uv\in E(G)}\frac{1}{\sqrt{d_u d_v}}$.

Sum connectivity index belongs to a family of Randic like indices. It was introduced by Zhou and Trinajstic [13]. Further studies on Sum connectivity index can be found in [14] [15].

Definition 1.4. *For a simple connected graph G, its sum connectivity index* $S(G)$ *is defined as,* $S(G)=\sum_{uv\in E(G)}\frac{1}{\sqrt{d_u+d_v}}$.

The Geometric-arithmetic index, $GA(G)$ index of a graph *G* was introduced by D. Vukicevic *et al*. [16]. Further studies on *GA* index can be found in [17]-[19].

Definition 1.5. *Let G be a graph and* $e=uv$ *be an edge of G then,* $GA(G)=\sum_{uv\in E(G)}\frac{2\sqrt{d_u d_v}}{d_u d_v}$.

The fifth Geometric-arithmetic index, $GA_5(G)$ was introduced by A.Graovac *et al.* [20] in 2011.

Definition 1.6. *For a Graph G, the fifth Geometric-arithmetic index is defined as* $GA_5(G)=\sum_{uv\in E(G)}\frac{2\sqrt{S_u S_v}}{S_u+S_v}$, *Where* S_u *is the sum of the degrees of all neighbors of the vertex u in G, similarly for* S_v.

2. Main Results

Theorem 2.1. *The Atom bond connectivity index of Dutch windmill graph is* $ABC\left(D_n^{(m)}\right)=\frac{mn}{\sqrt{2}}$.

Proof. Consider the Dutch windmill graph $D_n^{(m)}$. We partition the edges of $D_n^{(m)}$ into edges of the type $E_{(d_u,d_v)}$ where *uv* is an edge. In $D_n^{(m)}$ we get edges of the type $E_{(2,2)}$ and $E_{(2n,2)}$. Edges of the type $E_{(2,2)}$ and $E_{(2n,2)}$ are colored in red and black respectively as shown in the figure [18]. The number of edges of these types are given in the **Table 1**.

We know that $ABC(G)=\sum_{uv\in E(G)}\sqrt{\frac{d_u+d_v-2}{d_u d_v}}$

i.e., $ABC\left(D_n^{(m)}\right)=\left|E_{(2,2)}\right|\sum_{uv\in E_{(2,2)}(G)}\sqrt{\frac{d_u+d_v-2}{d_u d_v}}+\left|E_{(2m,2)}\right|\sum_{uv\in E_{(2m,2)}(G)}\sqrt{\frac{d_u+d_v-2}{d_u d_v}}$

$$ABC\left(D_n^{(m)}\right)=(n-2)m\sqrt{\frac{2+2-2}{2\cdot 2}}+2m\sqrt{\frac{2m+2-2}{2m\cdot 2}}$$

[From **Table 1** and **Figure 4**]

$$=(n-2)m\frac{1}{\sqrt{2}}+2m\frac{1}{\sqrt{2}}=\frac{mn}{\sqrt{2}}.$$ □

Theorem 2.2. *The Randic Index of Dutch windmill graph is* $\chi\left(D_n^{(m)}\right)=\frac{(n-2)m+2\sqrt{m}}{2}$

Proof. We know that $\chi(G)=\sum_{uv\in E(G)}\frac{1}{\sqrt{d_u d_v}}$

Table 1. Edge partition based on degrees of end vertices of each edge.

Edges of the type $E_{(d_u,d_v)}$	Number of edges
$E_{(2,2)}$	(n − 2)m
$E_{(2m,2)}$	2m

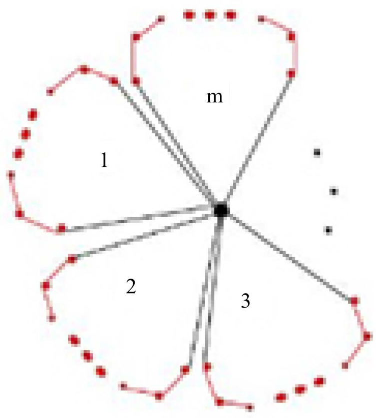

Figure 4. $D_n^{(m)}$.

i.e., $\chi\left(D_n^{(m)}\right)=\left|E_{(2,2)}\right|\sum_{uv\in E_{(2,2)}(G)}\frac{1}{\sqrt{d_u d_v}}+\left|E_{(2m,2)}\right|\sum_{uv\in E_{(2m,2)}(G)}\frac{1}{\sqrt{d_u d_v}}$

$=(n-2)m\frac{1}{\sqrt{2\cdot 2}}+2m\frac{1}{\sqrt{2m\cdot 2}}$ [From **Table 1** and **Figure 4**]

$=\frac{(n-2)m+2\sqrt{m}}{2}$. □

Theorem 2.3. *The Geometric-arithmetic index (GA) of Dutch windmill graph is*

$$GA\left(D_n^{(m)}\right)=\frac{m\left(mn-2m+n-2+4\sqrt{m}\right)}{m+1}.$$

Proof. We know that $GA(G)=\sum_{uv\in E(G)}\frac{2\sqrt{d_u d_v}}{(d_u+d_v)}$

$$GA\left(D_n^{(m)}\right)=\left|E_{(2,2)}\right|\sum_{uv\in E_{(2,2)}(G)}\frac{2\sqrt{d_u d_v}}{(d_u+d_v)}+\left|E_{(2m,2)}\right|\sum_{uv\in E_{(2m,2)}(G)}\frac{2\sqrt{d_u d_v}}{(d_u+d_v)}$$

$=(n-2)m\frac{2\sqrt{2\cdot 2}}{2+2}+2m\frac{2\sqrt{2m\cdot 2}}{2m+2}$ [From **Table 1** and **Figure 4**]

$=\frac{m\left(mn-2m+n-2+4\sqrt{m}\right)}{m+1}$. □

Theorem 2.4. *The Sum connectivity index* $S(G)$ *of Dutch windmill graph is* $S(G)=\frac{(n-2)m}{2}+\frac{m\sqrt{2}}{\sqrt{m+1}}$.

Proof. We know that $S(G)=\sum_{uv\in E(G)}\frac{1}{\sqrt{d_u+d_v}}$

i.e., $S\left(D_n^{(m)}\right)=\left|E_{(2,2)}\right|\sum_{uv\in E_{(2,2)}(G)}\frac{1}{\sqrt{d_u+d_v}}+\left|E_{(2m,2)}\right|\sum_{uv\in E_{(2m,2)}(G)}\frac{1}{\sqrt{d_u+d_v}}$

$=(n-2)m\frac{1}{\sqrt{2+2}}+2m\frac{1}{\sqrt{2m+2}}$ [From **Table 1** and **Figure 4**]

$=\frac{(n-2)m}{2}+\frac{m\sqrt{2}}{\sqrt{m+1}}$. □

Theorem 2.5. *The fourth atom bond connectivity index of Dutch windmill graph is*

$$ABC_4\left(D_m^{(n)}\right)=\begin{cases}\frac{m}{\sqrt{m+1}}\left[(n-4)\sqrt{6(m+1)}+\sqrt{m+2}+\sqrt{3}\right] & \text{if } n\geq 4\\ \frac{m}{\sqrt{m+1}}\left[\sqrt{\frac{2m+1}{2(m+1)}}+\sqrt{3}\right] & \text{if } n=3\end{cases}$$

Proof. Any Dutch windmill graph $D_n^{(m)}$ contains $(n-1)m+1$ vertices and *mn* edges. Let d_u denote the degree of the vertex *u*. We partition the edges of $D_n^{(m)}$ into edges of the type $E^*_{(S_u,S_v)}$ where *uv* is an edge and S_u is the sum of the degrees of all neighbours of vertex u in G. In other words, $S_u=\sum_{uv\in E(G)}d_v$, Similarly for S_v.

Case (1) If $n\geq 4$: In $D_n^{(m)}$ we get edges of the type $E^*_{(4,4)}$, $E^*_{(4,2m+2)}$ and $E^*_{(2m+2,4m)}$. Edges of the type $E^*_{(4,4)}$, $E^*_{(4,2m+2)}$ and $E^*_{(2m+2,4m)}$ are colored in red, green and black respectively as shown in the figure [1]. The number of edges of these types are given in the **Table 2**.

We know that $ABC_4(G)=\sum_{uv\in E(G)}\sqrt{\frac{S_u+S_v-2}{S_uS_v}}$

i.e.,

$$ABC_4\left(D_n^{(m)}\right)=\left|E^*_{(4,4)}\right|\sum_{uv\in E^*_{(4,4)}(G)}\sqrt{\frac{S_u+S_v-2}{S_uS_v}}+\left|E^*_{(4,2m+2)}\right|\sum_{uv\in E^*_{(4,2m+2)}(G)}\sqrt{\frac{S_u+S_v-2}{S_uS_v}}$$
$$+\left|E^*_{(2m+2,4m)}\right|\sum_{uv\in E^*_{(2m+2,4m)}(G)}\sqrt{\frac{S_u+S_v-2}{S_uS_v}}$$

[From **Table 2** and **Figure 5**]

$$=(n-4)m\sqrt{\frac{4+4-2}{4\cdot 4}}+2m\sqrt{\frac{4+2m+2-2}{4(2m+2)}}+2m\sqrt{\frac{2m+2+4m-2}{4m(2m+2)}}$$

$$=\frac{(n-4)m\sqrt{6}}{4}+m\sqrt{\frac{m+2}{m+1}}+m\sqrt{\frac{3}{m+1}}$$

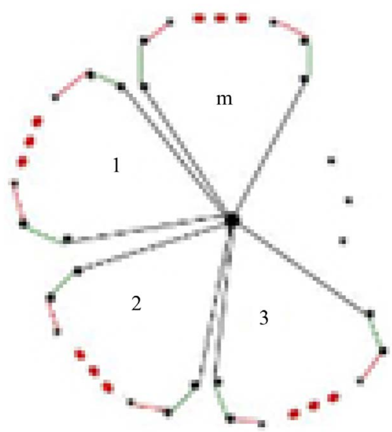

Figure 5. $D_n^{(m)}$.

Table 2. Edge partition based on degree sum of neighbors of end vertices of each edge.

Edges of the type	Number of edges
$E^*_{(4,4)}$	(n − 4)m
$E^*_{(4,2m+2)}$	2m
$E^*_{(2m+2,4m)}$	2m

$$= \frac{m}{\sqrt{m+1}}\left[(n-4)\sqrt{6(m+1)} + \sqrt{m+2} + \sqrt{3}\right].$$

Case (2) If $n = 3$: In $D_3^{(m)}$ we get edges of the type $E^*_{(2m+2,2m+2)}$ and $E^*_{(2m+2,4m)}$. The number of edges of these types are given in the **Table 3**.

We know that $ABC_4(G) = \sum_{uv\in E(G)} \sqrt{\frac{S_u + S_v - 2}{S_u S_v}}$

i.e.,

$$ABC_4\left(D_n^{(m)}\right) = \left|E^*_{(2m+2,2m+2)}\right| \sum_{uv\in E^*_{(2m+2,2m+2)}(G)} \sqrt{\frac{S_u + S_v - 2}{S_u S_v}} + \left|E^*_{(2m+2,4m)}\right| \sum_{uv\in E^*_{(2m+2,4m)}(G)} \sqrt{\frac{S_u + S_v - 2}{S_u S_v}}$$

$$= m\sqrt{\frac{2m+2+2m+2-2}{(2m+2)(2m+2)}} + 2m\sqrt{\frac{2m+2+4m-2}{(2m+2)4m}}$$

$$= m\frac{\sqrt{2(2m+1)}}{2(m+1)} + m\sqrt{\frac{3}{m+1}} = \frac{m}{\sqrt{m+1}}\left[\sqrt{\frac{2m+1}{2(m+1)}} + \sqrt{3}\right].$$ □

Theorem 2.6. *The fifth Geometric-arithmetic index* (GA_5) *of Dutch windmill graph is*

$$GA_5\left(D_n^{(m)}\right) = \begin{cases} (n-4)m + \frac{4m\sqrt{2(m+1)}}{3} + \frac{2m\sqrt{2m(m+1)}}{3m+1} & \text{if } n \geq 4 \\ m\left[1 + \frac{4\sqrt{2m(m+1)}}{3m+1}\right] & \text{if } n = 3 \end{cases}$$

Proof. We know that $GA_5(G) = \sum_{uv\in E(G)} \frac{2\sqrt{S_u S_v}}{(S_u + S_v)}$

Case (1) If $n \geq 4$:

$$GA_5\left(D_n^{(m)}\right) = \left|E^*_{(4,4)}\right| \sum_{uv\in E^*_{(4,4)}(G)} \frac{2\sqrt{S_u S_v}}{(S_u + S_v)} + \left|E^*_{(4,2m+2)}\right| \sum_{uv\in E^*_{(4,2m+2)}(G)} \frac{2\sqrt{S_u S_v}}{(S_u + S_v)} + \left|E^*_{(2m+2,4m)}\right| \sum_{uv\in E^*_{(2m+2,4m)}(G)} \frac{2\sqrt{S_u S_v}}{(S_u + S_v)}$$

[From **Table 2** and **Figure 5**]

$$= (n-4)m\frac{2\sqrt{4\cdot 4}}{4+4} + 2m\frac{2\sqrt{4(2m+2)}}{4+2m+2} + 2m\frac{2\sqrt{(2m+2)4m}}{2m+2+4m}$$

$$= (n-4)m + \frac{4m\sqrt{2(m+1)}}{3} + \frac{2m\sqrt{2m(m+1)}}{3m+1}.$$

Case (2) If $n = 3$:

$$GA_5\left(D_n^{(m)}\right) = \left|E^*_{(2m+2,2m+2)}\right| \sum_{uv\in E^*_{(2m+2,2m+2)}(G)} \frac{2\sqrt{S_u S_v}}{(S_u + S_v)} + \left|E^*_{(2m+2,4m)}\right| \sum_{uv\in E^*_{(2m+2,4m)}(G)} \frac{2\sqrt{S_u S_v}}{(S_u + S_v)}$$

Table 3. Edge partition based on degree sum of neighbors of end vertices of each edge.

Edges of the type	Number of edges
$E^*_{(2m+2,2m+2)}$	m
$E^*_{(2m+2,4m)}$	2m

[From Table 3]

$$= m\frac{2\sqrt{(2m+2)(2m+2)}}{2m+2+2m+2} + 2m\frac{2\sqrt{(2m+2)4m}}{2m+2+4m}$$

$$= m\left[1+\frac{4\sqrt{2m(m+1)}}{3m+1}\right]. \qquad \square$$

3. Conclusion

The problem of finding the general formula for *ABC* index, ABC_4 index, Randic connectivity index, Sum connectivity index, *GA* index and GA_5 index of Dutch Windmill Graph is solved here analytically without using computers.

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Conflict of Interests

The authors declare that there are no conflicts of interests regarding the publication of this paper.

References

[1] Estrada, E., Torres, L., Rodriguez, L. and Gutman, I. (1998) An Atom-Bond Connectivity Index: Modelling the Enthalpy of Formation of Alkanes. *Indian Journal of Chemistry*, **37A**, 849-855.

[2] Chen, J., Liu, J. and Guo, X. (2012) Some Upper Bounds for the Atom-Bond Connectivity Index of Graphs. *Applied Mathematics Letters*, **25**, 1077-1081. http://dx.doi.org/10.1016/j.aml.2012.03.021

[3] Chen, J. and Guo, X. (2012) The Atom-Bond Connectivity Index of Chemical Bicyclic Graphs. *Applied Mathematics—A Journal of Chinese Universities*, **27**, 243-252. http://dx.doi.org/10.1007/s11766-012-2756-4

[4] Xing, R., Zhou, B. and Dong, F. (2011) On Atom-Bond Connectivity Index of Connected Graphs. *Discrete Applied Mathematics*, **159**, 1617-1630. http://dx.doi.org/10.1016/j.dam.2011.06.004

[5] Furtula, B., Gravoc, A. and Vukicevic, D. (2009) Atom-Bond Connectivity Index of Trees. *Discrete Applied Mathematics*, **157**, 2828-2835. http://dx.doi.org/10.1016/j.dam.2009.03.004

[6] Gutman, I., Furtula, B. and Ivanovic, M. (2012) Notes on Trees with Minimal Atom-Bond Connectivity Index. *MATCH Communications in Mathematical and in Computer Chemistry*, **67**, 467-482.

[7] Xing, R., Zhou, B. and Du, Z. (2010) Further Results on Atom-Bond Connectivity Index of Trees. *Discrete Applied Mathematics*, **157**, 1536-1545. http://dx.doi.org/10.1016/j.dam.2010.05.015

[8] Xing, R. and Zhou, B. (2012) Extremal Trees with Fixed Degree Sequence for Atom-Bond Connectivity Index. *FILOMAT*, **26**, 683-688. http://dx.doi.org/10.2298/FIL1204683X

[9] Ghorbani, M. and Hosseinzadeh, M.A. (2010) Computing ABC_4 Index of Nanostar Dendrimers. *Optoelectronics and Advanced Materials*: *Rapid Communications*, **4**, 1419-1422.

[10] Farahani, M.R. (2013) Computing Fourth Atom-Bond Connectivity Index of V-Phenylenic Nanotubes and Nanotori. *Acta Chimica Slovenica*, **60**, 429-432.

[11] Farahani, M.R. (2013) On the Fourth Atom-Bond Connectivity Index of Armchair Polyhex Nanotube. *Proceedings of the Romanian Academy—Series B*, **15**, 3-6.

[12] Randic, M. (1975) On Characterization of Molecular Branching. *Journal of the American Chemical Society*, **97**, 6609-6615. http://dx.doi.org/10.1021/ja00856a001

[13] Zhou, B. and Xing, R. (2011) On Atom-Bond Connectivity Index. *Zeitschrift für Naturforschung*, **66a**, 61-66. http://dx.doi.org/10.5560/ZNA.2011.66a0061

[14] Zhou, B. and Trinajstic, N. (2009) On a Novel Connectivity Index. *Journal of Mathematical Chemistry*, **46**, 1252-1270. http://dx.doi.org/10.1007/s10910-008-9515-z

[15] Zhou, B. and Trinajstic, N. (2010) On General Sum-Connectivity Index. *Journal of Mathematical Chemistry*, **47**, 210-

218. http://dx.doi.org/10.1007/s10910-009-9542-4

[16] Vukicevic, D. and Furtula, B. (2009) Topological Index Based on the Ratios of Geometrical and Arithmetical Means of End-Vertex Degrees of Edges. *Journal of Mathematical Chemistry*, **46**, 1369-1376. http://dx.doi.org/10.1007/s10910-009-9520-x

[17] Chen, S. and Liu, W. (2010) The Geometric-Arithemtic Index of Nanotubes. *Journal of Computational and Theoretical Nanoscience*, **7**, 1993-1995. http://dx.doi.org/10.1166/jctn.2010.1573

[18] Das, K.C. and Trinajstic, N. (2010) Comparision between First Geometric-Arithmetic Index and Atom-Bond Connectivity Index. *Chemical Physics Letters*, **497**, 149-151. http://dx.doi.org/10.1016/j.cplett.2010.07.097

[19] Xiao, L., Chen, S., Guo, Z. and Chen, Q. (2010) The Geometric-Arithmetic Index of Benzenoidsystems and Phenylenes. *International Journal of Contemporary Mathematical Sciences*, **5**, 2225-2230.

[20] Graovac, A.. Ghorbani, M. and Hosseinzadeh, M.A. (2011) Computing Fifth Geometric-Arithmetic Index for Nanostar Dendrimers. *Journal of Mahematical Nanoscience*, **1**, 33-42.