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New Bounds for Zagreb Eccentricity Indices

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ABSTRACT

The Zagreb eccentricity indices are the eccentricity version of the classical Zagreb indices. The first Zagreb eccentricity index ($E_1(G)$) is defined as sum of squares of the eccentricities of the vertices and the second Zagreb eccentricity index ($E_2(G)$) is equal to sum of product of the eccentricities of the adjacent vertices. In this paper we give some new upper and lower bounds for first and second Zagreb eccentricity indices.

Keywords: Vertex Degree; Eccentricity; Zagreb Eccentricity Indices

1. Introduction

Let G be a simple connected graph with vertex set $V(G)$ and edge set $E(G)$ so that $|V(G)| = n$ and $|E(G)| = m$. For any vertex $v \in V(G)$, let deg(v) denote the degree of the vertex v. For vertices u and $v \in V(G)$, the distance between u and v is defined as length of the shortest path connecting u and v and is denoted by $d(u,v)$. The eccentricity of a vertex $v \in V(G)$, denoted by $\varepsilon(v)$, is the distance between v and a vertex farthest from v *i.e.* $\varepsilon(v) = \max\{d(v,x) : x \in V(G)\}$. The radius $r = r(G)$ and diameter $d = d(G)$ of a graph is the minimum and maximum eccentricity among the vertices of G *i.e.*

$$r = r(G) = \min\{\varepsilon(v) : v \in V(G)\}$$

and

$$d = d(G) = \max\{\varepsilon(v) : v \in V(G)\}$$

respectively. Also the total eccentricity of a graph, denoted by $\theta(G)$, is the sum of all the eccentricities of G [1] *i.e.*

$$\theta(G) = \sum_{v \in V(G)} \varepsilon(v).$$

The first and second Zagreb index of a graph were first introduced by Gutman in [2] which are the most known and widely used topological indices, defined as respectively

$$M_1(G) = \sum_{v \in V(G)} d(v)^2,$$

$$M_2(G) = \sum_{(u,v) \in E(G)} d(u)d(v)$$

Recently several Graph invariants based on vertex eccentricities subject to large number of studies. Analogues to Zagreb indices M. Ghorbani *et al*. [3] and D. Vukičević *et al*. [4] defined the Zagreb eccentricity indices by replacing degrees by eccentricity of the vertices. Thus the first and second Zagreb eccentricity indices of a graph G are defined as

$$E_1(G) = \sum_{v \in V(G)} \varepsilon(v)^2,$$

$$E_2(G) = \sum_{(u,v) \in E(G)} \varepsilon(u)\varepsilon(v)$$

The lower and upper bounds of n-vertex trees with fixed diameter and matching number and extremal trees with respect to Zagreb eccentricity indices were studied by R. Xing *et al*. [5] and recently K. C. Das *et al*. in [6] presented some properties, upper and lower bounds of Zagreb eccentricity indices and also characterize the extremal graphs.

Another useful eccentricity and degree based topological index called eccentric connectivity index was first introduced by Sharma, Goswami and Madan [7] and is defined as

$$\xi^c(G) = \sum_{v \in V(G)} \deg(v)\varepsilon(v) = \sum_{(u,v) \in E(G)} [\varepsilon(u) + \varepsilon(v)]$$

There was a vast research regarding various properties of this topological index [8-10].

The study of determining extremal properties such as upper bounds and lower bounds of some graph invariants were subject to a large number of investigations [11-15]. The aim of this paper is to study similar extremal properties for Zagreb eccentricity indices. In this paper we present some new upper and lower bounds of Zagreb eccentricity indices in terms of number of vertices (n), number of edges (m), radius (r), diameter (d), total eccentricity

$\left(\theta\left(G\right)\right)$, the first Zagreb indices ($M_1(G)$), the second Zagreb indices ($M_2(G)$) and the eccentric eccentricity index $\xi^c\left(G\right)$.

2. Bounds for the First Zagreb Eccentricity Index

We now give some lower and upper bounds of first Zagreb eccentricity index. In [6] Das *et al*. proved the following upper bound of $E_1(G)$.

Theorem 2.1. *Let G be a simple connected graph with n vertices and m edges, then*

$$E_1\left(G\right) \le M_1\left(G\right) - 4mn + n^3$$

with equality if and only if $G \cong P_4$ *or* $G \cong K_n$ *or G is isomorphic to a* $(n-1, n-2)$*-semiregular graph.*

Also, in [5] Xing *et al*. give the following result.

Theorem 2.2. *Let G be a simple connected graph with n vertices and m edges, then*

$$nr^2 \le E_1\left(G\right) \le nd^2$$

with equality if and only if G all the vertices of G are of same eccentricity.

Also in [6] Das *et al.* give lower bounds for $E_1(G)$ in terms of n and d. Now we prove some new upper and lower bounds of $E_1(G)$.

Theorem 2.3. *Let G be a simple connected graph with n vertices and m edges, then*

$$E_1\left(G\right) \ge \frac{\theta\left(G\right)^2}{n}$$

with equality if and only if G all the vertices of G are of same eccentricity.

Proof Using the Cauchy-Schwartz inequality, we get

$$\left[\varepsilon\left(v_1\right)^2 + \varepsilon\left(v_2\right)^2 + \cdots + \varepsilon\left(e_n\right)^2\right]\left[1^2 + 1^2 + \cdots + 1^2\right]$$
$$\ge \left[\varepsilon\left(v_1\right)\cdot 1 + \varepsilon\left(v_2\right)\cdot 1 + \cdots + \varepsilon\left(e_n\right)\cdot 1\right]^2$$

and hence using the definition of total eccentricity index and first Zagreb eccentricity index the desired result follows. Clearly in the above inequality equality holds when all the vertices of G are of same eccentricity.

Theorem 2.4. *Let G be a simple connected*, *then*

$$E_1\left(G\right) \ge \frac{M_1\left(G\right)}{\left(n-1\right)^2}$$

with equality if and only if $G \cong K_n$.

Proof We have, for all $v \in V\left(G\right)$, $\varepsilon\left(v\right) \ge \frac{D\left(v\right)}{n-1}$, with equality if an only if $G \cong K_n$, where $D\left(v\right)$ is the degree distance of the vertex v and is defined as

$$D\left(v\right) = \sum\nolimits_{u \in V(G)} d\left(v, u\right).$$

Hence from the definition of first Zagreb eccentricity index, we can write

$$E_1\left(G\right) \ge \frac{1}{\left(n-1\right)^2} \sum_{v \in V(G)} D\left(v\right)^2 \qquad (2.1)$$

Now since $D\left(v\right) \ge \deg\left(v\right)$ for all $v \in V\left(G\right)$, so from (2.1) we get the desired result with equality if and only if $\deg\left(v\right) = \left(n-1\right)$ and $\varepsilon\left(v\right) = 1$, that is $G \cong K_n$.

Corollary 2.1. *Let G be a simple connected graph with n vertices and m edges*, *then*

$$E_1\left(G\right) \ge \frac{M_1\left(G\right)}{\left(n-1\right)^2} - \frac{8m}{n-1} + 4n$$

with equality if and only if G is a path of length one.

Proof Again since, $D\left(v\right) \ge 2n - 2 - \deg\left(v\right)$, for all $v \in V\left(G\right)$, with equality if and only if $G \cong K_{1,n-1}$, so from (2.1) we get the desired result. Since the equality (2.1) holds if and only if $G \cong K_n$ so the equality holds in this result if and only if G is a path of length one which is the only complete graph as well as complete bipartite graph.

Theorem 2.5. *Let G be a simple connected graph on n vertices and* n_0 *be the number of vertices with eccentricity one in G*, *then*

$$E_1\left(G\right) \ge \left(4n - 3n_0\right)$$

with equality if and only if $G \cong K_n - \frac{n-n_0}{2}$, *where* $\left(n - n_0\right)$ *is even.*

Proof Since n_0 be the number of vertices with eccentricity one in G, so the remaining $\left(n - n_0\right)$ vertices are of eccentricity at least two. Let $S = \left\{v_i : 1 \le i \le n_0\right\}$ be the set of vertices such that $\varepsilon\left(v_i\right) = 1$ for $i = 1, 2, \cdots, n_0$. Then from the definition of first Zagreb eccentricity index, we have

$$E_1\left(G\right) = n_0 + \left(n - n_0\right) \sum_{u \in V(G) \setminus S} \varepsilon\left(u\right)^2 \ge n_0 + 4\left(n - n_0\right)$$

from where the desired result follows. Clearly, in this theorem equality holds if and only if $G \cong K_n - \frac{n-n_0}{2}$, where $\left(n - n_0\right)$ is even.

Theorem 2.6. *Let G be a simple connected graph with n vertices and m edges*, *then*

$$E_1\left(G\right) \ge \left(r + d\right)\theta\left(G\right) - nrd$$

with equality if and only if all the vertices of G are of same eccentricity.

Proof To prove this theorem, using the following Diaz-Metcalf inequality, we have, if a_i and b_i, $i = 1, 2, \cdots, n$ are real numbers such that $ha_i \le b_i \le Ha_i$ for $i = 1, 2, \cdots, n$, then

$$\sum_{i=1}^{n} b_i^2 + hH\sum_{i=1}^{n} a_i^2 \le (h+H)\sum_{i=1}^{n} a_i b_i \tag{2.2}$$

In the above inequality equality holds if and only if $b_i = ha_i$ or $b_i = Ha_i$ for every $i = 1,2,\cdots,n$. By setting $b_i = \varepsilon(v_i)$ and $a_i = 1$, for $i = 1,2,\cdots,n$, in (2.2) from above inequality we get

$$\sum_{i=1}^{n} \varepsilon(v_i)^2 + hH\sum_{i=1}^{n} 1^2 \le (h+H)\sum_{i=1}^{n} \varepsilon(v_i)$$

Now using the definition of first Zagreb eccentricity index and total eccentricity index we get

$$E_1(G) \le (h+H)\theta(G) - hHn$$

Since, $r \le \varepsilon(v_i) \le d$ for $i = 1,2,\cdots,n$, so we have $h = r$ and $H = d$. Hence the desired result follows from above. Clearly in the above inequality equality holds if and only if all the vertices of *G* are of same eccentricity.

Theorem 2.7. *Let G be a simple connected graph, then*

$$E_1(G) \le (n+r-1)\theta(G) - n(n-1)r$$

In the above inequality equality holds if and only if all the vertices of G are of same eccentricity.

Proof Let, sum of eccentricities of the vertices adjacent to

$$v_i = \varepsilon(v_i)m_i = \sum_{(v_i,v_j)\in E(G)} \varepsilon(v_j)$$
$$\le \theta(G) - \varepsilon(v_i) - (n-1-\varepsilon(v_i))r$$

so that

$$E_1(G) = \sum_{v_i\in V(G)} \varepsilon(v_i)^2 = \sum_{i=1}^{n} \varepsilon(v_i)m_i$$
$$\le n\theta(G) - \theta(G) - n(n-1)r + r\theta(G)$$

from where we get the desired result. Obviously in the above inequality equality holds if and only if all the vertices of *G* are of same eccentricity.

Theorem 2.8. *Let G be a simple connected graph on n vertices and m edges, then*

$$E_1(G) \ge n\theta(G)\left[\frac{\theta(G) - r(n-1)}{\theta(G) - n(r-1)}\right]$$

In the above inequality equality holds if and only if all the vertices of G are of same eccentricity.

Proof We will prove this theorem using the following Chebyschev's inequality: Let a_i and b_i are real numbers, then

$$n\sum_{i=1}^{n} a_i b_i \ge \sum_{i=1}^{n} a_i \sum_{i=1}^{n} b_i$$

with equality holds if and only if

$$a_1 = a_2 = \cdots = a_n \text{ or } b_1 = b_2 = \cdots = b_n.$$

Now setting $a_i = \varepsilon(v_i)$ and $b_i = \varepsilon(v_i)m_i$, for $i = 1,2,\cdots,n$, we get from (2.3)

$$n\sum_{i=1}^{n} \varepsilon(v_i)\varepsilon(v_i)m_i \ge \sum_{i=1}^{n} \varepsilon(v_i)\sum_{i=1}^{n} \varepsilon(v_i)m_i = \theta(G)E_1(G) \tag{2.4}$$

Again since

$$\sum_{i=1}^{n} \varepsilon(v_i)m_i \le \theta(G) - n\varepsilon(v_i) - (n-1-\varepsilon(v_i))r$$

with equality holding if and only if

$$\varepsilon(v_1) = \varepsilon(v_2) = \cdots = \varepsilon(v_n),$$

we get from (2.4)

$$\theta(G)E_1(G) \le n\theta(G)^2 - nr(n-1)\theta(G) + n(r-1)E_1(G)$$

from where we get the desired result. Clearly in this inequality equality holds if and only if all the vertices are of same eccentricity.

Theorem 2.9. *Let G be a simple connected graph where all the vertices must not be of equal eccentricity, then*

$$E_1(G) \le (d+r)\theta(G) - ndr - (n-a-b)(d-r-1)$$

where G consists of a number of vertices with eccentricity d and b number of vertices with eccentricity r. In the above inequality equality holds if and only if eccentricities of the vertices are equal to r + 1, r, d – 1, d.

Proof For any vertex $v_i \in V(G)$, we have

$$(d-r-1) \le (d-\varepsilon(v_i))(\varepsilon(v_i)-r) = -\varepsilon(v_i)^2 + (d+r)\varepsilon(v_i) - rd$$

with equality holds if and only if $\varepsilon(v_i) = r+1$ or $\varepsilon(v_i) = d-1$ for $i = 1,2,\cdots,n$. Now summing the above inequality for $r < \varepsilon(v_i) < d$, we get

$$\sum_{r<\varepsilon(v_i)<d} \varepsilon(v_i)^2 \le -(n-a-b)(d-r-1) - (n-a-b)rd + (d+r)(\theta(G) - ad - br)$$

So,

$$E_1(G) = \sum_{r<\varepsilon(v_i)<d} \varepsilon(v_i)^2 + ad^2 + br^2 \le ad^2 + br^2 - (n-a-b)(d-r-1) - nrd + (a+b)rd + \theta(G)(d+r) - (ad+br)(d+r)$$

from where we get the desired result. In the above inequality equality holds if and only if

$$\varepsilon(v_i) \in \{r, r+1, d-1, d\}.$$

From the above theorem the following result directly follows.

Corollary 2.2. *Let G be a simple connected graph with n vertices and m edges, then*

$$E_1(G) \le (d+r)\theta(G) - ndr - (n-k)(d-r-1)$$

where k is the number of vertices having eccentricity equal to d or r.

3. Bounds for the Second Zagreb Eccentricity Index

In [6] Das *et al*. give lower bounds for $E_2(G)$ in terms of m, d and proved the following upper bound of $E_1(G)$.

Theorem 3.1. *Let G be a simple connected graph with n vertices and m edges, then*

$$E_2(G) \le M_2(G) - nM_1(G) + mn^2$$

with equality if and only if $G \cong P_4$ *or* $G \cong K_n$ *or G is isomorphic to a* $(n-1, n-2)$*-semiregular graph.*

Also, in [5] Xing *et al*. proved the following result.

Theorem 3.2. *Let G be a simple connected graph with n vertices and m edges, then*

$$mr^2 \le E_2(G) \le md^2$$

with equality if and only if G all the vertices of G are of same eccentricity.

Now we prove some new upper and lower bounds of $E_2(G)$.

Theorem 3.3. *Let G be a simple connected graph with n vertices and m edges, then*

$$E_2(G) \ge \frac{m}{n^2}\theta(G)^2$$

with equality if and only if G all the vertices of G are of same eccentricity.

Proof Using the inequality between arithmetic and geometric mean, we get

$$\frac{1}{m}\sum_{(v_i,v_j)\in E(G)} \varepsilon(v_i)\varepsilon(v_j) \ge \left[\prod_{(v_i,v_j)\in E(G)} \varepsilon(v_i)\varepsilon(v_j)\right]^{\frac{1}{m}} = \left[\prod_{i=1}^{m} \varepsilon(v_i)^{d(v_i)}\right]^{\frac{1}{m}}$$

Now let, $P = \prod_{i=1}^{m} \varepsilon(v_i)^{d(v_i)}$, so that taking natural logarithm on both sides and using Jensen's inequality, we get

$$\ln P = \sum_{i=1}^{n} d(v_i)\ln\varepsilon(v_i) \ge \sum_{i=1}^{n} d(v_i)\ln\frac{1}{n}\sum_{i=1}^{n}\varepsilon(v_i) = 2m\ln\frac{\theta(G)}{n}$$

Thus $P \ge \ln\left(\frac{\theta(G)}{n}\right)^{2m}$, so that $E_2(G) \ge \frac{m}{n^2}\theta(G)^2$, which is the desired result. In this inequality equality holds if and only if all the vertices of G are of same eccentricity.

Theorem 3.4. *Let G be a simple connected graph, then*

$$E_2(G) \ge \xi^c(G) - m$$

where, $\xi^c(G)$ *is the eccentric connectivity index of G. In the above inequality equality holds if and only if* $\varepsilon(u)=1$ *or* $\varepsilon(v)=1$ *for all* $(u,v)\in E(G)$.

Proof Since $\varepsilon(v)\ge 1$ for all $v\in V(G)$, we have $\varepsilon(u)\varepsilon(v) \ge \varepsilon(u)+\varepsilon(v)-1$. So from the definition of second Zagreb eccentricity index, we can write

$$E_2(G) = \sum_{(u,v)\in E(G)} \varepsilon(u)\varepsilon(v) \ge \sum_{(u,v)\in E(G)} \left[\varepsilon(u)+\varepsilon(v)\right] - m$$

Now since

$$\xi^c(G) = \sum_{v\in V(G)} \deg(v)\varepsilon(v) = \sum_{(u,v)\in E(G)} \left[\varepsilon(u)+\varepsilon(v)\right],$$

the desired result follows from above. In the above inequality equality holds if and only if $\varepsilon(u)=1$ or $\varepsilon(v)=1$ for all $(u,v)\in E(G)$, for example if and only if $G\cong K_n$ or $G\cong K_{1,n-1}$.

Theorem 3.5. *Let G be a simple connected graph with n vertices and m edges, then*

$$E_2(G) \ge \frac{M_2(G)}{(n-1)^2}$$

with equality if and only if $G \cong K_n$.

Proof Since, for all $v\in V(G)$, $\varepsilon(v) \ge \frac{D(v)}{n-1}$, with equality if an only if $G\cong K_n$, like Theorem 2.4, using definition of second Zagreb eccentricity index the desired result follows.

Corollary 3.1. *Let G be a simple connected graph with n vertices and m edges, then*

$$E_2(G) \ge \frac{M_2(G)}{(n-1)^2} - \frac{2M_1(G)}{n-1} + 4m$$

with equality if and only if G is a path of length one.

Proof Since, $D(v) \ge 2n-2-\deg(v)$ for all $v\in V(G)$, with equality if and only if $G\cong K_{1,n-1}$, like Corollary 2.1, from the Theorem 3.5 we have

$$E_2(G) \ge 4m - \frac{2}{(n-1)}\sum_{(u,v)\in E(G)}\left[\deg(u)+\deg(v)\right] - \frac{1}{(n-1)^2}\sum_{(u,v)\in E(G)}\left[\deg(u)\deg(v)\right]$$

from where we get the desired result. Like Corollary 2.1, in this inequality equality holds if and only if G is a path of length one.

Theorem 3.6. *Let G be a simple connected graph, then*

$$E_2(G) \le m(\theta(G) - n + 1) + \frac{1}{2}(r-1)\xi^c(G)$$

In the above inequality equality holds if and only if all the vertices of G are of same eccentricity.

Proof Since,

$$\varepsilon(v_i)m_i \le \theta(G) - \varepsilon(v_i) - (n - 1 - \varepsilon(v_i))r$$

for $i = 1, 2, \cdots, n$, we have

$$E_2(G) = \frac{1}{2}\sum_{i=1}^{n}\deg(v_i)\sum_{(v_i,v_j)\in E(G)}\varepsilon(v_i)$$
$$= \frac{1}{2}\sum_{i=1}^{n}\deg(v_i)\varepsilon(v_i)m_i$$
$$\le \frac{1}{2}\left[2m\theta(G) - \xi^c(G) - 2m(n-1)r + r\xi^c(G)\right]$$

from where we get the desired result. Obviously in the above inequality equality holds if and only if all the vertices of G are of same eccentricity.

Corollary 3.2. *Let G be a simple connected graph with n vertices and m edges, then*

$$E_1(G) \ge m\theta(G) - \frac{1}{2}(r-1)M_1(G) + m(nr - 2n + 1)$$

with equality if and only if $G \cong K_n$ *or G is obtained from* K_n *by removing a perfect matching.*

Proof Since we have [8], $\xi^c(G) \le 2mn - M_1(G)$, with equality if and only if $G \cong K_n - ke$ for $k = 0, 1, 2, \cdots, \lfloor n/2 \rfloor$ or $G \cong P_4$, from the above theorem the desired result follows.

4. Conclusion

In this paper we have established some sharp upper and lower bounds of the Zagreb eccentricity indices in terms of some graph parameters such as order, size, radius, diameter, eccentric connectivity index, total eccentricity, first and second Zagreb indices. It may be useful to give the bounds for $E_1(G)$ and $E_2(G)$ indices in terms of other graph invariants.

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