Open Journal of Discrete Mathematics, 2012, 2, 173-178
http://dx.doi.org/10.4236/ojdm.2012.24035 Published Online October 2012 (http://www.SciRP.org/journal/ojdm)

Existence of Nonoscillatory Solutions of a Class of Nonlinear Dynamic Equations with a Forced Term

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Received June 26, 2012; revised July 19, 2012; accepted July 30, 2012

ABSTRACT

In this paper, we consider the following forced higher-order nonlinear neutral dynamic equation
$\left[x(t)+p(t)x(\tau(t))\right]^{\Delta^m}+f\left(t,x(\tau_1(t)),x(\tau_2(t)),\cdots,x(\tau_k(t))\right)=q(t),t\in[t_0,\infty)_{\mathbb{T}}$
on time scales. By using Banach contraction principle, we obtain sufficient conditions for the existence of nonoscillatory solutions for general $p(t)$ and $q(t)$ which means that we allow oscillatory $p(t)$ and $q(t)$. We give some examples to illustrate the obtained results.

Keywords: Dynamic Equation; Higher Order; Non-Oscillation; Time Scale; Neutral

1. Introduction

The study of dynamic equations on time scales, which has recently received a lot of attention, was introduced by Stefan Hilger in his Ph.D. thesis in 1988 in order to unify continuous and discrete analysis [1]. Dynamic equations on time scales have an enormous potential for modelling a variety of applications such as in population dynamics. Several authors have expounded on various aspects of this new theory, see the survey paper by Agarwal, Bohner, O'Regan and Peterson [2] and references cited therein. A book on the subject of time scales, by Bohner and Peterson [3], summarizes and organizes much of the time scale calculus. We refer also to the last book by Bohner and Peterson [4] for advances in dynamic equations on time scales.

Recently, much attention is concerned with oscillation and nonoscillatory solutions for dynamic equations on time scales [5-12].

In Li and Zhang [6] studied the existence of nonoscillatory solutions to neutral dynamic equation

$$\left[x(t)+p(t)x(\tau(t))\right]^{\Delta^n}+f_1\left(t,x(\tau_1(t))\right)-f_2\left(t,x(\tau_2(t))\right)=0.$$

Li, Han, Sun and Yang [10] established the existence of nonoscillatory solutions to the following second order neutral delay dynamic equation

$$\left[x(t)+p(t)x(\tau_0(t))\right]^{\Delta\Delta}+q_1(t)x(\tau_1(t))-q_2(t)x(\tau_2(t))=\mathrm{e}(t).$$

Zhang and Sun [13] studied the existence of nonoscillatory solutions of the forced nonlinear difference equation

$$\Delta\left(x_n-p_nx_{\tau(n)}\right)+f\left(n,x_{\sigma(n)}\right)=q_n.$$

Zhou and Zhang [14] obtained some sufficient conditions of nonoscillatory solutions for the higher order delay difference equation with positive and negative coefficients

$$\Delta^m\left(x_n+cx_{n-k}\right)+p_nx_{n-r}-q_nx_{n-l}=0.$$

Lu [15] obtained some necessary and sufficient conditions for the existence of nonoscillatory solutions for the following first order neutral equation

$$\left(x(t)-\sum_{i=1}^{m}p_i(t)x(h_i(t))\right)'+\sum_{j=1}^{n}f_j\left(t,x(g_j(t))\right)=Q(t).$$

Motivated by these works, in this paper, we consider the higher-order nonlinear neutral dynamic equation

$$\left[x(t)+p(t)x(\tau(t))\right]^{\Delta^m}+f\left(t,x(\tau_1(t)),\ x(\tau_2(t)),\cdots,\ x(\tau_k(t))\right)=q(t), \tag{1}$$

where $t\in[t_0,\infty)_{\mathbb{T}}$, $m\in\mathbb{N}$, $\sup\mathbb{T}=\infty$. We assume $p,q\in C_{rd}\left([t_0,\infty)_{\mathbb{T}},\mathbb{R}\right)$ and allow $p(t)$ and $q(t)$ to be oscillatory. $\tau,\tau_i\in C_{rd}\left([t_0,\infty)_{\mathbb{T}},\mathbb{T}\right)$ satisfy $\lim_{t\to\infty}\tau(t)=\lim_{t\to\infty}\tau_i(t)=+\infty, i=1,2,\cdots,k$,. $f(t,u_1,u_2,\cdots u_k)\in C\left(\mathbb{T}\times\mathbb{R}^k,\mathbb{R}\right)$ is nondecreasing for

u_j and $u_1 f(t,u_1,u_2,\cdots u_k) \ge 0$ for

$$u_1 u_j \ge 0,\ j = 1,2,\cdots,k.$$

We recall x is a solution of Equation (1) provided that $x(t)+p(t)x(\tau(t))$ is m times differentiable, and x satisfies Equation (1), A solution x of Equation (1) is called nonoscillatory if x is of one sign when eventually.

2. Existence Results for Nonoscillatory Solutions

In this section, we establish sufficient conditions of the existence of nonoscillatory solutions for Equation (1). First we define a sequence of functions $g_k(s,t)$, $k \in \mathbb{N}_0$ as follows:

$$g_0(s,t) \equiv 1, g_{k+1}(s,t) = \int_t^s g_k(\sigma(\tau),t)\Delta\tau.$$

For $g_k(s,t)$, we have the following Lemma.

Lemma 2.1. (Li and Zhang [6]) Assume s is fixed, and let $g_k^\Delta(s,t)$ be the derivative $g_k(s,t)$ with respect to t. Then

$$g_k^\Delta(s,t) = -g_{k-1}(s,t), k \in \mathbb{N}, t \in \mathbb{T}^k.$$

Let BC denote the Banach space of all bounded functions $x(t), t \ge t_0$, with the norm $\|x\| = \sup_{t\ge t_0}, |x(t)| < \infty$. We will use the following assumptions:

(i) there exists $\alpha > 0$ such that

$$|f(t,u_1,u_2,\cdots u_k) - f(t,v_1,v_2,\cdots v_k)| \le L(t)\max_{1\le i\le k}|u_i - v_i|$$

for $t \ge t_0$ and $0 \le u_i, v_i \le \alpha, j = 1,2,\cdots,k$, where $L(t) \in C_{rd}(\mathbb{T},\mathbb{T})$;

(ii) $\int_{t_0}^\infty g_{m-1}(\sigma(s),0)L(s)\Delta s < \infty$;

(iii) $\int_{t_0}^\infty g_{m-1}(\sigma(s),0)|q(s)|\Delta s < \infty$;

(iv) there exists $p \in \left(\frac{1}{2},1\right)$ such that

$$|p(t)| \le 1-p, t \ge t_0;$$

(v) there exists $p \in (-1,0]$ such that

$$p \le p(t) \le 0, t \ge t_0;$$

(vi) there exist $p_1, p_2 \in (-\infty,-1)$ such that

$$p_1 \le p(t) \le p_2, t \ge t_0;$$

(vii) there exists $p \in (0,1)$ such that

$$0 < p(t) \le p, t \ge t_0;$$

(viii) there exist $p_1, p_2 \in (1,+\infty)$ such that

$$p_1 \le p(t) \le p_2, t \ge t_0.$$

Theorem 2.1. Assume that (i), (ii), (iii) and (iv) hold, then Equation (1) has a bounded nonoscillatory solution which is bounded away from zero.

Proof. Choose d_1, c_1 such that $0 < d_1 < (2p-1)\alpha$ and $d_1 + (1-p)\alpha < c_1 < p\alpha$. Let $c = \min\left\{\frac{p}{2}\alpha, p\alpha - c_1, c_1 - d_1 - (1-p)\alpha\right\}$. There exists a $t_1 \ge t_0$ large enough such that when $t \ge t_1$, we have $\tau(t), \tau_i(t) \ge t_0, i = 1,2,\cdots,k$ and

$$\int_{t_1}^\infty g_{m-1}(\sigma(s),0)[\alpha L(s) + |q(s)|]\Delta s \le c. \quad (2)$$

By condition (i) and the hypotheses on $f(t,u_1,\cdots,u_k)$, for any $t \ge t_0, 0 \le u_i \le \alpha, i = 1,2,\cdots,k$, we have

$$f(t,u_1,\cdots,u_k) \le \alpha L(t). \quad (3)$$

We define a set $\Omega \subset BC$ as follows:

$$\Omega = \{x \in BC : d_1 \le x(t) \le \alpha, t \ge t_0\}. \quad (4)$$

Then Ω is a closed, bounded and convex subset of BC. Define a map Γ on Ω as follows:

$$(\Gamma x)(t) = \begin{cases} c_1 - p(t)x(\tau(t)) + (-1)^{m-1}\int_t^\infty g_{m-1}(\sigma(s),t) \\ \cdot\left[f(s,x(\tau_1(s)),(\tau_2(s)),\cdots,(\tau_k(s))) - q(s)\right]\Delta s, \\ t \ge t_1, \\ (\Gamma x)(t_1), t_0 \le t \le t_1. \end{cases}$$

First, we shall show that for any $x \in \Omega$ and $t \ge t_0$, $(\Gamma x)(t) \in \Omega$. For any $x \in \Omega$ and $t \ge t_1$, by (2), (3) and (4), we get

$$\begin{aligned}(\Gamma x)(t) &\ge c_1 - p(t)x(\tau(t)) - \int_t^\infty g_{m-1}(\sigma(s),t) \\ &\quad [f(s,x(\tau_1(s)),\ x(\tau_2(s)),\cdots,\ x(\tau_k(s))) + |q(s)|]\Delta s \\ &\ge c_1 - |p(t)|x(\tau(t)) \\ &\quad -\int_t^\infty g_{m-1}(\sigma(s),0)[\alpha L(s) + |q(s)|]\Delta s \\ &\ge c_1 - (1-p)\alpha - [c_1 - d_1 - (1-p)\alpha] = d_1.\end{aligned}$$

Furthermore, we have

$$\begin{aligned}(\Gamma x)(t) &\le c_1 - |p(t)|x(\tau(t)) - \int_t^\infty g_{m-1}(\sigma(s),t) \\ &\quad [f(s,x(\tau_1(s)),x(\tau_2(s)),\cdots,x(\tau_k(s))) + |q(s)|]\Delta s \\ &\le c_1 + (1-p)\alpha \\ &\quad +\int_t^\infty g_{m-1}(\sigma(s),0)[\alpha L(s) + |q(s)|]\Delta s \\ &\le c_1 + (1-p)\alpha + p\alpha - c_1 = \alpha.\end{aligned}$$

Hence when $t \ge t_0$, we obtain $d_1 \le (\Gamma x)(t) \le \alpha$, so $(\Gamma x)(t) \in \Omega$ for any $x \in \Omega$.

Next, we show that Γ is a contraction mapping on Ω. In fact for any $x, y \in \Omega$ and $t \ge t_1$, we have

$$
\begin{aligned}
&(\Gamma x)(t) \\
&\le |p(t)||x(\tau(t)) - y(\tau(t))| \\
&+\int_t^\infty g_{m-1}(\sigma(s),t)\left|f(s,x(\tau_1(s)),x(\tau_2(s)),\cdots,x(\tau_k(s))) \right.\\
&\left. -f(s,y(\tau_1(s)),\ y(\tau_2(s)),\cdots,y(\tau_k(s)))\right|\Delta s \\
&\le (1-p)\|x-y\| + \int_t^\infty g_{m-1}(\sigma(s),0)L(s)\|x-y\|\Delta s \\
&\le \left(1-p+\frac{p}{2}\right)\|x-y\| = \left(1-\frac{p}{2}\right)\|x-y\|.
\end{aligned}
$$

Since $0 < 1-\frac{p}{2} < 1$, we conclude that Γ is a contraction mapping on Ω. By the Banach fixed point theorem, Γ has a fixed point $x^* \in \Omega$. By Lemma 2.1, it is easy to see that $x^*(t)$ is a bounded nonoscillatory solution of the Equation (1). This completes the proof of Theorem 2.1.

Theorem 2.2. Assume that (i), (ii), (iii) and (v) hold, then Equation (1) has a bounded nonoscillatory solution which is bounded away from zero.

Proof. Choose $\beta > 0$, such that $\beta \le \frac{2(1+p)\alpha}{3}$. Obviously $(1+p)\alpha - \beta \ge \frac{\beta}{2}$. There exists a $t_1 \ge t_0$ sufficiently large such that when $t \ge t_1$, we have $\tau(t)$, $\tau_i(t) \ge t_0, i = 1,2,\cdots,k$ and

$$\int_{t_1}^\infty g_{m-1}(\sigma(s),0)\left[\alpha L(s) + |q(s)|\right]\Delta s \le \frac{\beta}{2}.$$

We define a closed, bounded and convex subset Ω of BC as follows:

$$\Omega = \left\{x \in BC : \frac{\beta}{2} \le x(t) \le \alpha, t \ge t_0\right\}.$$

Define a map Γ on Ω as follows:

$$
(\Gamma x)(t) = \begin{cases}
\beta - p(t)x(\tau(t)) + (-1)^{m-1}\int_t^\infty g_{m-1}(\sigma(s),t) \\
\cdot\left[f(s,x(\tau_1(s)),\ x(\tau_2(s)),\cdots,\ x(\tau_k(s))) - q(s)\right]\Delta s, \\
t \ge t_1, \\
(\Gamma x)(t_1), t_0 \le t \le t_1.
\end{cases}
$$

The rest of the proof is similar to that of Theorem 2.1 and hence omitted. The proof is complete.

Theorem 2.3. Assume that (i), (ii), (iii) and (vi) hold. τ has the inverse $\tau^{-1} \in C(\mathbb{T},\mathbb{T})$, then Equation (1) has a bounded nonoscillatory solution which is bounded away from zero.

Proof. We choose positive constants M_1, M_2, β, such that $M_2 \le \alpha$, $-p_1M_1 < \beta < (-p_2-1)M_2$. Let

$$c = \min\left\{\frac{\beta + p_1M_1}{p_1}p_2, (-p_2-1)M_2 - \beta, -\frac{1+p_2}{2}\alpha\right\}.$$

There exists a $t_1 \ge t_0$ large enough such that when $t \ge t_1$, we have $\tau^{-1}(\tau_i(t)) \ge t_0, i = 1,2,\cdots,k$, and

$$\int_{t^{-1}(t)}^\infty g_{m-1}(\sigma(s),0)\left[\alpha L(s) + |q(s)|\right]\Delta s \le c.$$

We define a closed, bounded and convex subset Ω of BC as follows:

$$\Omega = \{x \in BC : M_1 \le x(t) \le M_2, t \ge t_0\}.$$

Define a map $\Gamma : \Omega \to BC$ as follows:

$$
(\Gamma x)(t) = \begin{cases}
-\frac{\beta}{p(\tau^{-1}(t))} - \frac{x(\tau^{-1}(t))}{p(\tau^{-1}(t))} + \frac{(-1)^{m-1}}{p(\tau^{-1}(t))}\int_{t^{-1}(t)}^\infty g_{m-1}(\sigma(s),t) \\
\cdot\left[f(s,x(\tau_1(s)),\ x(\tau_2(s)),\cdots,\ x(\tau_k(s))) - q(s)\right]\Delta s, \\
t \ge t_1, \\
(\Gamma x)(t), t_0 \le t \le t_1.
\end{cases}
$$

First, we shall show that $\Gamma\Omega \subset \Omega$. For any $x \in \Omega$ and $t \ge t_1$, note that

$$
\begin{aligned}
&(\Gamma x)(t) \\
&\ge -\frac{\beta}{p(\tau^{-1}(t))} \\
&+\frac{1}{p(\tau^{-1}(t))}\int_{t^{-1}(t)}^\infty g_{m-1}(\sigma(s),t)\left[\alpha L(s) + |q(s)|\right]\Delta s \\
&\ge -\frac{\beta}{p_1} + \frac{1}{p_2}\int_{\tau^{-1}(t)}^\infty g_{m-1}(\sigma(s),0)\left[\alpha L(s) + |q(s)|\right]\Delta s \\
&\ge -\frac{\beta}{p_1} + \frac{(\beta + p_1M_1)p_2}{p_1p_2} = M_1
\end{aligned}
$$

and

$$(\Gamma x)(t) \le -\frac{\beta}{p_2} - \frac{M_2}{p_2} - \frac{(-p_2-1)M_2 - \beta}{p_2} = M_2.$$

Thus $(\Gamma x)(t) \in \Omega$ for $x \in \Omega$, this is $\Gamma\Omega \subset \Omega$.

Next, we show that Γ is a contraction mapping on Ω. In fact for any $x, y \in \Omega$ and $t \ge t_1$, we have

$$(\Gamma x)(t) \le -\frac{1}{p(\tau^{-1}(t))}\left|x(\tau^{-1}(t)) - y(\tau^{-1}(t))\right|$$
$$-\frac{1}{p(\tau^{-1}(t))}\int_{\tau^{-1}(t)}^{\infty} g_{m-1}(\sigma(s),t)\Big|f(s,x(\tau_1(s)),x(\tau_2(s)),$$
$$\cdots,x(\tau_k(s))) - f(s,y(\tau_1(s)),y(\tau_2(s)),\cdots,y(\tau_k(s)))\Big|\Delta s$$
$$\le -\frac{1}{p_2}\|x-y\| - \frac{1}{p_2}\int_{\tau^{-1}(t)}^{\infty} g_{m-1}(\sigma(s),0)L(s)\|x-y\|\Delta s$$
$$\le \frac{1}{p_2}\left(-1+\frac{1+p_2}{2}\right)\|x-y\| = \frac{p_2-1}{2p_2}\|x-y\|.$$

Since $0 < \frac{p_2-1}{2p_2} < 1$, we conclude that Γ is a contraction mapping on Ω. By the Banach fixed point theorem, Γ has a fixed point $x^* \in \Omega$. By Lemma 2.1, it is easy to see that $x^*(t)$ is a bounded nonoscillatory solution of the Equation (1). This completes the proof of Theorem 2.3.

Theorem 2.4. Assume that (i), (ii), (iii) and (vii) hold, then equation (1) has a bounded nonoscillatory solution which is bounded away from zero.

Proof. Choose $\beta > 0$, such that $p\alpha < \beta < \alpha$. Let $c = \min\left\{\alpha-\beta, \frac{\beta - p\alpha}{2}\right\}$. There exists a $t_1 \ge t_0$ large enough such that when $t \ge t_1$, we have

$$\tau(t),\tau_i(t) \ge t_0, i = 1,2,\cdots,k,$$

and

$$\int_{t_1}^{\infty} g_{m-1}(\sigma(s),0)\left[\alpha L(s) + |q(s)|\right]\Delta s \le c.$$

Easily to know

$$\Omega = \left\{x \in BC : \frac{\beta - p\alpha}{2} \le x(t) \le \alpha, t \ge t_0\right\}$$

is a closed, bounded and convex subset of BC. Define a map $\Gamma : \Omega \to BC$ as follows:

$$(\Gamma x)(t) = \begin{cases} \beta - p(t)x(\tau(t)) + (-1)^{m-1}\int_t^{\infty} g_{m-1}(\sigma(s),t) \\ \cdot\left[f(s,x(\tau_1(s)),x(\tau_2(s)),\cdots,x(\tau_k(s))) - q(s)\right]\Delta s, \\ t \ge t_1, \\ (\Gamma x)(t_1), t_0 \le t \le t_1. \end{cases}$$

The rest of the proof is similar to that of Theorem 2.1 and hence omitted. The proof is complete.

Theorem 2.5. Assume that $\alpha \ge 1$, (i), (ii), (iii) and (viii) hold. τ has the inverse $\tau^{-1} \in C(\mathbb{T},\mathbb{T})$, then Equation (1) has a bounded nonoscillatory solution which is bounded away from zero.

Proof. We choose β, such that $1 < \beta < p_1$. Let $c = \min\left\{\beta-1, \frac{p_1-\beta}{2}, \frac{\beta-1}{2}\alpha\right\}$. There exists a $t_1 \ge t_0$ large enough such that when $t \ge t_1$, we have $\tau^{-1}(\tau_i(t)) \ge t_0, i = 1,2,\cdots,k$, and

$$\int_{\tau^{-1}(t)}^{\infty} g_{m-1}(\sigma(s),0)\left[\alpha L(s) + |q(s)|\right]\Delta s \le c.$$

We define a closed, bounded and convex subset Ω of BC as follows:

$$\Omega = \left\{x \in BC : \frac{p_1-\beta}{2p_2} \le x(t) \le \frac{p_1+\beta}{2p_1}, t \ge t_0\right\}.$$

Define a map $\Gamma : \Omega \to BC$ as follows:

$$(\Gamma x)(t) = \begin{cases} \frac{\beta}{p(\tau^{-1}(t))} - \frac{x(\tau^{-1}(t))}{p(\tau^{-1}(t))} + \frac{(-1)^{m-1}}{p(\tau^{-1}(t))}\int_{\tau^{-1}(t)}^{\infty} g_{m-1}(\sigma(s),t) \\ \cdot\left[f(s,x(\tau_1(s)),x(\tau_2(s)),\cdots,x(\tau_k(s))) - q(s)\right]\Delta s, \\ t \ge t_1, \\ (\Gamma x)(t_1), t_0 \le t \le t_1. \end{cases}$$

The rest of the proof is similar to that of Theorem 2.3 and hence omitted. The proof is complete.

Remark 2.1. Theorem 1 - 5 not only unify the known results for differential and difference equations corresponding to Equation (1), but also generalize and improve essentially the existing results of [13-15] using the time scale theory.

We will give the following examples to illustrate our mainresults.

Example 2.1. Consider the forth-order dynamic equation on the time scale $\mathbb{T} = \{q^n : n \in \mathbb{N}_0, q > 1\}$

$$\left(x(t) - \frac{1}{\sqrt{q}}x\left(\frac{t}{q}\right)\right)^{\Delta^4} + \frac{(1-\sqrt{q})(q+1)^2(q^2+1)}{q^{10}} \times \frac{(q^2+q+1)}{t^2(t+q^3)^3}x^3\left(\frac{t}{q^3}\right) = 2\frac{(1-\sqrt{q})(q+1)^2(q^2+1)(q^2+q+1)}{q^{10}t^5} \tag{5}$$

Here $m = 4, p(t) = -\frac{1}{\sqrt{q}}, \tau(t) = \frac{t}{q}$,

$$q(t) = 2\frac{(1-\sqrt{q})(q+1)^2(q^2+1)(q^2+q+1)}{q^{10}t^5},$$

$$L(t) = 3\alpha^2\frac{\left|(1-\sqrt{q})(q+1)^2(q^2+1)(q^2+q+1)\right|}{q^{10}t^2(t+q^3)^3}.$$

By the definition of $g_k(s,t)$, we have

$$g_{4-1}(\sigma(s),0)L(s)$$
$$\le s^3\frac{3\alpha^2(\sqrt{q}-1)(q+1)^2(q^2+1)}{q^{10}}\frac{(q^2+q+1)}{s^2(s+q^3)^3}$$
$$\le\frac{3\alpha^2(\sqrt{q}-1)(q+1)^2(q^2+1)(q^2+q+1)}{q^{10}s^2},$$
$$g_{4-1}(\sigma(s),0)|q(s)|$$
$$\le s^3 2\frac{(\sqrt{q}-1)(q+1)^2(q^2+1)(q^2+q+1)}{q^{10}s^5}$$
$$=2\frac{(\sqrt{q}-1)(q+1)^2(q^2+1)(q^2+q+1)}{q^{10}s^2}.$$

Then

$$\int_{t_0}^{\infty}\frac{3\alpha^2(\sqrt{q}-1)(q+1)^2(q^2+1)(q^2+q+1)}{q^{10}s^2}\Delta s<\infty,$$
$$\int_{t_0}^{\infty}2\frac{(\sqrt{q}-1)(q+1)^2(q^2+1)(q^2+q+1)}{q^{10}s^2}\Delta s<\infty.$$

It is obvious that Equation (5) satisfies all conditions of Theorem 2.2. Hence Equation (5) has a bounded nonoscillatory solution which is bounded away from zero. In fact $x(t)=1+\frac{1}{t}$ is a solution of Equation (5). However, to the best of our knowledge, there are no results dealing with the existence of nonoscillatory solutions for Equation (5).

Example 2.2. Consider the third-order dynamic equation on the time scale $\mathbb{T}=\mathbb{N}$

$$\left(x(t)-2x(t-1)\right)^{\Delta^3}+\frac{1}{2^t}x(t-1)=\frac{11(2^t)+16}{8(2^{2t})}, \quad t\ge 2. \qquad (6)$$

Here $m=3, p(t)=-2, \tau(t)=t-1$,

$$f(t,x(\tau_1(t)))=\frac{1}{2^t}x(t-1)$$

and $q(t)=\frac{11(2^t)+16}{8(2^{2t})}$. It is easy to see that all conditions of Theorem 2.3 are satisfied and hence Equation (6) has a bounded nonoscillatory solution which is bounded away from zero. In fact $x(t)=1+\frac{1}{2^t}$ is a solution of Equation (6).

3. Acknowledgements

The authors sincerely thank the reviewers for their valuable suggestions and useful comments that have lead to the present improved version of the original manuscript.

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