Open Journal of Discrete Mathematics, 2012, 2, 109-113
http://dx.doi.org/10.4236/ojdm.2012.23021 Published Online July 2012 (http://www.SciRP.org/journal/ojdm)

A Note on Edge-Domsaturation Number of a Graph

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Received April 27, 2012; revised May 30, 2012; accepted June 12, 2012

ABSTRACT

The edge-domsaturation number $ds'(G)$ of a graph $G = (V, E)$ is the least positive integer k such that every edge of G lies in an edge dominating set of cardinality k. In this paper, we characterize unicyclic graphs G with $ds'(G) = q - \Delta'(G) + 1$ and investigate well-edge dominated graphs. We further define γ'^{-}-critical, γ'^{+}-critical, ds'^{-}-critical, ds'^{+}-critical edges and study some of their properties.

Keywords: Edge-Dominating Set; Edge-Domination Number; ds'-Critical; Edge-Domsaturation Number; Well Edge Dominated Graph

1. Introduction

Throughout this paper, G denotes a graph with order p and size q. By a graph we mean a finite undirected graph without loops or multiple edges. For graph theoretic terms we refer Harary [1] and in particular, for terminology related to domination theory we refer Haynes *et al.* [2].

1.1. Definition

Let $G = (V, E)$ be a graph. A subset D of E is said to be an edge dominating set if every edge in E-D is adjacent to at least one edge in D. An edge dominating set D is said to be a minimal edge dominating set if no proper subset of D is an edge dominating set of G. The edge domination number $\gamma'(G)$ of a graph G equals the minimum cardinality of an edge dominating set of G. An edge dominating set of G with cardinality $\gamma'(G)$ is called a $\gamma'(G)$-set or γ'-set.

Acharya [3] introduced the concept of domsaturation number $ds(G)$ of a graph. For any graph G of order p, and for any integer r such that $\gamma(G) \le r \le p$, we call the set $DC_r(G) = \{u \in V(G) / u \in D \text{ forsome } D \in \mathfrak{A}_r(G)\}$ the r-level domination core of G. We say that G is r-level domination-saturated (or in short, "r-domsaturated") if $DC_r(G) - V(G)$. The domsaturation number $ds(G)$ is then defined by $ds(G) = \min\{r / G \text{isr-domsaturated}\}$. Arumugam and Kala [4] observed that for any graph G, $ds(G) = \gamma(G)$ or $ds(G) = \gamma(G) + 1$ and obtained several results on $ds(G)$. We now extend the concept of domsaturation number of a graph to edges.

1.2. Definition

The least positive integer k such that every edge of G lies in an edge dominating set of cardinality k is called the edge-domsaturation number of G and is denoted by $ds'(G)$.

If G is a graph with edge set E and D is a γ'-set of G, then for any edge $e \in E$-D, $D \cup \{e\}$ is also an edge dominating set and hence $ds'(G) = \gamma'(G)$ or $\gamma'(G) + 1$.

Thus we have the following definition.

1.3. Definition

A graph G is said to be of class 1 or class 2 according as $ds'(G) = \gamma'(G)$ or $\gamma'(G) + 1$.

1.4. Definition

An edge e of G is

1) γ'-critical if $\gamma'(G - e) \ne \gamma'(G)$;
2) γ'^{+}-critical if $\gamma'(G - e) > \gamma'(G)$;
3) γ'^{-}-critical if $\gamma'(G - e) < \gamma'(G)$;
4) γ'-fixed if every γ'-set contains e;
5) γ'-free if there exists γ'-sets containing e and also γ'-sets not containing e;
6) γ'-totally free if there is no γ'-set containing e.

We use the following theorem.

1.5. Theorem [5]

For any connected unicyclic graph $G = (V, E)$ with cycle C, $\gamma'(G) = q - \Delta'(G)$ if and only if one of the following holds.

1) $C = C_3$;
2) $G = C_3 = (u_1, u_2 . u_3, u_1)$, $\deg u_1 \ge 3$, $\deg u_2 = \deg u_3 = 2$, $\deg(u_1, w) \le 2$ for all vertices w not on C and $\deg w \ge 3$ for at most one vertex w not on C;
3) $G = C_3 = (u_1, u_2 . u_3, u_1)$, $\deg u_1 \le 3$, $\deg u_2 \le 3$, $\deg u_3 = 2$

all the vertices not on C adjacent to u_1 have degree at most 2 and all vertices whose distance from u_1 is 2 are pendent vertices;

4) $C = C_3 = (u_1, u_2, u_3, u_1)$, $\deg u_1$=3, $\deg u_2$≤3, $\deg u_3$≤3 and all vertices not on C are pendent vertices;

5) $C = C_4$;

6) $C = C_4$, either exactly one vertex of C has degree at least 3 and all vertices not on C are pendent vertices.

2. Main Results

2.1. Lemma

An edge e of G is γ'^{-}-critical if and only if $\gamma'(G-e) = \gamma'(G) - 1$

Proof

For any edge e, we observe that $\gamma'(G-e) = \gamma'(G) - 1$ or $\gamma'(G)$ or $\gamma'(G)+1$. Now, suppose e is γ'^{-}-critical. Then $\gamma'(G-e) < \gamma'(G)$. Hence $\gamma'(G-e) = \gamma'(G) - 1$. The converse is obvious.

2.2. Theorem

An edge e is γ'^{-}-critical if and only if

$$N(e) \subset \cup_{f \in D-e} N(f) \qquad (1)$$

for some γ'-set D containing e.

Proof

If e is γ'^{-}-critical, $\gamma'(G-e) = \gamma'(G) - 1$ by lemma 2.1. Let S be a γ'-set of $G - e$. If S contains an edge of $N(e)$, then S will be an edge dominating set of G and hence $\gamma'(G) \le \gamma'(G-e)$, a contradiction. Thus S does not contain any edge of $N(e)$. Since $\gamma'(G-e) = \gamma'(G) - 1$, $D = S \cup \{e\}$ is a γ'-set of G and so Equation (1) holds. Conversely, suppose e is an edge such that (1) is true. Then $G - e$ is an edge dominating set of $G - e$ and hence $\gamma'(G-e) = \gamma'(G) - 1$. Thus e is γ'^{-}-critical.

2.3. Theorem

Let G be a graph without isolated edges. An edge e in G is γ'^{-}-critical if and only if

1) e is γ'-free, and

2) no γ'-set of $G - e$ contains any edge of $N(e)$.

Proof

If e is γ'^{-}-critical, then $\gamma'(G-e) = \gamma'(G) - 1$ by Lemma 2.1. As in theorem 2.2, if S is any γ'-set of $G - e$, then S will not contain any edge of $N(e)$ and $S \cup \{f\}$ is a γ'-set of G for every $f \in N[e]$. This implies that e is γ'-free. Conversely, suppose (1) and (2) are true. Let S be a γ'-set of $G - e$. By (2) S does not contain any edge of $N[e]$. Hence S cannot be an edge dominating set of G. But, for any edge $f \in N[e]$, $S \cup \{f\}$ is an edge dominating set of G. Since S is a minimum edge dominating set for $G - e$, $S \cup \{f\}$ is also a minimum edge dominating set for G and hence $\gamma'(G) = |S \cup \{f\}| = \gamma'(G-e) + 1$. Thus e is γ'^{-}-critical.

2.4. Theorem

Let G be a graph and $e \in E(G)$. Then

1) e is γ'-fixed if and only if there exists no edge dominating set of $G - e$ with $\gamma'(G)$ edges which is also an edge dominating set of G.

2) e is γ'-totally free if and only if every γ'-set of G is a γ'-set of $G - e$.

Proof

1) Assume that e is γ'-fixed. Suppose there exists an edge dominating set S of $G - e$ with $|S| = \gamma'(G)$ which is also an edge dominating set of G. Then S is a γ'-set not containing e which is impossible as e is γ'-fixed. The converse is obvious.

2) Let e be γ'-totally free. Then e does not belong to any γ'-set of G and so every γ'-set D of G is an edge dominating set of $G - e$. Thus $\gamma'(G-e) \le \gamma'(G)$. If $\gamma'(G-e) \le \gamma'(G)$, then by theorem 2.3, e is γ'-free and so $\gamma'(G-e) = \gamma'(G)$, D is a γ'-set of $G - e$. The converse is obvious.

2.5. Theorem

Let G be a connected graph. If a cut edge e of G is γ'-fixed, then e is γ'^{+}-critical

Proof

Let S be a γ'-set of G. Let e be a cut edge that is γ'-fixed. Then e belongs to every γ'-set. Since e is a cut edge, $G - e$ is a disconnected graph with at least two components G' and G'' . Let e' and e'' be the neighbors of e in G' and G'' respectively. Therefore $D = (S - e) \cup \{e', e''\}$ is a minimum edge dominating set of $G - e$ so that $\gamma'(G-e) = \gamma'(G) + 1$. Hence e is γ'^{+}-critical.

2.6. Theorem

An edge e in a graph G is γ'^{+}-critical if and only if

1) e is not isolated edge

2) e is γ'-fixed and

3) There is no edge dominating set for $G - N[e]$ having $\gamma'(G)$ edges which also dominates $N[e]$.

Proof

If e is γ'^{+}-critical, then $\gamma'(G-e) = \gamma'(G) + 1$, by lemma 2.1. Clearly e is not an isolated edge. If S is a γ'-set of $G - N[e]$ having $\gamma'(G)$ edges which also dominates $N(e)$ then $\gamma'(G-e) \le \gamma'(G)$, a contradiction. Thus no edge dominating set of $G - N[e]$ having $\gamma'(G)$ edges can dominate $N(e)$. By Theorem 2.4, e is γ'-fixed. The converse is obvious.

We now investigate relationships between, γ'-free edges, γ'-totally free edges and graphs which are class 1 and class 2.

2.7. Theorem

If G is a graph without isolated edges, then G is of class 2 if and only if G has γ'-totally free edges.

Proof

Suppose G has a γ'-totally free edge e. By Theorem 2.4 (2), G is of class 2. Conversely, suppose G is of class 2. Then there exists an edge e which is not in any γ'-set. Hence every γ'-set of G is also a γ'-set of $G - e$ so that e is γ'-totally free.

2.8. Theorem

Proof

Let G be a connected graph. If G has a γ'-fixed edge, then it has a γ'-totally free edge.

Suppose G has a γ'-fixed edge e. Then e belongs to every γ'-set.

Claim: No neighbor of e belongs to any γ'-set of G. Suppose at least one of its neighbor say e' belongs to a γ'-set D. Let $e = uv$ and e' be incident with u. Then $D_1 = D - \{e\} \cup \{e''\}$, where e'' is any edge incident with v is an edge dominating set of $G - e$ with γ'-edges which is also an edge dominating set of G. But by Theorem 2.6, this is a contradiction, since e is a γ'-fixed edge. Therefore no neighbor of e belongs to any γ'-set of G. Thus neighbors of e are all γ'-totally free in G.

We now investigate the class of graphs which are ds'^{+}, ds'^{-}-critical.

2.9. Lemma

Let $e \in E(G)$. If e is γ'-totally free and $G - e$ is of class 1, then $ds'(G) = ds'(G-e)+1$.

Proof

Since e is γ'-totally free, by Theorem 2.4,

$$\gamma'(G) = \gamma'(G-e) \quad (1)$$

Since e is γ'-totally free, by Theorem 2.3, G is of class 2 and so

$$ds'(G) = \gamma'(G)+1 \quad (2)$$

Since $G - e$ is of class 1, we have

$$ds'(G-e) = \gamma'(G-e) \quad (3)$$

From Equations (1), (2) and (3), we have

$$ds'(G) = ds'(G-e)+1.$$

2.10. Lemma

Let $e \in E(G)$. If e is γ'-totally free and $G - e$ is of class 2, then $ds'(G) = ds'(G-e)$.

Proof

If e is γ'-totally free, then by Theorem 2.4,

$$\gamma'(G) = \gamma'(G-e) \quad (1)$$

Since G and $G - e$ are of class 2, we have

$$ds'(G) = \gamma'(G)+1 \quad (2)$$

and

$$ds'(G-e) = \gamma'(G-e)+1 \quad (3)$$

From equations (1), (2) and (3), we have

$$ds'(G) = ds'(G-e).$$

2.11. Lemma

Let e be an edge of G. If e is γ'-free and $G - e$ is of class 1, then $ds'(G) = ds'(G-e)$ or $ds'(G) = ds'(G-e)+1$.

Proof

Suppose e is a γ'-free edge. In any case G is either of class 1 or class 2.

Case (1). G is of class 1.

Let S be a γ'-set of $G - e$. If S does not contain any neighbor of e, then every neighbor of e is γ'-totally free in $G - e$. This implies that $G - e$ is of class 2. But this is a contradiction and so S must contain a neighbor of e. Then by theorem 2.4, $\gamma'(G) = \gamma'(G-e)$. Since G and $G - e$ are of class 1, we have

$$ds'(G) = \gamma'(G) = \gamma'(G-e) = ds'(G-e).$$

Case (2). G is of class 2.

Since $G - e$ is of class 1, then by a similar argument, S must contain a neighbor of e. Since G is of class 2, we have $ds'(G) = \gamma'(G)+1 = \gamma'(G-e)+1 = ds'(G-e)+1$.

2.12. Lemma

Let e be an edge of G. If e is γ'-free and $G - e$ is of class 2, then $ds'(G) = ds'(G-e)$, $ds'(G) = ds'(G-e)+1$ or $ds'(G) = ds'(G-e)-1$.

Proof

Case (1). G is of class 1.

Let S be a γ'-set of $G - e$. We have the following cases:

Subcase (1). S contains a neighbor of e.

Now $\gamma'(G) = \gamma'(G-e)$. Since G is of class 1 and $G - e$ is of class 2, we have $ds'(G-e) = ds'(G)+1$.

Subcase (2). S does not contain a neighbor of e.

Now $\gamma'(G) = \gamma'(G-e)+1$. Since $G - e$ is of class 2 and G is of class 1, we have $ds'(G) = ds'(G-e)$.

Case (2). G is of class 2.

By an argument similar to that in case (1), we have $ds'(G) = ds'(G-e)$ or $ds'(G) = ds'(G-e)+1$.

2.13. Lemma

Let e be an edge of G. If e is γ'-fixed and $G - e$ is of class 1, then $ds'(G) = ds'(G-e)$.

Proof

If e is γ'-fixed, then by Theorem 2.8, all of its neighbors are γ'-totally free. Then by Theorem 2.7, G is of class 2 and hence

$$ds'(G) = \gamma'(G) + 1 \quad (1)$$

As e is γ'-fixed, by Theorem 2.4, $\gamma'(G) \neq \gamma'(G-e)$. If $\gamma'(G) > \gamma'(G-e)$, then e is γ'-critical. Then by Lemma 2.11, e is γ'-free and this is a contradiction. Therefore $\gamma'(G) = \gamma'(G-e) - 1$. Since G is of class 2 and $G - e$ is of class 1, we have $ds'(G-e) = ds'(G)$.

2.14. Lemma

Let $e \in E(G)$. If e is γ'-fixed and $G - e$ is of class 2, then $ds'(G) = ds'(G-e) - 1$.

Proof

By an argument analogous to that in Lemma 2.13, since $G - e$ is of class 2, we have $ds'(G) = ds'(G-e) - 1$.

2.15. Theorem

Let G be a graph without isolated edges. An edge e in G is ds'^{-}-critical if and only if one of the following holds.

1) e is γ'-totally free and $G - e$ is of class 1.

2) e is γ'-free, G is of class 2 and $G - e$ is of class 1.

3) e is γ'-free and both G and $G - e$ are of class 2.

Proof

Suppose e is ds'^{-}-critical. Then

$$ds'(G) = ds'(G-e) + 1 \quad (1)$$

Let S be a γ'-set of G. Then we have the following cases:

Case (1). G and $G - e$ are of class 1.

By (1), $\gamma'(G) = \gamma'(G-e) + 1$. By theorem 2.3, e is γ'-free and no γ'-set of G-e contains any edge of $N(e)$. Now every neighbor of e is γ'-totally free in $G - e$. Therefore $G - e$ is of class 2, which is a contradiction.

Case (2). G is of class 1 and $G - e$ is of class 2.

Then Equation (1) becomes $\gamma'(G) = \gamma'(G-e) + 2$. But this is not possible.

Case (3). G is of class 2 and $G - e$ is of class 1.

Then Equation (1) becomes $\gamma'(G) = \gamma'(G-e)$. Then either e is γ'-free or γ'-totally free.

Case (4). G and $G - e$ are of class 2.

In this case, Equation (1) becomes $\gamma'(G) = \gamma'(G-e) + 1$. Then by theorem 2.3, e is γ'-free.

From Lemmas 2.9, 2.11 and 2.12, the converse is true.

2.16. Theorem

Let G be a graph without isolated edges. An edge e in G is ds'^{+}-critical if and only if one of the following holds.

1) e is γ'-free, G is of class 1 and $G - e$ is of class 2.

2) e is γ'-fixed and $G - e$ is of class 2.

Proof

Suppose e in G is ds'^{+}-critical. Hence

$$ds'(G) = ds'(G-e) - 1 \quad (1)$$

Let S be a γ'-set of G. Then we have the following cases:

Case (1). G and G-e are of class 1.

From equation (1) $\gamma'(G) = \gamma'(G-e) - 1$ and so G is γ'^{+}-critical. Hence by Theorem 2.6, e is γ'-fixed, which is a contradiction.

Case (2). G is of class 1 and $G - e$ is of class 2.

Now equation (1) becomes $\gamma'(G) = \gamma'(G-e)$. Then S must contain a neighbor of e. Since G is of class 1, e is γ'-free.

Case (3). G is of class 2 and $G - e$ is of class 1.

Then Equation (1) becomes $\gamma'(G) = \gamma'(G-e) - 2$, which is not possible.

Case (4). G is of class 2 and $G - e$ is of class 2.

In this case, Equation (1) becomes $\gamma'(G) = \gamma'(G-e) - 1$. Then by Theorem 2.4, e is γ'-fixed.

Conversely, suppose if (1) or (2) is true. Then by case (1) of Lemma 2.12 and Lemma 2.14, the result follows.

3. Edge-Domsaturation Number of a Graph

Theorem

For any connected unicyclic graph $G = (V, E)$ with cycle C, $ds'(G) = q - \Delta'(G) + 1$ if and only if one of the following holds.

1) $C = C_3 = (u_1, u_2, u_3, u_1)$, $\deg u_1 \geq 3$, $\deg u_2 \geq 3$, $\deg u_3 = 2$, $\deg u_{u \in N[u_1] \cap (V-C)} \leq 2$ and there exists $w \in V - C$ such that $d(u_i, w) \leq 2$, $i = 1, 2$.

2) $C = C_3 = (u_1, u_2, u_3, u_1)$, $\deg u_1 \geq 4$, $\deg u_2 = 2$, $\deg u_3 = 2$, exactly one vertex w not on C has $\deg w \geq 2$ and remaining vertices are pendent vertices.

Proof

Suppose $ds'(G) = q - \Delta'(G) + 1$.

Let $C = C_k = (u_1, u_2, \cdots, u_k, u_1)$ be the unique cycle in G.

If $C = C_k$, then $ds'(G) = \lceil q/3 \rceil < q - 1$ for all $n \geq 3$ and so $G \neq C_k$.

Let S denote the set of all pendent edges of G and let $|S| = t$.

Claim 1: $t \leq \Delta'(G) - 2$. Since $E - (S \cup \{e\})$ is an edge dominating set for any edge e of C, $\gamma'(G) \leq q - t - 1$. For any pendent edge f, $E - (S \cup \{g, e\}) \cup \{f\}$ is an edge dominating set of G containing f. Here g is an edge adjacent to f and e is any edge of the cycle. Hence $ds'(G) \leq q - t - 1$, so that $t \leq \Delta'(G) - 2$.

Claim 2: $e = uv$ is an edge with degree Δ'. Then either u or v lies on C_k.

Now let $G \neq C_k$ and $e = uv$ be an edge of maximum degree Δ'. If $e \in C_k$, then for some edge $e' \in C_k, G - e'$ is a tree T of G with at least $(\Delta'(G) + 1)$ pendent edges. If X is the set of all pendent edges of $G - e$, then $|X| \geq \Delta'(G) + 1$. Then $E(T) - X$ is an edge dominating set of cardinality at most $q - \Delta'(G) - 1$. Therefore $ds'(G) < q - \Delta'(G) + 1$, which is a contradiction.

Case (1). u or v lies on C.

Claim 3: $G-C_k$ is the union of P_1 and P_2. Suppose not. Then, $G-C_k$ contains $P_k = x_1x_2\cdots x_K, k \geq 3$. Suppose $u = u_1$ lies on C_K. Let T_{u_1} be the maximal tree rooted at u_1 not containing any edge of C_k. Clearly T_{u_1} has at least $\Delta'(G) - 2$ pendent edges, say S. Then $E(G)-(S \cup \{u_1u_2,\ u_ku_1,\ u_ix_1\}),\ i = 1,2,3,\cdots,k$ is an edge dominating set of cardinality less than $q-\Delta'(G)$. Therefore $ds'(G) < q-\Delta'(G)+1$, which is a contradiction.

In this case, G has at least $\Delta' - 2$ pendent edges. Let W be the set of these pendent edges. Further $ds'(C_k) = \lceil k/3 \rceil$ and let Y denote a γ'-set of C_k. Let $Z = E(C_k) - Y$. If $k >$ 4, then $E(G)-W-Z$ is an edge dominating set of cardinality less than $q-\Delta'(G)$. Hence $C_k = C_3$ or C_4. Since $t \geq \Delta'(G)-2$. By claim 1, $t = \Delta'(G)-2$.

Subcase (1). $C = C_3 = (u_1,u_2,u_3,u_1)$

$G-C_3$ is the union of P_1 and P_2. Also u or v lies on C. Let $u = u_1$. Therefore $G-C_3$ contains at least one P_2. Since $t = \Delta'(G)-2$, no other vertex other than u and v has degree > 3.

If $G - C_3$ is the union of $P_2's$ alone, then $\{x_1x_2,\ u_iu_j\}$ or $\{u_1x_1, u_iu_j\}, i \neq ji, j = 1,2,3$ is an edge dominating set and every edge lies in a γ'-set. Therefore $ds'(G) = q-\Delta'(G)$.

If $G-C_3$ is the union of $P_1's$ and $P_2's$, then from Theorem 1.5, $\gamma'(G) = q-\Delta'(G)$. But pendent edges adjacent to u_1 does not lie in any γ'-set. Therefore $ds'(G) = q-\Delta'(G)+1$.

Subcase (2). $\boldsymbol{C = C_4 = (u_1,u_2,u_3,u_4,u_1)}$

As in subcase (1), $G - C_4$ also contains P_2. Then $E(G)-W-\{u_1u_2,\ u_1u_4,\ u_2u_3\}$ is an edge dominating set of cardinality $< q-\Delta'(G)$. Therefore $ds'(G) < q-\Delta'(G)+1$.

Case (2). u and v lies on C.

Claim 4: $G - C_k$ is the union of P_1 and P_2.

Suppose not. Then, G – C_k contains $P_k = x_1x_2\cdots x_k$, $k \geq 3$. Suppose $e = u_1u_2$ lies on C_k. Let $T_{u_1u_2} = T_{u_1} \cup T_{u_2} \cup \{u_1,\ u_2\}$.

Clearly $T_{u_1u_2}$ has at least $\Delta'(G)-2$ pendent edges, say P.

Then $E(G)-P-\{u_ix_1,\ u_ku_1,\ u_2u_3\}, i = 1,2\cdots k$ is an edge dominating set of cardinality less than $q-\Delta'(G)$. Therefore $ds'(G) < q-\Delta'(G)+1$, which is a contradiction.

As in case (1), $t = \Delta'(G)-2$. Let $e = u_1u_2$ be an edge of maximum degree.

Subcase (1). $C = C_3 = (\ u_1,u_2,u_3,u_1)$

In this case, from Theorem 1.5, (3), u_3u_1 does not belong to any γ'-set. Therefore $ds'(G) = q-\Delta'(G)+1$.

Subcase (2). $C = C_4 = (\ u_1,u_2,u_3,u_4,u_1)$

From Theorem 1.5, there does not exist an edge dominating set of cardinality $q-\Delta'(G)$.

The converse is obvious.

4. Well-Edge Dominated Graph

A graph G is called well dominated if all minimal dominating sets have the same cardinality. This concept was introduced by Finbow, Hartnell and Nowakowski [6].

4.1. Definition

A graph G is well-edge dominated if every minimal edge dominating set of G has the same cardinality.

4.2. Lemma

If G is a well-edge dominated graph and e is an edge of G, then there exists a minimum edge dominating set containing e and a minimum edge dominating set not containing e.

Proof

To obtain an edge dominating set containing e, place e in the set D, delete $N[e]$ from G and continue in this greedy fashion until there are no edges left. Then D is minimal and since G is well-edge dominated, it is minimum.

To obtain a minimum edge dominating set not containing e, we use the same greedy method except that we use a neighbor of e as our initial edge in D.

4.3. Theorem

If G is well-edge dominated, then G is of class 1.

Proof

From the above lemma, it is clear that every edge belongs to any one of the γ'-set. Therefore G is of class 1.

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