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Age-Structured Population Projection of Bangladesh by Using a Partial Differential Model with Quadratic Polynomial Curve Fitting

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Abstract

In this paper, the age-specific population of Bangladesh based on a linear first order (hyperbolic) partial differential equation which is known as Von-Foerster Equation is studied. Applying quadratic polynomial curve fitting, the total population and population density of Bangladesh are projected for the years 2001 to 2050 based on the explicit upwind finite difference scheme for the age-structured population model based on given data (source: BBS & ICDDR, B) for initial value in the year 2001. For each age-group, the future birth rates and death rates are estimated by using quadratic polynomial curve fitting of the data for the years 2001 to 2012. Quadratic polynomial curve fitting is also used for the boundary value as the (0 - 4) age-group population based on the population size of the age-group for the years 2001 to 2012.

Keywords

Von-Foerster Equation, Birth Rate, Death Rate, Curve Fitting

1. Introduction

The fast growth of population during the past decades has frustrated the development efforts in Bangladesh. In 1971 the population of the country was around 75 million. According to the 5th census of Bangladesh Bureau of Statistics (BBS) in 2011, the total population of Bangladesh is 150 million. The area of Bangladesh is 147,570 square kilometers only and it is one of the most densely populated countries all over the world. To handle such

mass population in such short land is a huge problem for the government to take any developing steps. Due to scarcity of resources, it is not possible to provide educational, health, medical, transport and housing facilities to the entire population. A rapidly increasing population plugs the economy into mass unemployment and under employment. As a result, the actual development is just getting being delayed a hampered a lot. With the help of population model, we can predict what the number of population will be by the year of 2050. Therefore, the government should take immediate steps to keep the population under control and the people themselves should adopt family planning for their own benefit. In order to make an efficient planning for the demands of different age-group, it is important to predict the age-structured population of the country. Therefore, in this paper we project the future age-structured population of Bangladesh based on a partial differential equation model. The information we get from age-specified population group can help us in future planning of social and economical development. For example, if we can predict the population of children of 0 - 4 years old, we can provide necessary medical care and baby food to reduce the mortality rate and keep children healthy and nourished. After 50 years of liberation of Bangladesh, the government of Bangladesh is going to celebrate the year 2021 as "apotheosis of liberation". The government has already declared the year as "vision 2021". To educate the people of Bangladesh within 2021, we can afford necessary support for children of 4 - 15 years old. We can provide human resource development program for young people to reduce the unemployment problem and also proper health care to elder people.

2. Age-Structured Population Model

For the prediction of age-structured population, various differential equation models have been formulated by different Mathematicians in different time. In [1], Murray studies the linear age structure population model where he used time-independent death rate. An extensive study of linear and nonlinear age-dependent population dynamics can be found in the works of Webb [2], Gurtin [3] and Iannelli [4].

In this paper, we study a linear first order hyperbolic partial differential equation to predict age–dependent population. We project the future age-structured population in Bangladesh based on this linear model of population which is known as **Von-Foerster** equation, given as follows:

$$\frac{\partial u(x,t)}{\partial t} + \frac{\partial u(x,t)}{\partial x} = -\mu(x,t)u(x,t), \quad x,t \geq 0 \tag{1}$$

with $u(x,0) = u_0(x), x \geq 0$

$$u(0,t) = \int_0^\infty \lambda(x,t)u(x,t)\mathrm{d}x, \quad t \geq 0$$

where, $u(x,t)$ is the age density function, $u(x,t)$ the age specific death rate, $\lambda(x,t)$ the age specific fertility rate so that $\int_0^\infty \lambda(x,t)u(x,t)\mathrm{d}x$ is the birth rate and $u_0(x)$ is the initial age distribution.

3. Numerical Scheme for the Model

The age-structured population model can be written as

$$u_t + u_x = -\mu(t,x)u(t,x), \quad x \geq 0, t \geq 0 \tag{2}$$

with initial condition

$$u(x,0) = u_0(x), \quad x \geq 0 \tag{3}$$

$u_0(x)$ a nonnegative function, and a boundary condition at $x = 0$ (the inflow boundary of the domain)

$$u(0,t) = b(t), \quad t \geq 0 \tag{4}$$

Based on [5] and [6] we discretize the time derivative by a forward difference formula, and the age derivative with a backward difference on a discrete mesh $x_i = i\Delta x, i \in N^+$ and $t^n = n\Delta t, n \in N^+$. The forward difference approximation for $\frac{\partial u}{\partial t}$ obtained by the Taylor series formula is

$$\frac{\partial u(t,x)}{\partial t} \approx \frac{u(t+k,x)-u(t,x)}{k} \tag{5}$$

and the backward difference approximation for $\dfrac{\partial u}{\partial x}$ is

$$\frac{\partial u(t,x)}{\partial x} \approx \frac{u(t,x)-u(t,x-h)}{h} \tag{6}$$

We consider uniform grid spacing with step size h and k for space and time respectively, $x_{i+1} = x_i + h$ and $t^{n+1} = t^n + k$. Using the approximation U_i^n for $u(x_i, t^n)$ in (5) and (6), Equation (2) approximates as

$$\frac{U_i^{n+1}-U_i^n}{\Delta t} + \frac{U_i^n - U_{i-1}^n}{\Delta x} = -\mu(x,t)U_i^n \tag{7}$$

$$\therefore U_i^{n+1} = U_i^n - v\left(U_i^n - U_{i-1}^n\right) - \Delta t\,\mu U_i^n$$

which is the explicit upwind difference scheme for the age-structured population model.

Here $v := \dfrac{\Delta t}{\Delta x}$. The boundary value $U_0^n, n \geq 0$; must be obtained from the boundary condition (4) at $x = 0$ and initial value U_i^0 for $i \in N^+$, obtain from the initial condition (3) at $t = 0$.

4. Numerical Experiments

We implement the explicit upwind difference scheme and introduce quadratic polynomial curve fitting procedure for the age-structured population model.

4.1. Curve Fitting

Curve fitting is the process of constructing a curve, or mathematical function that has the best fit to a series of data points, possibly subject to constraints. Curve fitting can involve either interpolation, where an exact fit to the data is required, or smoothing, in which a "smooth" function is constructed that approximately fits the data.

4.2. Quadratic Polynomial Models

Given *n* data points $(x_1, y_1), (x_2, y_2), \cdots, (x_n, y_n)$. Consider the Quadratic Polynomial Modelsas

$$y = a_0 + a_1 x + a_2 x^2 \tag{8}$$

The residual at each data point is given by

$$e_i = y_i - a_0 - a_1 x_i - a_2 x_i^2 \tag{9}$$

The sum of the square of the residuals is given by

$$S = \sum_{i=1}^n e_i^2 = \sum_{i=1}^n \left(y_i - a_0 - a_1 x_i - a_2 x_i^2\right)^2 \tag{10}$$

To find the constants of the polynomial regression model, we put the derivatives successively with respect to a_0, a_1 and a_2 to zero, that is,

$$\frac{\partial S}{\partial a_0} = \sum_{i=1}^n 2\left(y_i - a_0 - a_1 x_i - a_2 x_i^2\right)(-1) = 0 \tag{11}$$

$$\frac{\partial S}{\partial a_1} = \sum_{i=1}^n 2\left(y_i - a_0 - a_1 x_i - a_2 x_i^2\right)(-x_i) = 0 \tag{12}$$

$$\frac{\partial S}{\partial a_2} = \sum_{i=1}^n 2\left(y_i - a_0 - a_1 x_i - a_2 x_i^2\right)\left(-x_i^2\right) = 0 \tag{13}$$

Setting those equations in matrix form gives

$$
\begin{bmatrix}
n & \left(\sum_{i=1}^{n} x_i\right) & \left(\sum_{i=1}^{n} x_i^2\right) \\
\left(\sum_{i=1}^{n} x_i\right) & \left(\sum_{i=1}^{n} x_i^2\right) & \left(\sum_{i=1}^{n} x_i^3\right) \\
\left(\sum_{i=1}^{n} x_i^2\right) & \left(\sum_{i=1}^{n} x_i^3\right) & \left(\sum_{i=1}^{n} x_i^4\right)
\end{bmatrix}
\begin{bmatrix}
a_0 \\
a_1 \\
a_2
\end{bmatrix}
=
\begin{bmatrix}
\sum_{i=1}^{n} y_i \\
\sum_{i=1}^{n} x_i y_i \\
\sum_{i=1}^{n} x_i^2 y_i
\end{bmatrix}
\tag{14}
$$

The above are solved for a_0, a_1, a_2.

4.3. Incorporation of Data into Explicit Upwind Difference Scheme

To predict the Age Distributed Population we incorporate the initial and boundary data into the Explicit Upwind Scheme with respect to the assumptions and considerations below:

We assume that $u(t,x)$ is the total population distribution function rather than a density distribution function.

- We have considered the age of people of Bangladesh in between 0 to 85+ years. We divide this age-group (0 to 85 years) into 18 sub-groups, each sub-group contains 5 years interval *i.e.* 0 - 4, 5 - 9, 10 - 14, $\cdots$, 80 - 84, 85+.
- We have used age distributed population of 2001 as initial data $u_0(x)$ and we have assumed the population of the (0 - 4) age-group as the newborns baby (population) and the population of this first age-group of the years of 2001 to 2012 of data (**Table 1**) [7] is the basis to formulate the birth rate $b(t)$. We have estimated the birth rate by fitting quadratic polynomials. To evaluate birth rate $b(t)$, we have been fitting quadratic polynomials

Table 1. Mid-year population distribution in percentage by age-group from 2001 to 2012.

Age	Population in percentage in the years 2001-2012											
	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012
0 - 4	11.8	12.1	12.1	12.0	12.1	11.8	11.5	11.4	11.2	11.0	10.7	10.8
5 - 9	11.4	11.2	11.2	11.2	11.1	11.3	11.6	11.7	11.7	11.6	11.4	11.2
10 - 14	12.8	12.2	11.8	11.3	11.1	10.7	10.7	10.7	10.7	10.6	10.7	11.0
15 - 19	10.7	10.8	10.6	10.7	10.6	10.4	10.0	9.7	9.3	9.1	8.9	8.9
20 - 24	8.6	8.2	8.2	8.3	8.0	8.1	8.2	8.0	8.1	8.2	8.0	7.7
25 - 29	6.6	6.9	6.8	6.5	6.8	7.0	6.7	6.6	6.7	6.6	6.7	6.6
30 - 34	6.7	6.5	6.5	6.5	6.2	6.1	6.2	6.1	5.9	6.2	6.4	6.2
35 - 39	6.7	6.7	6.5	6.5	6.5	6.4	6.2	6.1	6.1	5.9	5.8	5.9
40 - 44	5.9	6.2	6.4	6.5	6.6	6.4	6.4	6.3	6.2	6.2	6.1	5.9
45 - 49	4.1	4.3	4.7	5.0	5.3	5.7	5.9	6.1	6.3	6.3	6.2	6.2
50 - 54	3.3	3.4	3.5	3.6	3.8	3.9	4.1	4.5	4.8	5.1	5.5	5.8
55 - 59	3.4	3.4	3.2	3.2	3.2	3.2	3.3	3.4	3.5	3.6	3.8	4.0
60 - 64	2.7	2.8	3.0	3.0	3.1	3.1	3.1	3.1	3.0	3.0	2.9	3.0
65 - 69	2.4	2.3	2.3	2.4	2.3	2.3	2.4	2.6	2.6	2.7	2.7	2.7
70 - 74	1.4	1.5	1.6	1.6	1.7	1.9	1.8	1.8	1.9	1.9	1.9	1.9
75 - 79	0.9	0.8	0.8	0.9	1.0	1.0	1.0	1.1	1.1	1.2	1.3	1.3
80 - 84	0.4	0.4	0.4	0.4	0.5	0.5	0.5	0.5	0.5	0.6	0.6	0.6
85+	0.2	0.2	0.2	0.2	0.2	0.3	0.3	0.3	0.3	0.3	0.4	0.3

(Source: ICDDR, B).

$$b(t) = b(0) + b(1)*t + b(2)*t^2 \tag{15}$$

Then we have used $b(t)$ as the boundary condition $u_a(t)$. For this we have used data of birth rate from **Table 1** [7].

• We have estimated death rate, which has been parameterized by μ in the age specific population model, by fitting quadratic polynomials. To evaluate death rate $\mu(x,t)$, we have been fitting quadratic polynomials,

$$\mu(t) = \mu(0) + \mu(1)*t + \mu(2)*t^2 \tag{16}$$

For this we have used data of death rate from **Table 2** [8].

Using theses age and time dependent death rate, initial value and boundary condition on Explicit Upwind Finite Difference Scheme, we can forecast the age specific population distribution.

4.4. Total Population Projection

Figure 1 shows our estimated total population projection marked by "Age-Structured Population Model (By Quadratic Polynomial)" for the years 2001 to 2050. Here it is observed that the initial population in our model is 131 million for the year 2001 and the predicted population for the year 2050 is 246.3280 million. In the year 2021, in our model the predicted population will be 170.2044 million whereas the total population was 75 million in 1971. So the total population in 2021 will be 2.26 multiple of the total population of 1971.

Figure 2 shows the comparison of the predicted population of Bangladesh from the years 2001 to 2050 with the projection of Bangladesh Bureau of Statistics (BBS) which is made by Md. Kabir [9], the predicted population by Dutta and Andallah [10] and the predicted population by Md. Minarul Haque, Faruque Ahmed, Sayedul Anam, Md. Rashed Kabir [11]. Here, the population of Bangladesh is calculated by three mathematical methods. Dutta and Andallah have used "Linear Equation", Md. Minarul Haque, Faruque Ahmed, Sayedul Anam, Md.

Table 2. Death rate by age-group and year (per 1000 population).

| Age	Death rate (per 1000 population) in the years 2001-2012											
	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012
0 - 4	54.4	55.6	48.7	47.0	43.5	37.9	36.6	31.1	31.7	32.4	30.6	22.4
5 - 9	1.3	1.1	0.8	0.9	0.9	0.7	0.9	0.8	0.5	0.5	0.5	0.3
10 - 14	0.8	0.6	0.8	0.7	0.7	0.4	0.6	0.4	0.2	0.4	0.6	0.3
15 - 19	0.9	0.7	1.1	1.0	0.9	0.9	0.8	1.0	0.8	0.8	0.7	0.7
20 - 24	0.8	1.1	1.1	1.1	0.9	0.9	0.5	1.1	1.0	1.0	1.1	0.9
25 - 29	1.0	1.0	1.0	0.8	0.8	0.8	0.8	1.0	0.9	0.6	1.1	1.5
30 - 34	1.2	1.9	1.1	1.3	1.4	0.7	1.5	0.9	1.2	1.4	1.2	0.4
35 - 39	1.9	1.3	1.9	2.2	2.2	2.0	2.1	1.8	1.5	1.4	1.2	0.7
40 - 44	3.1	3.5	2.8	3.0	2.6	2.6	1.8	2.0	2.3	2.2	1.7	1.4
45 - 49	3.9	3.7	3.5	4.3	4.1	4.1	3.6	4.0	3.6	3.5	2.7	4.8
50 - 54	6.0	8.0	5.5	5.7	6.0	6.9	6.7	6.2	7.7	4.3	5.1	7.0
55 - 59	9.5	9.9	11.0	8.7	9.2	7.9	10.3	8.1	10.4	10.9	9.6	10.3
60 - 64	19.9	22.7	17.2	18.7	18.9	17.2	19.4	17.7	15.0	18.8	15.7	15.1
65 - 69	27.2	28.4	35.5	35.5	33.7	29.4	30.2	34.5	30.2	28.3	23.2	24.5
70 - 74	51.4	49.3	43.7	53.6	54.6	46.2	56.6	53.0	47.3	46.9	41.5	47.2
75 - 79	84.3	92.6	87.2	98.6	93.9	84.1	84.4	67.8	78.9	77.8	74.8	91.7
80 - 84	105.7	142.5	148.5	128.6	127.2	94.1	124.4	128.9	125.0	119.2	99.8	108.2
85+	166.7	161.0	204.3	196.1	194.2	162.2	190.5	222.4	165.1	169.7	166.9	176.2

(Source: ICDDR, B).
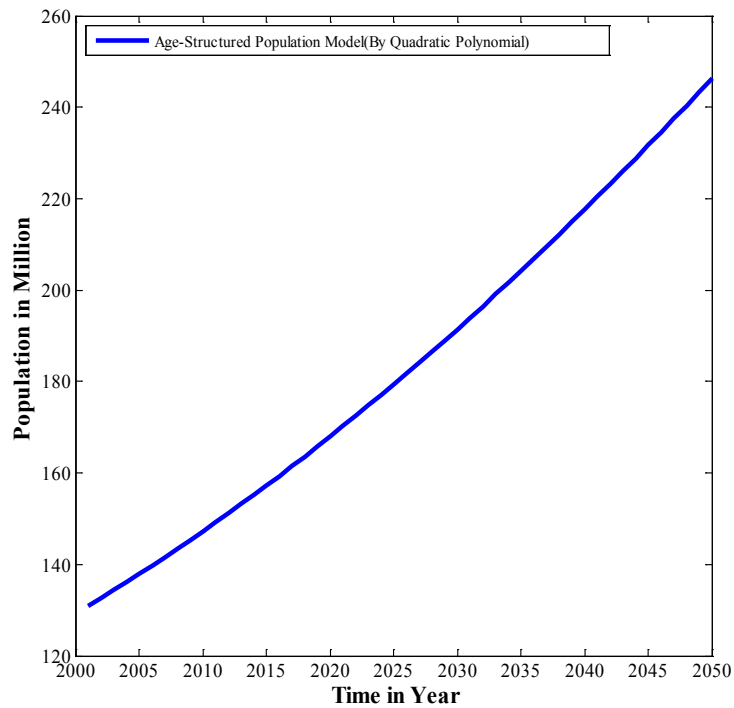
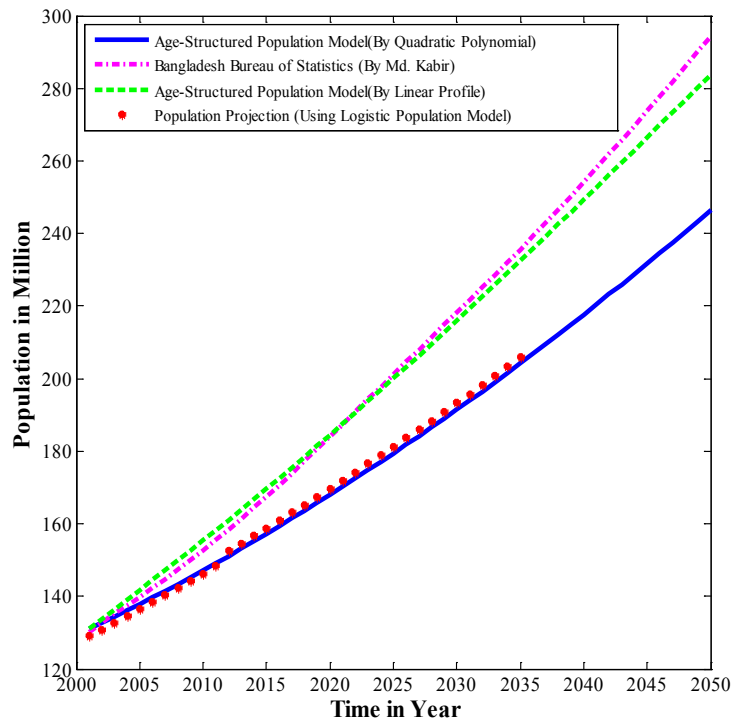**Figure 1.** Example of a figure caption (figure caption).

Figure 2. Comparison of our population projection with different methods.

Rashed Kabirhave used "Logistic Population Model" and we have used "Quadratic Polynomial Curve Fitting". Our curve is going to most nearer to the curve of Logistic Population Model with the increased period of time. Here it is observed that the initial population is 131 million in 2001 in our projection as well as same in the projection by Linear Equation where as it is 130.02 in the projection of BBS and 129.090 million in the projection

by Logistic Population Model. It is also observed that, for the year 2050 our predicted population is 246.3280 million, but it is 294.38 million corresponding to BBS and 283.8058 corresponding to the projection by Linear Equation.

In 2035, in our model the predicted population will be 204.2385 million whereas the predicted population will be 235.67 million corresponding to the projection of BBS, 232.5276 million corresponding to the projection by Linear Equation and 205.979 million corresponding to the projection by Logistic Population Model. By **Figure 2** one can observe that the inclination of the increasing population of our Age-Structured Population Model is more consistent than the demographical method of BBS and the population model by Linear Equation. From the comparison it is also observed that the prediction by the Quadratic Polynomial Curve Fitting method is much closer to the prediction by Logistic Population Model.

4.5. Population Projection for Different Age-Groups

Figure 3 is showing the population projection for different age-groups of Bangladesh from the years 2001 to 2050. It is clearly observed that, the age of the people in the scale (0 - 15) will increase rapidly. The middle aged people will also increase but the increasing rate will be partially sloth and the increasing rate of aged people will be very sloth.

Figure 4 shows the comparison of our estimated population per-square kilometer projection marked by "Age-Structured Population Density (By Quadratic Polynomial)" with that of marked by "Bangladesh Bureau of Statistics Density (by Md. Kabir)" for the years 2001 to 2050. Here it is observed that the initial population per-square kilometer is 888 in 2001 in our projection whereas it is 881 according to the projection of BBS. It is also observed that, for the year 2050 our predicted population per-square kilometer is 1669, but it is 1995 according to BBS. In 2021, in our model the predicted population per-square kilometer will be 1153 whereas the predicted population per-square kilometer will be 1271 according to the projection of BBS.

The total predicted population for the years 2001 to 2050 is presented in **Table 3** and the predicted population per-square kilometer is presented in **Table 4**.

5. Conclusion

We have considered a continuous and deterministic mathematical model known as Von-Foerster model, which is a linear first order partial differential equation used to predict population distribution by age at any time, given the initial distribution and the variation of birth and death rates with age and time. Although this is a linear equation, it is not easy to solve the difficulty of enforcing boundary condition. For this, we have used finite difference
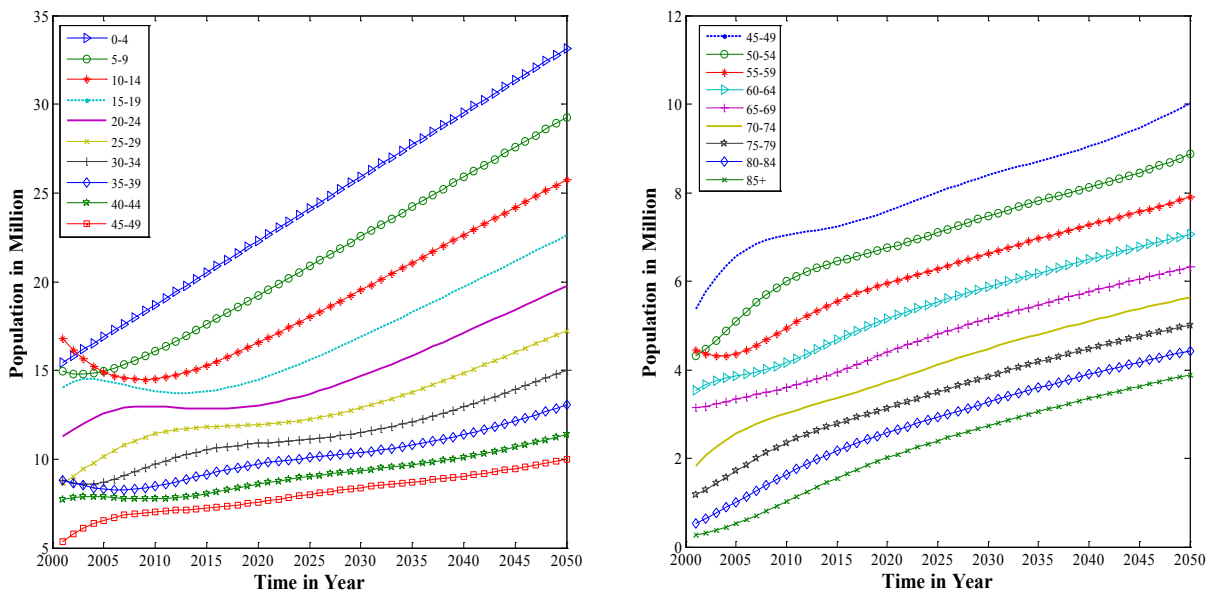

Figure 3. Population projection of different age-groups.

Table 3. Population projection of Bangladesh from 2001 to 2050 (in millions).

Year	Projection by Quadratic Polynomial Curve Fitting	Projection by BBS	Projection by Linear Equation	Projection by Logistic Population Model
2001	**131.0000**	**130.02**	**131.0000**	**129.090**
2002	132.6495	132.60	133.6691	130.927
2003	134.3484	135.12	136.3539	132.784
2004	136.0868	137.54	139.0453	134.662
2005	137.8620	139.90	141.7436	136.559
2006	139.6718	142.21	144.4503	138.478
2007	141.5142	144.75	147.1678	140.416
2008	143.3875	147.34	149.8988	142.376
2009	145.2902	150.00	152.6461	144.355
2010	147.2211	152.73	155.4129	146.356
2011	**149.1792**	**155.53**	**158.2017**	**148.378**
2012	151.1639	158.41	161.0150	152.484
2013	153.1747	161.37	163.8548	154.569
2014	155.2113	164.41	166.7226	156.676
2015	**157.2739**	**167.53**	**169.6193**	**158.804**
2016	159.3624	170.73	172.5453	160.953
2017	161.4771	173.99	175.5003	163.124
2018	163.6183	177.31	178.4838	165.316
2019	165.7864	180.68	181.4949	167.530
2020	167.9816	184.08	184.5324	169.766
2021	**170.2044**	**187.49**	**187.5950**	**172.024**
2022	172.4549	190.91	190.6813	174.304
2023	174.7334	194.34	193.7901	176.606
2024	177.0399	197.76	196.9201	178.931
2025	179.3745	201.18	200.0703	181.277
2026	181.7373	204.60	203.2396	183.646
2027	184.1280	208.01	206.4274	186.037
2028	186.5467	211.42	209.6328	188.451
2029	188.9930	214.83	212.8554	190.887
2030	191.4668	218.25	216.0947	193.345
2031	193.9678	221.69	219.3503	195.827
2032	196.4957	225.14	222.6218	198.331
2033	199.0503	228.62	225.9087	200.857
2034	201.6313	232.13	229.2108	203.407
2035	**204.2385**	**235.67**	**232.5276**	**205.979**
2036	206.8715	239.27	235.8587	
2037	209.5302	242.91	239.2037	
2038	212.2142	246.59	242.5621	
2039	214.9235	250.32	245.9336	
2040	217.6576	254.10	249.3177	
2041	220.4165	257.93	252.7142	
2042	223.2000	261.81	256.1225	
2043	226.0078	265.74	259.5426	
2044	228.8397	269.71	262.9743	
2045	231.6957	273.73	266.4174	
2046	234.5753	277.78	269.8719	
2047	237.4786	281.88	273.3378	
2048	240.4053	286.01	276.8153	
2049	243.3552	290.18	280.3045	
2050	**246.3280**	**294.38**	**283.8058**	

Table 4. Population per-square kilometer of Bangladesh from 2001 to 2050.

| Year	Our density	Density by BBS
2001	888	881
2002	899	899
2003	910	916
2004	922	932
2005	934	948
2006	947	964
2007	959	981
2008	972	998
2009	985	1016
2010	998	1035
2011	**1011**	**1054**
2012	1024	1073
2013	1038	1094
2014	1052	1114
2015	1066	1135
2016	1080	1157
2017	1094	1179
2018	1109	1202
2019	1123	1224
2020	1138	1247
2021	**1153**	**1271**
2022	1169	1294
2023	1184	1317
2024	1200	1340
2025	1216	1363
2026	1232	1386
2027	1248	1410
2028	1264	1433
2029	1281	1456
2030	1298	1479
2031	1314	1502
2032	1332	1526
2033	1349	1549
2034	1366	1573
2035	1384	1597
2036	1402	1621
2037	1420	1646
2038	1438	1671
2039	1456	1696
2040	1475	1722
2041	1494	1748
2042	1513	1774
2043	1532	1801
2044	1551	1828
2045	1570	1855
2046	1590	1882
2047	1609	1910
2048	1629	1938
2049	1649	1966
2050	**1669**	**1995**
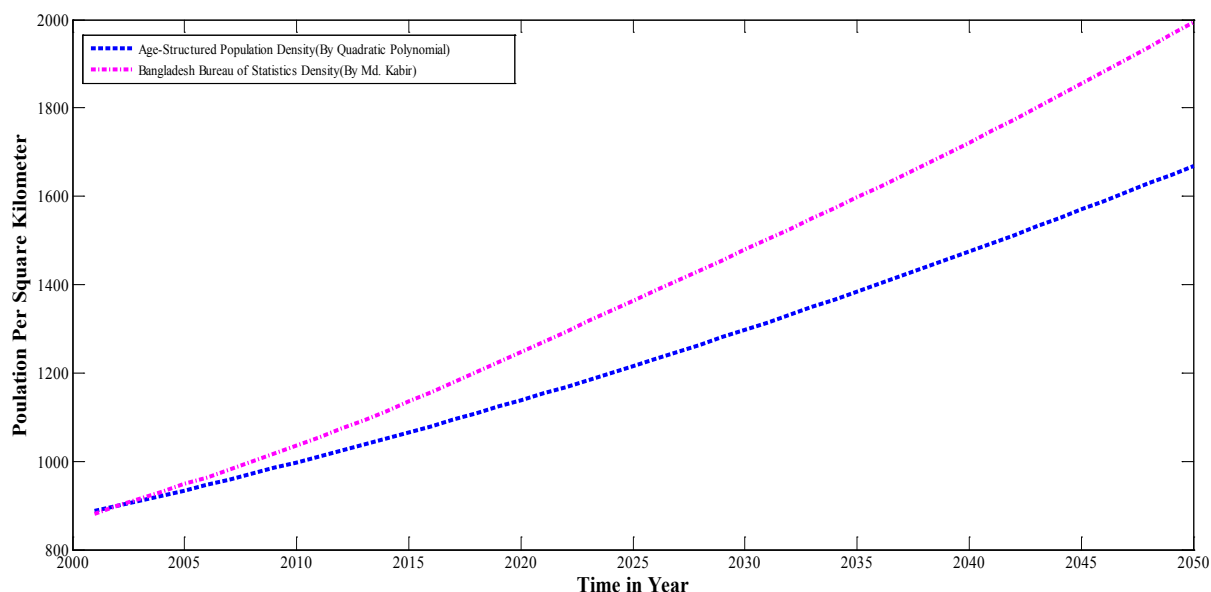
Figure 4. Comparison of our population density projection with Bangladesh Bureau of Statistics for the years 2001 to 2050.

method for the numerical solution of the age-structured population model. For the numerical experiment, the year 2001 is used as the initial time. We have predicted the age distribution population up to the year 2050. In this experiment we have provided the data for birth rate and death rate up to 2012 from [7] and [8] respectively. The results have showed a "very good agreement" with the age distribution population size up to 2012 given in [7]. These have justified the correctness of our implementation of the explicit upwind finite difference scheme for the age-structured population projection in Bangladesh and in other countries as well. We have predicted the total population and population density up to 2050 and these results have showed a very good agreement with the results according to the projection by Logistic Population Model [11] which have been predicted up to 2035. We have observed that, in the year 2050, our population will be 246.3280 million whereas it is 294.38 million according to Kabir [9] and 283.8058 million according to the projection by Linear Equation [10]. In [10], the calculation is based on a linear modeling of birth and death rate and our calculation is based on a quadratic polynomial curve fitting of birth and death rate. We have estimated the birth rate and death rate by a non-linear profile. So, the results are more realistic. This motivates us for further study of the model.

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