

An Interval Matrix Based Generalized Newton Method for Linear Complementarity Problems

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Abstract

The penalty equation of LCP is transformed into the absolute value equation, and then the existence of solutions for the penalty equation is proved by the regularity of the interval matrix. We propose a generalized Newton method for solving the linear complementarity problem with the regular interval matrix based on the nonlinear penalized equation. Further, we prove that this method is convergent. Numerical experiments are presented to show that the generalized Newton method is effective.

Keywords

Linear Complementarity Problem, Nonlinear Penalized Equation, Interval Matrix, Generalized Newton Method

1. Introduction

The linear complementarity problem, denoted by $LCP(M, q)$, is to find a vector $x \in R^n$ such that

$$x \geq 0, \quad Mx - q \geq 0, \quad x^T (Mx - q) = 0 \quad (1.1)$$

where $M \in R^{n \times n}$ is a given matrix and $q \in R^n$ is a given vector. This problem serves as a unified formulation of linear and quadratic programming problems as well as of two-person (noncooperative) matrix-games, and has several important applications in economics and engineering sciences; see Cottle, Pang, and Stone [1] and its references.

There exist several methods for solving $LCP(M, q)$, such as projection method, multi splitting method, interior point method, and the nonsmooth Newton method, smoothing Newton method, homotopy method etc. See [1]-[6] and its references.

In [7], it given a nonlinear penalized Equation (1.2) corresponding to linear complementarity problem (1.1).

Find $x_\lambda \in R^n$ such that

$$Mx_\lambda + \lambda [x_\lambda]_+^{\frac{1}{k}} = q \tag{1.2}$$

where $\lambda > 1$ is the penalized parameter, $k > 0$, $[u]_+ = \max\{u, 0\}$ and $y^\alpha = (y_1^\alpha, y_2^\alpha, \dots, y_n^\alpha)^T$ for any $y = (y_1, y_2, \dots, y_n)^T \in R^n$.

The nonlinear penalized problems (1.2) corresponding to the linear complementarity problem (1.1), which its research has achieved good results. In 1984, Glowinski [1] studied nonlinear penalized Equation (1.2) in R^n , and proved the convergence of penalized equation that matrix A was symmetric positive definite. In 2006, Wang *et al.* [8] presented a power penalty function approach to the linear complementarity problem arising from pricing American options. It is shown that the solution to the penalized equation converges to that of the linear complementarity problem with matrix is positive definite. In 2008, Yang [7] proved that solution to this penalized Equations (1.2) converged to that of the LCP at an exponential rate for a positive definite matrix case where the diagonal entries were positive and off-diagonal entries were not greater than zero. The same year, Wang and Huang [9] presented a penalty method for solving a complementarity problem involving a second-order nonlinear parabolic differential operator, and defined a nonlinear parabolic partial differential equation (PDE) approximating the variational inequality using a power penalty term with a penalty constant $\lambda > 1$, a power parameter $k > 0$ and a smoothing parameter ε . And prove that the solution to the penalized PDE converges to that of the variational inequality in an appropriate norm at an arbitrary exponential rate of the form $O([\lambda^{-k} + \varepsilon(1 + \lambda\varepsilon^{1/k})]^{1/2})$. Under some assumptions, Li [10] [11] proved that the solution to this equation converges to that of the linear complementarity problem with A is a strict row diagonally dominant upper triangular P -matrix when the penalty parameter approaches to infinity and the convergence rate was also exponential. It is worth mentioning that the penalty technique has been widely used solving nonlinear programming, but it seems that there is a limited study for the LCP.

Although the studies solving for the linear complementarity problem based on the nonlinear penalized equation have good results. But there is no method that is given for solving the nonlinear penalized equation. Throughout the paper, we propose a generalized Newton method for solving the nonlinear penalized equation with under the suppose $[M, M + \lambda I]$ is regular. So the method can better to solve linear complementarity problem. We will show that the proposed method is convergent. Numerical experiments are also given to show the effectiveness of the proposed method.

2. Preliminaries

Some words about our notation: I refers to the identity matrix, and $\|\cdot\|$ represents the 2-norm. For $x \in R^n$, all vectors are column vectors, x^T refers to the transpose of the x , $[x]_+ = \max\{x, 0\}$, that generalized Jacobian $\partial[x]_+ = D(x)$, where $D(x)$ denotes diagonal matrix, On the diagonal elements with component 1.0 or $\sigma \in [0, 1]$ corresponding to the component of x which is positive, negative or zero, respectively.

The definition of interval matrix arises from the linear interval equations [12], given two matrices $\underline{A} = (A_{ij})$ and $\bar{A} = (\bar{A}_{ij})$, an interval matrix $A^I = [\underline{A}, \bar{A}] = \{A : \underline{A} \leq A \leq \bar{A}\}$, where $\underline{A} \leq \bar{A}$ refers to $\underline{A}_{ij} \leq \bar{A}_{ij}$ for each i, j . A^I is called regular if each $A \in A^I$ is nonsingular.

Lemma 1: Assume that interval matrix $[M, M + \lambda I]$ is regular. Then for diagonal matrix $D(x_\lambda)$ and real number $\lambda > 1$, $M + \lambda D(x_\lambda)$ is nonsingular.

Proof: By definition of $D = \text{diag}(x)$, for every x , we have $D \in [0, I]$ then

$$M + \lambda D \in [M, M + \lambda I]$$

By the assumptions, we have $M + \lambda D(x_\lambda)$ is nonsingular. □

Lemma 2: Let $x, y \in R^n$. Then

$$\|[x]_+ - [y]_+\| \leq \|x - y\|$$

Definition 1 [1]: A matrix $M \in R^{n \times n}$ is said to be a P-matrix if all its principal minors are positive.

Lemma 3 [1]: A matrix $M \in R^{n \times n}$ is a P-matrix if and only if the $LCP(q, M)$ has a unique solution for all vectors $q \in R^n$.

3. Generalized Newton Method

In this section, we will propose that a new generalized Newton method based on the nonlinear penalized equation for solving the linear complementarity problem. Because when $k > 1$, penalty term of the nonlinear penalized equation (1.2) is none Lipschitz continuously, hence we only discusses a case that $k = 1$. So from nonlinear penalized equation (1.2), we have that

$$Mx_\lambda + \lambda [x_\lambda]_+ = q \tag{3.1}$$

These case penalty problems for the continuous Variational Inequality and the linear complementarity problems are discussed in [2] [13].

Let us note

$$f(x_\lambda) = Mx_\lambda + \lambda [x_\lambda]_+ - q, \lambda > 1 \tag{3.2}$$

Thus, nonlinear penalized equation (3.1) is equivalent to the equation $f(x_\lambda) = 0$.

A generalized Jacobian $\partial f(x_\lambda)$ of $f(x_\lambda)$ is given by

$$\partial f(x_\lambda) = M + \lambda D(x_\lambda)$$

where $D(x_\lambda) = \partial [x_\lambda]_+$ is a diagonal matrix whose diagonal entries are equal 1,0 or a real number $\sigma \in [0,1]$ depending on whether the corresponding component of x_λ is positive, negative, or zero. The generalized Newton method for finding a solution of the equation $f(x_\lambda) = 0$ consists of the following iteration:

$$f(x_\lambda^i) + \partial f(x_\lambda^i)(x_\lambda^{i+1} - x_\lambda^i) = 0 \tag{3.3}$$

Then

$$\begin{aligned} \partial f(x_\lambda^i)x_\lambda^{i+1} &= \partial f(x_\lambda^i)x_\lambda^i - f(x_\lambda^i) \\ [M + \lambda_k D(x_\lambda^i)]x_\lambda^{i+1} &= [M + \lambda_k D(x_\lambda^i)]x_\lambda^i - [Mx_\lambda^i + \lambda_k [x_\lambda^i]_+ - q] \\ &= \lambda_k D(x_\lambda^i)x_\lambda^i - \lambda_k [x_\lambda^i]_+ + q \end{aligned}$$

Since $D(x_\lambda^i)x_\lambda^i = [x_\lambda^i]_+$, consequently

$$[M + \lambda_k D(x_\lambda^i)]x_\lambda^{i+1} = q \tag{3.4}$$

Proposition 1 $Mx_\lambda + \lambda [x_\lambda]_+ = q$ equivalent to $0 \leq z \perp Az + b \geq 0$, where

$$z = Mx_\lambda - q, \quad A = (M + \lambda I)M^{-1}, \quad b = \lambda M^{-1}q$$

Proof: Since $|x_\lambda| + x_\lambda = \begin{cases} 2x_\lambda, & x_\lambda \geq 0 \\ 0, & x_\lambda < 0 \end{cases}$, then $[x_\lambda]_+ = \frac{1}{2}(|x_\lambda| + x_\lambda)$, we have

$$\begin{aligned}
 Mx_\lambda + \lambda [x_\lambda]_+ - q &= Mx_\lambda + \lambda \frac{1}{2}(|x_\lambda| + x_\lambda) - q = \left(M + \lambda \frac{1}{2}I\right)x_\lambda + \lambda \frac{1}{2}|x_\lambda| - q = 0 \\
 (2M + \lambda I)x_\lambda + \lambda |x_\lambda| - 2q &= 0
 \end{aligned}$$

By [14], its equivalent to

$$\begin{aligned}
 0 &\leq (2M + \lambda I)x_\lambda - \lambda x_\lambda - 2q \perp (2M + \lambda I)x_\lambda + \lambda x_\lambda - 2q \geq 0 \\
 0 &\leq Mx_\lambda - q \perp (M + \lambda I)x_\lambda - q \geq 0
 \end{aligned} \tag{3.5}$$

Let $z = Mx_\lambda - q$ then $(M + \lambda I)x_\lambda - q = (M + \lambda I)M^{-1}(z + q) - q = (M + \lambda I)M^{-1}z + \lambda M^{-1}q$. \square

Proposition 2 $Mx_\lambda + \lambda [x_\lambda]_+ = q$ has a unique solution if and only if the interval matrix $[M, M + \lambda I]$ is regular.

Proof: Since $M, M + \lambda I \in [M, M + \lambda I]$, by theorem 1.2 of [12], if $[M, M + \lambda I]$ is regular, then $(M + \lambda I)M^{-1}$ is P-matrix, which implies that the LCP has a unique solution for any $q \in R^n$ [1], from the relation between the (3.1) and the LCP (3.5), we can easily deduce that the (3.1) is uniquely solvable for any $q \in R^n$. \square

Algorithm 3

Step 1: Choose an arbitrary initial point $x_{\lambda_0}^0 \in R^n$, $\varepsilon > 0$ and given $\lambda_0 > 1$, $\mu > 1$, $\sigma \in [0, 1]$, let $k := 0$;

Step 2: for the λ_k , computer $x_{\lambda_k}^{i+1}$ by solving

$$\left[M + \lambda_k D(x_{\lambda_k}^i)\right]x_{\lambda_k}^{i+1} = b \tag{3.6}$$

Step 3: If $\|x_{\lambda_k}^{i+1} - x_{\lambda_k}^i\| < \varepsilon$, let $\bar{x}_k = x_{\lambda_k}^i$ go to step 4. Otherwise, $i = i + 1$ go to step 2.

Step4: If $\|\bar{x}_{k+1} - \bar{x}_k\| < \varepsilon$ and $\|M\bar{x}_k - q\| < \varepsilon$, terminate, $\bar{x} = \bar{x}_k$ is solution of LCP. Otherwise let $\lambda_{k+1} = \mu\lambda_k$, $x_{\lambda_{k+1}}^0 = \bar{x}_k$ let $k := k + 1$, go to 2.

4. The Convergence of the Algorithm

We will show that the sequence $\{x_{\lambda_k}^i\}_{i=1}^\infty$ generated by the generalized Newton iteration (3.6) converges to an accumulation point \bar{x}_k associated with λ_k . First, we establish boundness of the sequence $\{x_{\lambda_k}^i\}$ for any $\lambda_k > 0$ generated by the Newton iterates (3.6) and hence the existence of accumulation point at each generalized Newton iteration.

Theorem 3: Suppose that the interval matrix $[M, M + \lambda_k I]$ is regular. Then, the sequence $\{x_{\lambda_k}^i\}$ generated by Algorithm 3 is bounded. Consequently, there exists an accumulation points \bar{x}_k such that $[M + \lambda_k D(\bar{x}_k)]\bar{x}_k = b$.

Proof: Suppose that sequence $\{x_{\lambda_k}^i\}$ is unbounded, Thus, there exists an infinite nonzero subsequence $\{x_{\lambda_k}^{i_j}\} \subset \{x_{\lambda_k}^i\}$ such that

$$\{x_{\lambda_k}^{i_j}\} \rightarrow \infty, D(x_{\lambda_k}^{i_j}) = \tilde{D} \text{ and } \tilde{D} \in [0, I]$$

where \tilde{D} is main diagonal element of diagonal matrix which is $1, 0, \sigma \in [0, 1]$.

We know subsequence $\left\{\frac{x_{\lambda_k}^{i_j}}{\|x_{\lambda_k}^{i_j}\|}\right\}$ is bounded. Hence, exists convergence subsequence and assume that convergence point is \tilde{x} , and satisfy

$$(A + \lambda_k \tilde{D}) \frac{x_{\lambda_k}^{ij}}{\|x_{\lambda_k}^{ij}\|} = \frac{b}{\|x_{\lambda_k}^{ij}\|}$$

Letting $j \rightarrow \infty$ yields

$$(M + \lambda_k \tilde{D}) \tilde{x} = 0, \|\tilde{x}\| = 1.$$

Since $M + \lambda_k \tilde{D} \in [M, M + \lambda_k I]$ and the interval matrix $[M, M + \lambda_k I]$ is regular, we know that $(M + \lambda_k \tilde{D})^{-1}$ is exists and hence $\tilde{x} = 0$, contradicting to the fact that $\|\tilde{x}\| = 1$. Consequently, the sequence $\{x_{\lambda_k}^i\}$ is bounded and there exists an accumulation point \bar{x}_k of $\{x_{\lambda_k}^i\}$ such that

$$\bar{x}_k = (M + \lambda_k D(\bar{x}_k))^{-1} b \quad \square$$

Under a somewhat restrictive assumption we can establish finite termination of the generalized Newton iteration at a penalized equation solution as follows.

Theorem 4: Suppose that the interval matrix $[M, M + \lambda_k I]$ is regular and $\left\| (M + \lambda_k D(x_{\lambda_k}^i))^{-1} \right\| < \frac{1}{2\lambda_k}$ holds for all sufficiently large λ_k , then the generalized Newton iteration (3.6) linearly converges from any starting point $x_{\lambda_k}^0$ to a solution \bar{x}_k of the nonlinear penalized equation (3.1).

Proof: Suppose that \bar{x}_k is a solution of nonlinear penalized equation. By the lemma 1, $M + \lambda_k D(x_{\lambda_k}^i)$ is nonsingular. To simply notation, let $\bar{D} = D(\bar{x}_k)$, $D^i = D(x_{\lambda_k}^i)$ and since $(M + \lambda_k D(\bar{x}_k))\bar{x}_k = b$,

$$[M + \lambda_k D(x_{\lambda_k}^i)] x_{\lambda_k}^{i+1} = b \text{ have } \bar{D}\bar{x}_k = [\bar{x}_k]_+, \quad D^i x_{\lambda_k}^i = [x_{\lambda_k}^i]_+, \text{ hence}$$

$$\begin{aligned} M(x_{\lambda_k}^{i+1} - \bar{x}_k) &= -\lambda_k D^i x_{\lambda_k}^{i+1} + b + \lambda_k \bar{D}\bar{x}_k - b = -\lambda_k (D^i x_{\lambda_k}^{i+1} - [\bar{x}_k]_+) \\ &= -\lambda_k (D^i (x_{\lambda_k}^{i+1} - x_{\lambda_k}^i + x_{\lambda_k}^i) - [\bar{x}_k]_+) \\ &= -\lambda_k ([x_{\lambda_k}^i]_+ - [\bar{x}_k]_+) - \lambda_k D^i (x_{\lambda_k}^{i+1} - \bar{x}_k + \bar{x}_k - x_{\lambda_k}^i) \\ &= -\lambda_k ([x_{\lambda_k}^i]_+ - [\bar{x}_k]_+) - \lambda_k D^i (\bar{x}_k - x_{\lambda_k}^i) - \lambda_k D^i (x_{\lambda_k}^{i+1} - \bar{x}_k) \\ (M + \lambda_k D^i)(x_{\lambda_k}^{i+1} - \bar{x}_k) &= -\lambda_k ([x_{\lambda_k}^i]_+ - [\bar{x}_k]_+) - \lambda_k D^i (\bar{x}_k - x_{\lambda_k}^i) \\ (x_{\lambda_k}^{i+1} - \bar{x}_k) &= (M + \lambda_k D^i)^{-1} (-\lambda_k ([x_{\lambda_k}^i]_+ - [\bar{x}_k]_+) + \lambda_k D^i (x_{\lambda_k}^i - \bar{x}_k)) \end{aligned}$$

since $\left\| (M + \lambda_k D(x_{\lambda_k}^i))^{-1} \right\| < \frac{1}{2\lambda_k}$ and by the lemma 2, we have

$$\begin{aligned} \|(x_{\lambda_k}^{i+1} - \bar{x}_k)\| &\leq \|(M + \lambda_k D^i)^{-1}\| (2\lambda_k \|(x_{\lambda_k}^i - \bar{x}_k)\|) \\ &= 2\lambda_k \cdot \|(M + \lambda_k D^i)^{-1}\| \|(x_{\lambda_k}^i - \bar{x}_k)\| < \|(x_{\lambda_k}^i - \bar{x}_k)\| \end{aligned} \quad (4.2)$$

Letting $i \rightarrow \infty$ and taking limits in both sides of the last inequality above, we have

$$\frac{\|(x_{\lambda_k}^{i+1} - \bar{x}_k)\|}{\|(x_{\lambda_k}^i - \bar{x}_k)\|} < 1$$

So the iteration (3.6) linearly converges to a solution \bar{x}_k of nonlinear penalized equation (3.1). In here, we will focus on the convergence of Algorithm 3. □

Theorem 5: Suppose M is P-Matrix and that the interval matrix $[M, M + \lambda_k I]$ is regular and

$\left\| \left(M + \lambda_k D(x_{\lambda_k}^i) \right)^{-1} \right\| < \frac{1}{2\lambda_k}$ holds, then Algorithm 3 linearly converges from any starting point $x_{\lambda_0}^0$ to a solution x^* of the $LCP(M, q)$ (1.1).

Proof: Since M is P-Matrix, then the $LCP(M, q)$ has a unique solution, let the solution denote x^* , by the assumptions of the theorem, the generalized Newton iteration (3.6) linearly converges to a solution \bar{x}_k of the nonlinear penalized equation (3.1).

$$\left\| \bar{x}_{\lambda_{k+1}} - x^* \right\| = \left\| \bar{x}_{\lambda_{k+1}} - x_{\lambda_{k+1}}^i + x_{\lambda_{k+1}}^i - x_{\lambda_k}^i + x_{\lambda_k}^i - x^* \right\| \leq \left\| \bar{x}_{\lambda_{k+1}} - x_{\lambda_{k+1}}^i \right\| + \left\| x_{\lambda_{k+1}}^i - x_{\lambda_k}^i \right\| + \left\| x_{\lambda_k}^i - x^* \right\|$$

let $i \rightarrow \infty$, then

$$\left\| \bar{x}_{\lambda_{k+1}} - x^* \right\| \leq \left\| \bar{x}_{\lambda_{k+1}} - \bar{x}_{\lambda_k} \right\| + \left\| \bar{x}_{\lambda_k} - x^* \right\|$$

we have $\lim_{k \rightarrow \infty} \frac{\left\| \bar{x}_{\lambda_{k+1}} - x^* \right\|}{\left\| \bar{x}_{\lambda_k} - x^* \right\|} \leq 1$ □

5. Numerical Experiments

In this section, we give some numerical results in order to show the practical performance of Algorithm 2.1. Numerical results were obtained by using Matlab R2007 (b) on a 1G RAM, 1.86 Ghz Intel Core 2 processor. Throughout the computational experiments, the parameters were set as $\varepsilon = 1.0e-8$, $\lambda_0 = 10$, $\mu = 2$.

Example 1: The matrix A of linear complementarity problem $LCP(A, b)$ of as follows (This example appear in the Geiger and Kanzow [15], Jiang and Qi [16], YONG Long-quan, DENG Fang-an, CHEN Tao [17]):

$$A = \begin{pmatrix} 4 & -1 & 0 & \cdots & 0 & 0 \\ -1 & 4 & -1 & \cdots & 0 & 0 \\ 0 & -1 & 4 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 4 & -1 \\ 0 & 0 & 0 & \cdots & -1 & 4 \end{pmatrix}, \quad b = (-1, -1, \dots, -1, -1)^T$$

The computational results are shown in **Table 1**. This x^0 is initial point, k is number of inner iterations, the outer iteration number is m , x^* is iteration results.

Example 2: The matrix A of linear complementarity problem $LCP(A, b)$ of as follows (This example appear in the Geiger and Kanzow [15], Jiang and Qi [16], YONG Long-quan, DENG Fang-an, CHEN Tao [17]):

$$A = \text{diag} \left(\frac{1}{n}, \frac{2}{n}, \dots, 1 \right), \quad b = (-1, -1, -1, \dots, 1, -1)^T$$

The computational results are shown in **Table 2**. This x^0 is initial point, k is number of inner iterations, the outer iteration number is m , x^* is iteration results.

Table 1. Result from example 1.

n	x^0	k	m	$x^* = (\bar{x}_1, \bar{x}_2, \dots, \bar{x}_n)^T$
6	$(0, 0, 0, 0, 0, 0)^T$	3	2	$(-0.3659, -0.4634, -0.4878, -0.4878, -0.4634, -0.3659)^T$
6	$(-2, 0, -2, -2, 0, -2)^T$	3	2	$(-0.3659, -0.4634, -0.4878, -0.4878, -0.4634, -0.3659)^T$
7	$(0, 0, 0, 0, 0, 0, 0)^T$	3	2	$(-0.3659, -0.4639, -0.4896, -0.4948, -0.4896, -0.4639, -0.3660)^T$
7	$(1, 1, 1, 1, 1, 1, 1)^T$	3	2	$(-0.3659, -0.4639, -0.4896, -0.4948, -0.4896, -0.4639, -0.3660)^T$

Table 2. Result from example 2.

n	x^0	k	m	$x^* = (\bar{x}_1, \bar{x}_2, \dots, \bar{x}_n)^T$
6	$(1, -1, 1, -1, 1, -1)^T$	3	2	$(-6, -3, -2, -1.5, -1.2, -1)^T$
6	$(0, 0, 0, 0, 0, 0)^T$	3	2	$(-6, -3, -2, -1.5, -1.2, -1)^T$
8	$(0, 0, 0, 0, 0, 0, 0, 0)^T$	3	2	$(-8, -4, -2.67, -2, -1.6, -1.34, -1.14, -1)^T$
8	$(1, -1, 1, -1, 1, -1, 1, -1)^T$	3	2	$(-8, -4, -2.67, -2, -1.6, -1.34, -1.14, -1)^T$
16	$(1, -1, 1, -1, \dots, 1, -1)^T$	3	2	$(-16, -8, -5.3, -4, -3.2, -2.67, -2.28, -2, -1.78, -1.6, -1.45, -1.34, -1.23, -1.14, -1.06, -1)^T$
16	$(0, 0, 0, 0, \dots, 0, 0)^T$	3	2	$(-16, -8, -5.3, -4, -3.2, -2.67, -2.28, -2, -1.78, -1.6, -1.45, -1.34, -1.23, -1.14, -1.06, -1)^T$

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