

1-A Cosmological Model with Varying G and Λ in General Relativity

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Received 2013

ABSTRACT

In this paper homogeneous Bianchi type -I space-time with variable G and Λ containing matter in the form of a perfect fluid assuming the cosmological term proportional to H^2 (where H is Hubble Parameter). Initially the model has a point type singularity, gravitational constant G (t) is decreasing and cosmological constant Λ is infinite at this time. When time increases, Λ decrease. The model does not approach isotropy, if it is small. The model is quasi-isotropic for large value of it.

Keywords: Bianchi Type-I Universe; Varying G and Λ ; Cosmology; Hubble Parameter

1. Introduction

Cosmology is study of large scale structures of universe. The simplest homogeneous and anisotropic model is Bi-anchi type -I cosmological model which is generalization of Friedmann-Robertson-Walker (FRW) model. Cosmological model with cosmological constant are serious participants to describe the dynamics of the universe. The origin of universe is greatest cosmological mystery even today. As we are aware that the expansion of the universe is undergoing time acceleration Perlmutter *et al.*, (1997,1998,1999), Riess *et al.*, (1998,2004), Allen *et al.*, (2004), Peebles *et al.*, (2003), Padmanabhan. (2003) & Lima. (2004). To resolve the problem of a huge difference between the effective cosmological constant observed today and the vacuum energy density predicted by the quantum field theory, several mechanisms have been proposed by (Weinberg, 1989). A possible way is to consider a varying cosmological term due to the coupling of dynamic degree of freedom with the matter fields of the universe.

The cosmological constant is small because the universe is old. Models with dynamically decaying cosmological term representing the energy density of vacuum have been studied by Vishwakarma, R. G. (2000, 2001,2005), Arbab, A. I. (1998), Berman, M. S. (1991a, 1991b). Cosmological scenarios with a time varying cosmological constant were proposed by several researchers. A number of models with different decay laws for the variation of cosmological term were investigated during the last two decades Chen & Wu (1990); Pavan

(1991); Carvalho *et al.*, (1992); Lima & Maia (1994); Lima & Trodden (1996); Arbab & Abdel-Rahman (1994); Cunha & Santos (2004); Carneiro & Lima (2005). A number of authors investigated Bianchies models, using the approach that there is link between variation of gravitational constant and cosmological constant [Abdel-Rahman 1990; Berman 1991a; Kalligas *et al.* 1992; Abdussattar and Vishkarma 2005; Pradhan *et al.* 2006; Singh and Tiwari 2007]. Lot of work has been done by Saha (2005a, 2005b, 2006a, 2006b), in studying the anisotropic Bianchi type-I Cosmological Model in general relativity with varying G and Λ .

In this paper we study homogeneous Bianchi type -I space-time with variable G and Λ containing matter in the form of a perfect fluid. We obtain solution of the Einstein field equations assuming the cosmological term proportional to H^2 (where H is Hubble Parameter).

1.1. The Metric and Field Equations

We consider the Bianchi type - I metric in the orthogonal form

$$ds^2 = -dt^2 + A^2(t) dx^2 + B^2(t) dy^2 + C^2(t) dz^2 \quad (1)$$

The non-zero components of the Ricci tensor R_{ij} .

We assume that cosmic matter is taken to be perfect fluid given by the energy- momentum tensor

$$T_{ij} = (\rho + p) v_i v_j + p g_{ij} \quad (2)$$

where p , ρ are the isotropic pressure and energy density of the fluid. We take equation of state

$$p = w \rho, 0 \leq w \leq 1. \tag{2^1}$$

v_i is four velocity vector of the fluid satisfying

$$g_{ij} v^i v^j = -1 \tag{3}$$

Einstein's field equations with time dependent G and Λ are

$$R_{ij} - \frac{1}{2} R_1 g_{ij} = -8\pi G(t) T_{ij} + \Lambda(t) g_{ij} \tag{4}$$

For the metric (1) and energy-momentum tensor (2) in commoving system of coordinates, the field equation (4) yields.

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = -8\pi G\rho + \Lambda \tag{5}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} = -8\pi G\rho + \Lambda \tag{6}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -8\pi G\rho + \Lambda \tag{7}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} = 8\pi G\rho + \Lambda \tag{8}$$

In view of vanishing of the divergence of Einstein tensor, we have

$$8\pi G \left[\dot{\rho} + (\rho + p) \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) \right] + 8\pi \rho \dot{G} + \dot{\Lambda} = 0 \tag{9}$$

The usual energy conservation equation of general relativity quantities is

$$\dot{\rho} + (\rho + p) \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) = 0 \tag{10}$$

Equation (9) together with (10) puts G and Λ in some sort of coupled field given by

$$8\pi \rho \dot{G} + \dot{\Lambda} = 0 \tag{11}$$

implying that Λ is a constant whenever G is constant. Using equation (2¹) in equation (10) and then integrating, we get $k > 0$

$$\rho = \frac{k}{R^2} \tag{11^1}$$

We define, R as the average scale factor of Bianchi type- I universe.

$$R = (ABC)^{1/3} \tag{12}$$

The Hubble parameter H , volume expansion θ , shear σ

and deceleration parameter q are given by

$$\theta = 3H = \frac{3\dot{R}}{R}, \quad \sigma = \frac{k}{\sqrt{3}R^3}, \quad k > 0 \text{ (Constant)}$$

$$q = -1 - \frac{\dot{H}}{H^2} = -\frac{R\ddot{R}}{\dot{R}^2}$$

Einstein's field equations (5)-(8) can be also written in terms of Hubble parameter H , shear σ and deceleration parameter q as

$$H^2(2q - 1) - \sigma^2 = 8\pi G\rho - \Lambda \tag{13}$$

$$3H^2 - \sigma^2 = 8\pi G\rho + \Lambda \tag{14}$$

On integrating (5) - (8), we obtain

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = \frac{k_1}{R^3} \tag{15}$$

and

$$\frac{\dot{B}}{B} - \frac{\dot{C}}{C} = \frac{k_2}{R^3} \tag{16}$$

Where k_1 and k_2 are constants of integration. From (14), we obtain

$$\frac{3\sigma^2}{\theta^2} = 1 - \frac{24\pi G\rho}{\theta^2} - \frac{3\Lambda}{\theta^2} \tag{17}$$

Implying that $\Lambda \geq 0$

$$0 < \frac{\sigma^2}{\theta^2} < \frac{1}{3}, 0 < \frac{8\pi G\rho}{\theta^2} < \frac{1}{3}$$

Thus the presence of positive Λ lowers the upper limit of anisotropy whereas a negative Λ contributes to the anisotropy.

Equation (17) can also be written as

$$\frac{\sigma^2}{3H^2} = 1 - \frac{8\pi G\rho}{3H^2} - \frac{\Lambda}{3H^2} = 1 - \frac{\rho}{\rho_c} - \frac{\rho_v}{\rho_c} \tag{18}$$

where $\rho_c = \frac{3H^2}{8\pi G}$ is the critical density and

$\rho_v = \frac{\Lambda}{8\pi G}$ is the vacuum density.

From (13) and (14), we get,

$$\frac{d\theta}{dt} = -12\pi G\rho - \frac{\theta^2}{2} + \frac{3\Lambda}{2} - \frac{3}{2}\sigma^2 = -12\pi G(\rho + p) - 3\sigma^2 \tag{19}$$

Thus the universe will be in decelerating phase for negative Λ and for positive Λ universe will slow down the rate of decrease showing that the rate of volume expansion decreases during time evolution and presence of positive Λ , slows down the rate of this decrease whereas a negative Λ would promote it.

1.2. Solution of the Field Equation

The system of equations (5)-(8) and (11) supply only five equations in seven unknown parameters (A, B, C, ρ , Λ , and G). Two extra equations are needed to solve the system completely. For this purpose we take cosmological term is proportional to H^2 .

$$\text{i.e. } \Lambda = H^2 \tag{20}$$

This variation law was proposed by Olson, *et al.*, (1987), Pavon(1991), Maia, *et al.*, (1994); Silveira, *et al.*, (1994, 1997), Torres, *et al.*,(1996). Because observations suggest that Λ is very small in the present universe, a decreasing functional form permits Λ to be large in early universe.

Using equation (11¹) and equation (20) in equation (11) we get

$$G = \frac{-e^{\frac{3t_0}{2}}}{8k\pi\sqrt{t}} \tag{21}$$

From equations (13), (14), (20) and (20¹) we get

$$H' = -2H^2 \tag{22}$$

Find the time evolution of Hubble parameter, integrate (22), we get

$$\frac{\dot{R}}{R} = H = \frac{1}{2t} \tag{23}$$

where t_0 is a constant of integration. The integration constant is related to the choice of origin of time.

From equation (23), we obtain the scale factor

$$R = e^{\frac{t_0}{2}} \times \sqrt{t} \tag{24}$$

By using equation (24) in (15) and (16), the metric (1), we get

$$ds^2 = R^2 \left[m_1^2 \exp\left\{\left[\frac{-2}{M\sqrt{t}}\right]\left[\frac{2k_1+k_2}{3}\right]\right\} dx^2 + R^2 \left[m_2^2 \exp\left\{\left[\frac{-2}{M\sqrt{t}}\right]\left[\frac{k_2-k_1}{3}\right]\right\} dy^2 \right. \right. \\ \left. \left. + R^2 \left[m_3^2 \exp\left\{\left[\frac{-2}{M\sqrt{t}}\right]\left[\frac{-2k_2-k_1}{3}\right]\right\} dz^2 \right] \right] \tag{25}$$

where $M = \frac{3t_0}{2}$ and m_1, m_2 and m_3 are constants. For the model (25), the spatial V, density ρ , gravitational constant G and cosmological constant Λ are

$$V = R^3 = \left[e^{\frac{t_0}{2}} \times \sqrt{t} \right]^3 \tag{26}$$

$$\rho = k / \left[e^{\frac{t_0}{2}} \times \sqrt{t} \right]^3 \tag{27}$$

$$G = \frac{-e^{\frac{3t_0}{2}}}{8k\pi\sqrt{t}} \tag{28}$$

$$\Lambda = \frac{1}{4t^2} \tag{29}$$

Expansion scalar θ and shear σ are

$$\theta = \frac{3}{2t} \tag{30}$$

$$\sigma = \frac{k}{e^{\frac{3t_0}{2}} \sqrt{3}} \times \frac{1}{t^{\frac{3}{2}}} \tag{31}$$

$$R = e^{\frac{t_0}{2}} \times \sqrt{t} \tag{32}$$

$$q = 1 \tag{33}$$

$$\Omega = \frac{e^{\frac{3t_0}{2}}}{4kt} \tag{34}$$

$$\frac{\sigma}{\theta} = \frac{2k}{3\sqrt{3}} \times \frac{1}{e^{\frac{3t_0}{2}}} \times \frac{1}{\sqrt{t}} \tag{35}$$

2. Observations and Conclusions

1. Thus we observe that as spatial volume $V \rightarrow 0$ at $t = 0$ and expansion scalar θ is infinite, which shows that universe starts evolving with zero volume at $t = 0$ infinite rate of expansion. Hence the model has a point type singularity at initial epoch.

2. Initially at $t = 0$ the energy density ' ρ ', pressures ' p ', shear σ , cosmological term Λ tend all infinite.

3. As t increases the spatial volume increases but the expansion rate decreases. Thus the rate of expansion slows down with increase in time and tend to zero.

4. As $t \rightarrow \infty$ the spatial volume V becomes infinitely large. All parameters $\theta, \rho, p, \Omega, \sigma, \Lambda \rightarrow 0$ asymptotically but G is decreasing. Therefore at large value of t model gives empty universe. The cosmic scenario starts from a big bang at $t = 0$ and continues until $t = \infty$.

5. The ratio $\sigma/\theta \rightarrow 0$ as $t \rightarrow \infty$. So the model approach isotropy for large value of t .

6. We also see for model (25), if $t \rightarrow \infty$ then $q = 2$ implies universe is decelerating as q is positive.

7. The possibility of G increasing with time, at least in some stages of the development of the universe, has been investigated by Abdel-Rahman (1990), Chow (1981), Levit (1980) and Milne (1935). $\Lambda \propto \frac{1}{T^2}$ Include

Berman (1990), Ber-man and Som (1990), Berman *et al.* (1989), and Bertolami (1986b, 1986a). This form of Λ is physically reasonable as observations suggest that Λ is very small in the present universe. A decreasing functional form permits Λ to be large in the early universe.

8. In summary, we observed on investigation that Bianchi type-I cosmological model with variable G and Λ in presence of perfect fluid with cosmological term is proportional to H^2 (H is Hubble Parameter) suggested by Silveira *et al.*, (1994,1997) and others. Initially the model has a point type singularity, gravitational constant $G(t)$ is decreasing and cosmological constant Λ is infinite at this time when time increases Λ decrease. The model approach isotropy for large value of t , the model is quasi-isotropic *i.e.* $\frac{\sigma}{\theta} = 0$.

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