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2-A Cosmological Model with Varying G and Λ in General Relativity

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ABSTRACT

Spatially homogeneous and anisotropic Cosmological models play a significant role in the description of the early stages of evolution of the universe. The problem of the cosmological constant is still unsettled. The authors recently considered time dependent G and Λ with Bianchi type–I Cosmological model .We considered in this paper homogeneous Bianchi type -I space-time with variable G and Λ containing matter in the form of a perfect fluid assuming the cosmological term proportional to R^{-2} (where R is scale factor). Initially the model has a point type singularity, gravitational constant G (*t*) is decreasing and cosmological constant Λ is infinite at this time. When time increases Λ decreases. Unlike in some earlier works we have neither assumed equation of state nor particular form of G. The model does not approach isotropy, if '*t*' is small .The model is quasi-isotropic for large value of '*t*'.

Keywords: Bianchi Type-I Universe; Varying G and Λ; Cosmology

1. Introduction

Cosmology is the scientific study of large scale properties of the universe as a whole. Cosmology is study of motion of crystalline objects. The origin of universe is greatest cosmological mystery even today. As we are aware that the expansion of the universe is undergoing time acceleration [Perlmutter *et al*., (1997,1998,1999), Riess *et al*., (1998,2004), Allen *et al*., (2004), Peebles *et al*., (2003), Padmanabhan. (2003) &Lima. (2004)]. Present universe is suitably represented by Friedmann-Robertson-Walker model which is isotropic and homogeneous in nature. To resolve the problem of a huge difference between the effective cosmological constant observed today and the vacuum energy density predicted by the quantum field theory, several mechanisms have been proposed by Weinberg (1989). A possible way is to consider a varying cosmological term due to the coupling of dynamic degree of freedom with the matter fields of the universe. The cosmological constant is small because the universe is old. Models with dynamically decaying cosmological term representing the energy density of vacuum have been studied by R. G. Vishwakarma, (2000,2001,2005), A. I. Arbab, (1998) and Berman (1991,1991b). Cosmological scenarios with a time varying cosmological constant were proposed by several researchers. A number of models with different decay laws for the variation of cosmological term were investigated during the last two decades[Chen & Wu (1990); Pavan (1991); Carvalho *et al*.,(1992); Lima & Maia(1994); Lima & Trodden (1996); Arbab & Abdel-Rahman(1994); Cunha & Santos (2004); Carneiro & Lima(2005)].

A lot of work has been done by Saha (2005a, 2005b, 2006a, 2006b), in studying the anisotropic Bianchi type-I Cosmological Model in general relativity with varying G and Λ. In this paper we study homogeneous Bianchi type -I space-time with variable G and Λ containing matter in the form of a perfect fluid. We obtain solution of the Einstein field equations assuming the cosmological term proportional to R^{-2} (where R is scale factor).

2. The Metric and Field Equations

We consider the Bianchi type - I metric in the orthogonal form

$$ds^2 = -dt^2 + A^2(t)\,dx^2 + B^2(t)\,dy^2 + C^2(t)\,dz^2 \quad (1)$$

The non-zero components of the Ricci tensor R_{ij}

We assume that cosmic matter is taken to be perfect fluid given by the energy- momentum tensor

$$T_{ij} = (\rho + p)\,v_i v_j + p g_{ij} \quad (2)$$

where p, ρ are the isotropic pressure and energy density of the fluid. We take equation of state

$$p = w\rho .\ 0 \le w \le 1. \qquad (2^1)$$

v^i is four velocity vector of the fluid satisfying

$$g_{ij}\ v^i v^j = -1 \qquad (3)$$

Einstein's field equations with time dependent G and Λ are

$$R_{ij} - \tfrac{1}{2} R^l_l g_{ij} = -8\pi G(t) T_{ij} + \Lambda(t) g_{ij} \qquad (4)$$

For the metric (1) and energy-momentum tensor (2) in commoving system of coordinates, the field equation (4) yields.

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = -8\pi G p + \Lambda \qquad (5)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{B} + \frac{\dot{A}\dot{C}}{AC} = -8\pi G p + \Lambda \qquad (6)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -8\pi G p + \Lambda \qquad (7)$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} = 8\pi G \rho + \Lambda \qquad (8)$$

In view of vanishing of the divergence of Einstein tensor, we have

$$8\pi G\left[\dot{\rho} + (\rho + p)\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right)\right] + 8\pi\rho\dot{G} + \dot{\Lambda} = 0 \qquad (9)$$

The usual energy conservation equation of general relativity quantities is

$$\dot{\rho} + (\rho + p)\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = 0 \qquad (10)$$

Equation (9) together with (10) puts G and Λ in some sort of coupled field given by

$$8\pi\rho\,\dot{G} + \dot{\Lambda} = 0 \qquad (11)$$

implying that Λ is a constant whenever G is constant. Using equation(2^1) in equation (10) and then integrating, we get $k > 0$

$$\rho = \frac{k}{R^2} \qquad (11^1)$$

We define, R as the average scale factor of Bianchi type-I universe.

$$R = (ABC)^{1/3} \qquad (12)$$

The Hubble parameter *H*, volume expansion *θ*, shear *σ* and deceleration parameter q are given by

$$\theta = 3H = \frac{3\dot{R}}{R} \quad \sigma = \frac{k}{\sqrt{3}R^3},\ k > 0\ (\text{Constant})$$

$$q = -1 - \frac{\dot{H}}{H^2} = -\frac{R\ddot{R}}{\dot{R}^2}$$

Einstein's field equations (5)-(8) can be also written in terms of Hubble parameter *H*, shear *σ* and deceleration parameter *q* as

$$H^2(2q-1) - \sigma^2 = 8\pi G p - \Lambda \qquad (13)$$

$$3H^2 - \sigma^2 = 8\pi G\rho + \Lambda \qquad (14)$$

On integrating (5) - (8), we obtain

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = \frac{k_1}{R^3} \qquad (15)$$

and

$$\frac{\dot{B}}{B} - \frac{\dot{C}}{C} = \frac{k_2}{R^3} \qquad (16)$$

where k_1 and k_2 are constants of integration. From (14), we obtain

$$\frac{3\sigma^2}{\theta^2} = 1 - \frac{24\pi G\rho}{\theta^2} - \frac{3\Lambda}{\theta^2} \qquad (17)$$

Implying that $\Lambda \ge 0$

$$0 < \frac{\sigma^2}{\theta^2} < \frac{1}{3}, 0 < \frac{8\pi G\rho}{\theta^2} < \frac{1}{3}$$

Thus the presence of positive Λ lowers the upper limit of anisotropy whereas a negative Λ contributes to the anisotropy.

Equation (17) can also be written as

$$\frac{\sigma^2}{3H^2} = 1 - \frac{8\pi G\rho}{3H^2} - \frac{\Lambda}{3H^2} = 1 - \frac{\rho}{\rho_c} - \frac{\rho_v}{\rho_c} \qquad (18)$$

where $\rho_c = \frac{3H^2}{8\pi G}$ is the critical density and $\rho_v = \frac{\Lambda}{8\pi G}$ is the vacuum density.

From (13) and (14), we get,

$$\frac{d\theta}{dt} = -12\pi G p - \frac{\theta^2}{2} + \frac{3\Lambda}{2} - \frac{3}{2}\sigma^2 = -12\pi G(\rho + p) - 3\sigma^2 \qquad (19)$$

Thus the universe will be in decelerating phase for negative Λ and for positive Λ universe will slow down the rate of decrease,showing that the rate of volume expansion decreases during time evolution and presence of positive Λ, slows down the rate of this decrease whereas a negative Λ would promote it.

3. Solution of the Field Equation

The system of equations (5)-(8) and (11) supply only five equations in seven unknown parameters (A, B, C,ρ, p, Λ, and G). Two extra equations are needed to solve the system completely. For this purpose we take cosmological term is proportional to R^{-2} , where 'a' is a positive constant.i.e we take the decaying vacuum energy density

$$\text{i.e.} \quad \Lambda = \frac{a}{R^2} \tag{20}$$

This variation law was proposed by Olson *et al.*, (1987), Pavon (1991), Maia *et al.*, (1994); Silveira *et al.*, (1994,1997) and Torres *et al.*,(1996).Because observations suggest that Λ is very small in the present universe, a decreasing functional form permits Λ to be large in early universe.

Using equation (11^1) and equation (20) in eq (11) we get

$$G = \frac{aR}{4k\pi} \tag{21}$$

From equations (13), (14), (20) and (20^1) we get

$$\frac{\ddot{R}}{R} + 2\left[\frac{\dot{R}}{R}\right]^2 - \frac{2a}{R^2} = 0 \tag{22}$$

Find the time evolution of Hubble parameter, integrate (22), we get

$$\frac{\dot{R}}{R} = H = \sqrt{a}\,[\sqrt{a}\;t + t_0]^{-1} \tag{23}$$

where t_0 is a constant of integration. The integration constant is related to the choice of origin of time.

From eq (23),we obtain the scale factor

$$R = [\sqrt{a}\;t + t_0] \tag{24}$$

By using equation (24) in (15) and (16) in the metric (1), we get

$$ds^2 = R^2\left[m_1^2 exp\left\{\left[\frac{-1}{\sqrt{a}R^2}\right]\left[\frac{2k_1+k_2}{3}\right]\right\}\right]dx^2 + R^2\left[m_2^2 exp\left\{\left[\frac{-1}{\sqrt{a}R^2}\right]\left[\frac{k_2-k_1}{3}\right]\right\}\right]dy^2 + R^2\left[m_3^2 exp\left\{\left[\frac{-1}{\sqrt{a}R^2}\right]\left[\frac{-2k_2-k_1}{3}\right]\right\}\right]dz^2 \tag{25}$$

where m_1,m_2 and m_3 are constants.

For the model (25), the spatial V, density ρ, gravitational constant G and cosmological constant Λ are

$$V = R^3 = [\sqrt{a}\;t + t_0]^3 \tag{26}$$

$$\rho = k/[\sqrt{a}\;t + t_0]^3 \tag{27}$$

$$G = \frac{aR}{4k\pi} = a\,[\sqrt{a}\;t + t_0]/4k\pi \tag{28}$$

$$\Lambda = a/[\sqrt{a}\;t + t_0]^2 \tag{29}$$

Expansion scalar θ and shear σ are

$$\theta = 3\sqrt{a}\,[\sqrt{a}\;t + t_0]^{-1} \tag{30}$$

$$\sigma = \frac{k}{\sqrt{3}\left[\sqrt{a}t + t_0\right]^3},\; k > 0\;(\text{Constant}) \tag{31}$$

$$q = -1/2 \tag{32}$$

4. Observations and Conclusion

1) Thus we observe that as spatial volume V→0 at $t = t^1$ where $t^1 = \frac{-t_0}{\sqrt{a}}$ and expansion scalar θ is infinite, which shows that universe starts evolving with zero volume at $t = t^1$ with an infinite rate of expansion.

2) The scale factors also vanish at $t = t^1$ and hence the model has a point type singularity at initial epoch. Initially at $t = t^1$ the energy density 'ρ', pressures 'p', shear σ, cosmological term Λ tend all infinite but G is finite.

3) As t increases the spatial volume increases but the expansion rate decreases. Thus the rate of expansion slows down with increase in time.

4) As $t\to\infty$ the spatial volume V becomes infinitely large. All parameters θ, ρ, p, σ, Λ → 0 asymptotically but G is decreasing. Therefore at large value of t model gives empty universe. The cosmic scenario starts from a big bang at $t = t^1$ and continues until $t = \infty$.If $t = t^1$ then gravitation constant is zero and as t increase G also increases.

The possibility of G increasing with time, at least in some stages of the development of the universe, has been investigated by Abdel-Rahman (1990), Chow (1981), Levit (1980) and Milne (1935). $\Lambda\alpha\frac{1}{T^2}$ Include Berman (1990), Berman and Som (1990), Berman *et al.*, (1989), and Bertolami (1986b,1986a). This form of Λ is physically reasonable as observations suggest that Λ is very small in the present universe. A decreasing functional form permits Λ to be large in the early universe.

5) The ratio σ/θ→0 as $t\to\infty$ .So the model approach isotropy for large value of t.

In summary, we have investigated the Bianchi type-I cosmological model with variable G and Λ in presence of perfect fluid with cosmological term proportional to R^{-2} (R is scale factor) suggested by Silveira *et al.*, (1994, 1997) and others. Initially the model has a point type singularity, gravitational constant G (t) is decreasing and cosmological constant Λ is infinite at this time when time increases Λ decrease.

It is interesting that Beesham (1994) , Lima and Carvalho (1994), Kalligas, *et al.*, (1995) and Lima (1996) have also derived the Bianchi type I cosmological models with variable G and Λ assuming a particular from of G and by taking equation of state .But we have neither assumed equation of state nor particular form of G. The model approach isotropy for large value of t, the model is quasi-isotropic i.e. $\frac{\sigma}{\theta} = 0$.

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