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Periodic Solution of Impulsive Lotka-Volterra Recurrent Neural Networks with Delays

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ABSTRACT

In this paper, periodic solution of impulsive Lotka-Volterra recurrent neural networks with delays is studied. Using the continuation theorem of coincidence degree theory and analysis techniques, we establish criteria for the existence of periodic solution of impulsive Lotka-Volterra recurrent neural networks with delays.

Keywords: Lotka-Volterra; Delays; Periodic Solution; Impulsive

1. Introduction

In recent years, applications of theory differential equations in mathematical ecology have been developed rapidly. Various mathematical models have been proposed in the study of population dynamics. The Lotka-Volterra competition system is the most famous models for dynamics of population. Owing to its theoretical and practical significance, the Lotka-Volterra systems have been studied extensively [1,2]. The Lotka-Volterra type neural networks, derived from conventional membrane dynamics of competing neurons, provide a mathematical basis for understanding neural selection mechanisms. Recently, periodic solutions of impulsive Lotka-Volterra recurrent neural networks have been reported.

It is well known that delays are important phenomenon in neural networks [3]. Thus, studying the dynamic properties of neural networks with delays has interesting implications in both theory and applications [4-7]. In this paper, we will study the following impulsive Lotka-Volterra recurrent neural networks system with delays:

$$\begin{cases} x_i'(t) = x_i(t)[h_i(t) - x_i(t) + \sum_{j=1}^{n} a_{ij}(t)x_j(t) \\ \qquad + c_i(t)x_i(t-\tau_i(t))], t \neq t_k, \\ \Delta x_i(t_k) = b_{ik}x_i(t_k), i = 1,2,\dots n, k = 1,2,\dots,n, \end{cases} \tag{1}$$

where each $x_i(t)$ denotes the activity of neuron, $A = (a_{ij})_{n\times n}$ is real $n \times n$ matrices, each of their elements denotes a synaptic weight and represents the strength of the synaptic connection from neuron *j* to neuron *i*, $h_i(t)$ denotes external inputs. The variable delays $\tau_i(t)$ for $i = 1,2,\dots,n$ are nonnegative continuous functions satisfying $0 \le \tau_i(t) \le \tau$ for $t \ge 0$, where $\tau \ge 0$ is a constant. $h_i(t)$, $a_{ij}(t)$, $c_i(t)$ are all positive periodic continuous functions with period $T > 0$.

2. Existence of Positive Periodic Solutions

Lemma 1 [8] Let *X* and *Y* be two Banach spaces. Consider an operator equation $Lx = \lambda Nx$ where $L: \mathrm{Dom}\, L \cap X \to Y$ is a Fredholm operator of index zero and $\lambda \in [0,1]$ is a parameter. Let *P* and *Q* denote two projectors such that $P: X \to \mathrm{Ker}\, L$ and $Q: Y \to Y/\mathrm{Im}\, L$. Assume that $N: \overline{\Omega} \to Y$ is *L*-compact on $\overline{\Omega}$, where Ω is open bounded in *X*. Furthermore, assume that

(a) For each $\lambda \in (0,1)$, $x \in \partial\Omega \cap \mathrm{Dom}\, L$, $Lx \neq \lambda Nx$,

(b) For each $x \in \partial\Omega \cap \mathrm{Ker}\, L$, $QNx \neq 0$,

(c) $\deg\{JQN, \Omega \cap \mathrm{Ker}\, L, 0\} \neq 0$, where

$J: \mathrm{Im}\, Q \to \mathrm{Ker}\, L$ is an isomorphism and $\deg\{*\}$ represents the Brouwer degree.

Then the equation $Lx = Nx$ has at least one solution in $\overline{\Omega} \cap \mathrm{Dom}\, L$.

For the sake of convenience, we introduce the following notation:

$$\overline{u} = \frac{1}{T}\int_0^T u(t)dt,\ g_i^l = \min_{t\in[0,T]} g_i(t),$$

$$g_i^u = \max_{t\in[0,T]} g_i(t),\ (i = 1,2,\dots,n),$$

$$PC(J,R) = \left\{ \begin{array}{l} x: J \to R \big| x(t) \text{ is continuous} \\ \text{with respect to } t \neq t_1,\dots,t_p; x(t^+) \\ \text{ and } x(t^-) \text{ exsit at } t_1,\dots,t_p; \\ \text{and } x(t_k) = x(t_k^+), k = 1,2,\dots,p \end{array} \right\},$$

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where $u(t)$, $g(t)$ are *T*-periodic functions.

Lemma 2 $z_i(t)$ is an *T*-periodic solution of (1) if and only if $\ln\{z_i(t)\}$ is an T-periodic solution of

$$\begin{cases} z_i'(t) = h_i(t) - \exp\{z_i(t)\} + \sum_{j=1}^{n} a_{ij}(t)\exp\{z_j(t)\} \\ \quad + c_i(t)\exp\{z_i(t-\tau_i(t))\}, t \neq t_k, \\ \Delta x_i(t_k) = \ln(1+b_{ik}), i = 1,2,\ldots n, k = 1,2,\ldots,n. \end{cases} \quad (2)$$

where $\ln\{z_i(t)\} = (\ln\{z_1(t)\}, \ln\{z_2(t)\}, \ldots, \ln\{z_n(t)\})$

Now we are ready to state and prove the main results of the present paper.

Theorem Assume that $\bar{a}_{ij} + \bar{c}_i < 1$, then system (1) has at least one *T*-periodic solution.

Proof. To complete the proof, we only need to search for an appropriate open bounded subset verifying all the requirements in Lemma 1.

Let

$$z = (z_1(t), z_2(t), \ldots, z_n(t))^T,$$
$$Z = \{z \in PC(R, R^n) \mid z(t+T) = z(t)\},$$
$$Y = Z \times R^{2p},$$

then it is standard to show that both *Z* and *Y* are Banach space when they are endowed with the norms

$$\| z \|_c = \sup_{t\in[0,T]} | z(t) |$$

and

$$\| (z, c_1, \ldots, c_p) \| = (\| z \|_p^2 + | c_1 |^2 + \cdots + | c_p |^2)^{\frac{1}{2}}.$$

Set $L : \mathrm{Dom}\, L \to Y$ as

$$(Lz)(t) = (z'(t), \Delta z(t_1), \ldots, \Delta z(t_p)),$$

where $\mathrm{Dom}\, L = Z = \{z \in Z \mid z'(t) \in PC(R, R^n)\}$.

At the same time, we denote $N : Z \to Y$ as

$$(Nz)(t) = ((h_i(t) - \exp\{z_i(t)\} + \sum_{j=1}^{n} a_{ij}(t)\exp\{z_j(t)\} + c_i(t)\exp\{z_i(t-\tau_i(t))\}), (I_1, \ldots I_p))$$

It is easily to prove that *L* is a Fredholm mapping of index zero.

Consider the operator equation

$$Lz = \lambda Nz \quad \lambda \in (0,1). \quad (3)$$

Integrating (3) over the interval $[0,T]$, we obtain

$$\begin{aligned} \bar{h}_i T = & -\sum_{k=1}^{p} \ln(1+b_{ik}) + \int_0^T \exp\{z_i(t)\}dt \\ & -\int_0^T \sum_{j=1}^{n} a_{ij}(t)\exp\{z_j(t)\}dt \\ & -\int_0^T c_i(t)\exp\{z_i(t-\tau_i(t))\}dt, \\ & (i = 1,2,\ldots,n). \end{aligned} \quad (4)$$

Then, we can derive

$$\int_0^T | z_i'(t) | \le 2\bar{h}_i T + \sum_{k=1}^{p} \ln(1+b_{ik}), (i = 1,2,\ldots,n).$$

Since $z_i(t) \in PC([0,T], R^n)$, there exist ξ_i, $\eta_i \in [0,T] \cap [t_1^+, t_2^+, \ldots, t_p^+]$, such that

$$z_i(\xi_i) = \inf_{t\in[0,T]} z_i(t),\ z_i(\eta_i) = \sup_{t\in[0,T]} z_i(t),\ (i = 1,2,\ldots,n).$$

For (4) we can see

$$\begin{aligned} \bar{h}_i T \le & -\sum_{k=1}^{p} \ln(1+b_{ik}) + \int_0^T \exp\{z_i(t)\}dt \\ & -\int_0^T a_{ii}(t)\exp\{z_i(t)\}dt \\ & -\int_0^T c_i(t)\exp\{z_i(t-\tau_i(t))\}dt \\ \le & -\sum_{k=1}^{p} \ln(1+b_{ik}) + \int_0^T \exp\{z_i(\eta_i)\}dt \\ & -\int_0^T a_{ii}(t)\exp\{z_i(\eta_i)\}dt \\ & -\int_0^T c_i(t)\exp\{z_i(\eta_i)\}dt \\ \le & -\sum_{k=1}^{p} \ln(1+b_{ik}) - (\bar{a}_{ii} + \bar{c}_i - 1)\exp\{z_i(\eta_i)\}T, \end{aligned}$$

which implies $z_i(\eta_i) \ge \ln\left[\dfrac{\bar{h}_i + \frac{1}{T}\sum_{k=1}^{p}\ln(1+b_{ik})}{1 - \bar{a}_{ii} - \bar{c}_i}\right] := A$

Thus, $\forall t \in [0,T]$, we have

$$z_i(t) \ge z_i(\eta_i) + \sum_{k=1}^{p} \ln(1+b_{ik}) - \int_0^T | z_i'(t) | dt \ge A - 2\bar{h}_i T := M.$$

Similarly, according to (4), we have

$$\begin{aligned} \bar{h}_i T \ge & -\sum_{k=1}^{p} \ln(1+b_{ik}) + \int_0^T \exp\{z_i(t)\}dt \\ & -\int_0^T \sum_{j=1}^{n} a_{ij}(t)\exp\{M\}dt \\ & -\int_0^T c_i(t)\exp\{z_i(t-\tau_i(t))\}dt \\ \ge & -\sum_{k=1}^{p} \ln(1+b_{ik}) + \int_0^T \exp\{z_i(\xi_i)\}dt \\ & -\int_0^T \sum_{j=1}^{n} a_{ij}(t)\exp\{M\}dt \\ & -\int_0^T c_i(t)\exp\{z_i(\xi_i)\}dt \\ \ge & -\sum_{k=1}^{p} \ln(1+b_{ik}) - \sum_{j=1}^{n} \bar{a}_{ij}(t)\exp\{M\}T \\ & + (1 - \bar{c}_i)\exp\{z_i(\xi_i)\}T, \end{aligned}$$

which implies,

$$z_i(\xi_i) \le \ln\left[\frac{\bar{h}_i + \frac{1}{T}\sum_{k=1}^{p}\ln(1+b_{ik}) + \sum_{j=1}^{n}\bar{a}_{ij}\exp\{M\}}{1-\bar{c}_i}\right]$$

$$:= B.$$

Thus, $\forall t \in [0,T]$, we have

$$z_i(t) \le z_i(\xi_i) + \sum_{k=1}^{p}\ln(1+b_{ik}) + \int_0^T |z_i'(t)|dt$$

$$\le B + 2\bar{h}_i T + 2\sum_{k=1}^{p}\ln(1+b_{ik}) := N.$$

Now, we can derive

$$|z_i(t)| \le \max\{|M|,|N|\} := M_1.$$

Obviously, M_1 is independent of λ. Then, there exists a constant $F>0$, such that $\max\{|z_i|\} \le F$. Let $r > M_1 + F$, $\Omega = \{z \in Z : \|z\|_c < r\}$, then it is clear that Ω satisfies condition (a) of Lemma1 and N is L-compact on $\bar{\Omega}$. Let $J: \operatorname{Im} Q \to x, (d,0,\ldots,0) \to d$, a direct computation gives

$$\deg\{JQN, \Omega \cap KerL, 0\} \ne 0.$$

By now we have proved that Ω satisfies all the requirements in Mawhin's continuation theorem (Lemma 1). Hence, system (2) has at least one T-periodic solution $z(t) = (z_1(t), z_2(t), \ldots, z_n(t))^T$ in $\operatorname{Dom} L \cap \bar{\Omega}$. The proof is completes.

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REFERENCES

[1] Z. Jin and M. Zhen, "Periodic Solutions for Delay Differential Equations Model of Plankton Allelopathy," *Computers & Mathematics with Applications*, Vol. 44, No. 3-4, 2002, pp. 491-500. doi: 10.1016/S0898-1221(02)00163-3

[2] S. Battaaz and F. Zanolin, "Coexistence States for Periodic Competitive Kolmogorov Systems," *Journal of Mathematical Analysis and Applications*, Vol. 219, 1998, pp. 178-199.

[3] S. Arik, "Stability Analysis of Delayed Neural Network," *Fundam. Theory Applications*, Vol. 52, 2000, pp. 1089-1092.

[4] Y. Zhang and K. T. Kok, "Global Convergence of Lotka-Volterra Recurrent Neural Networks with delays," *Circuits and syetemsm*, Vol. 52, 2005, pp. 2482-2489.

[5] J. Zhang and Z. J. Gui, "Periodic Solutions of Nonautonomous Cellular Neural Networks with Impulses and Delays," *Real World Applications*, Vol. 1, No. 3, 2009, pp. 1891-1903. doi:10.1016/j.nonrwa.2008.02.029

[6] F. Y. Wei and S. H. Wang, "Almost Periodic Solution and Global Stability for Cooperative L-V Diffusion System," *Journal of Mathematical Research and Exposition*, Vol. 30, 2010, pp. 1108-1116.

[7] X. Z. Meng and L. S. A. Chen, "Permanence and Global Stability in an Impulsive Lotka-Volterra Species Competitive System with both Discrete Delays and Continuous Delays," *International Journal of Biomathematics*, Vol. 1, No. 2, 2008, pp. 179-196. doi: 10.1142/S1793524508000151

[8] R. E. Gaines and J. L. Mawhin, "Coincidence Degree and Nonlinear Deferential Equations," *Berlin: Springer Verlag*, 1977.

