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Mean-Field Solution of the Mixed Spin-2 and Spin-5/2 Ising Ferrimagnetic System with Different Single-Ion Anisotropies

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ABSTRACT

The mixed spin-2 and spin-5/2 Ising ferrimagnetic system with different anisotropies $\left(D_A/z|J|\right)$ for the spin-2 and $\left(D_B/z|J|\right)$ for the spin-5/2 is studied by the use of the mean-field theory based on the Bogoliubov inequality for the free energy. First, the ground state phase diagram of the system at zero temperature is obtained on the $\left(D_A/z|J|, D_B/z|J|\right)$ plane. Topologically, different kinds of phase diagrams are achieved by changing the temperature and the values of the single ion anisotropies $D_A/z|J|$ and $D_B/z|J|$. Besides second-order transition lines, first order phase transition lines terminating at tricritical points, are found. The existence and dependence of a compensation temperature on single-ion anisotropies is also investigated.

Keywords: Mixed Spin; Ising Model; Ferrimagnetic; Sublattice Magnetization; Tricritical Points

1. Introduction

In the last two decades, much attention has been paid to the study of the magnetic properties of two-sublattice mixed-spin ferrimagnetic Ising systems, because they are well adapted to consider some types of ferrimagnetism, namely the molecular-based magnetic materials [1-3] which have less translational symmetry than their single-spin counterparts since they consist of two interpenetrating sublattices and have increasing interest. In a ferrimagnetic material, the different temperature dependences of the sublattice magnetizations raise the possibility of the existence of a compensation temperature: a temperature below the critical point where the total magnetization is zero [4]. This interesting behaviour has important applications in the field of thermomagnetic recording [5, 6]. For this reason, in recent years, there have been many theoretical studies on the magnetic properties of systems formed by two sublattices with different spins and with different crystal field interactions.

One of the earliest and simplest of these models to be studied was the mixed-spin Ising system consisting of spin-1/2 and spin-S (S > 1/2) in a uniaxial crystal field. The model for different values of S (S > 1/2) has been investigated by acting on honey-comb lattice [7-9], as well as on Bethe lattice [10,11]), mean field approximation [12], effective field theory with correlations [13-17], cluster variational theory [11], renormalization-group technique [18] and Monte-Carlo simulation [19-21].

It should be mentioned that the effects of different sublattice crystal-field interactions on the magnetic properties of the mixed spin-1 and spin-3/2 Ising ferromagnetic system with different single-ion anisotropies have been investigated with the use of an effective field theory [22,23], mean field theory [24], a cluster variational method [25] and Monte Carlo simulation [26]. Recently, The attention was devoted to the high order mixed spin ferrimagnetic systems (mixed spin-3/2 and spin-2 ferrimagnetic system mixed spin-2 and spin-5/2 ferrimagnetic system and mixed spin-3/2 and spin-5/2 system) in order to construct their phase diagrams in the temperature-anisotropy plane and to consider their magnetic properties. Bobak and Dely investigated the effect of single-ion anisotropy on the phase diagram of the mixed spin-3/2 and spin-2 Ising system by the use of a mean-field theory based on the Bogoliubov inequality for the free energy

[27]. Albayrak also studied the mixed spin-3/2 and spin-2 Ising system with two different crystal-field interactions on Bethe lattice by using the exact recursion equations [28]. Bayram Deviren *et al.* have used the effective field theory to study the magnetic properties of the ferrimagnetic mixed spin-3/2 and spin-2 Ising model with crystal field in a longitudinal magnetic field on a honeycomb and a square lattice [29]. We should mention that an early attempt to study the mixed-spin-2 and spin-5/2 system on a honeycomb lattice was made by Kaneyoshi and co-workers [30] within the frame work of the EFT. Nakamura [31,32] applied Monte Carlo (MC) simulations to study the magnetic properties of a mixed spin-2 and spin-5/2 system on a honeycomb lattice. Li *et al*. [33, 34] studied the magnetic properties of the mixed spin-2 and spin-5/2 system on a layered honeycomb lattice by a multisublattice green-function technique to investigate the magnetic properties of a mixed $\mathrm{AFe^{II}Fe^{III}}\left(\mathrm{C_2O_4}\right)_3\left[\mathrm{A}=\mathrm{N}\left(n\text{-}\mathrm{C}_n\mathrm{H}_{2n+1}\right), n=3,5\right]$ and to consider the compensation behaviour of the system. Wei and co-worker [35] examined the internal energy, specific heat and initial susceptibility of the mixed spin-2 and spin-5/2 ferrimagnetic system with an interlayer coupling by the use of the EFT with correlations. Albayrak [36] studied the critical behaviour of the mixed spin-2 and spin-5/2 Ising ferrimagnetic system on Bethe lattice. And he also examined the critical and the compensation temperatures of the mixed spin-2 and spin-5/2 Ising ferrimagnetic system on Bethe lattice by using the exact recursion equations. Keskin and Ertas [37] investigated the Existence of a dynamic compensation temperature of a mixed spin-2 and spin-5/2 Ising ferrimagnetic system in an oscillating field.

In this paper, we studied the effects of two different single-ion anisotropies in the phase diagram and in the compensation temperature of the mixed spin-2 and spin-5/2 Ising ferrimagnetic system within the theoretical framework of the mean-field theory and we found some outstanding features in the temperature dependences of total and sublattice magnetizations.

The outline of this work is as follows. In Section 2, we define the model and present the mean-field theory based on the Bogoliubov inequality for the Gibbs free energy and then, we describe a Landau expansion of the free energy in the order parameter. In Section 3, we present the results and the discussion about the phase diagrams and compensation temperature for various values of the single-ion anisotropies, as well as the temperature dependences of the magnetizations in some particular cases. Finally, in Section 4, we present our conclusions.

2. The Model and Calculation

We consider a mixed Ising spin-2 and spin-5/2 system consisting of two sublattices *A* and *B*, which are arranged alternately. The sublattice *A* are occupied by spins S_i, which take the spin values of $\pm 2, \pm 1, 0$, while the sublattice *B* are occupied by spins S_j, which take the spin values of $\pm 5/2, \pm 3/2, 1/2$. In each site of the lattice, there is a single-ion anisotropy (D_A in the sublattices *A* and D_B in the sublattice *B*) acting in the spin-2 and spin-5/2. The Hamiltunian of the system according to the mean-field theory is given by

$$H = -J\sum_{(i,j)} S_i^A S_j^B - D_A \sum \left(S_i^A\right)^2 - D_B \sum \left(S_j^B\right)^2, \quad (1)$$

where the first summation is carried out only over nearest-neighbor pairs of spins on different sublattices and *J* is the nearest-neighbour exchange interaction.

The most direct way of deriving the mean-field theory is to use the variation principle for the Gibbs free energy,

$$G(H) \le \Phi \equiv G_0(H_0) + \langle H - H_0 \rangle_0, \quad (2)$$

where $G(H)$ is the true free energy described by Hamiltonian given in the relation (1), $G_0(H)$ is the free energy described by the trial Hamiltonian H_0 which depends on variational parameters and $\langle \cdots \rangle_0$ denotes a thermal average over the ensemble defined by H_0.

Depending on the choice of the trial Hamiltonian, one can construct approximate methods of different accuracy. However, owing to the complexity of the problem, we consider in this work the simple choice of H_0, namely:

$$\begin{aligned} H_0 = &-\sum_{i\in A}\left[\gamma_A S_i^A + D_A\left(S_i^A\right)^2\right] \\ &-\sum_{j\in B}\left[\gamma_B S_j^B + D_B\left(S_j^B\right)^2\right], \end{aligned} \quad (3)$$

where γ_A and γ_B are the two variational parameters related to the molecular fields acting on the two different sublattices, respectively. Through this approach, we found the free energy and the equations of state (sublattice magnetization per site m_A

$$\begin{aligned} g = \frac{\Phi}{N} = &\frac{-1}{2\beta}\ln\left[1 + 2\exp(4\beta D_A)\cosh(2\beta\gamma_A) + 2\exp(\beta D_A)\cosh(\beta\gamma_A)\right] \\ &-\frac{1}{2\beta}\ln\left[2\exp\left(\frac{25}{4}\beta D_B\right)\cosh\left(\frac{5}{2}\beta\gamma_B\right) + 2\exp\left(\frac{9}{4}\beta D_B\right)\cosh\left(\frac{3}{2}\beta\gamma_B\right) + 2\exp\left(\frac{1}{4}\beta D_B\right)\cosh\left(\frac{1}{2}\beta\gamma_B\right)\right] \\ &-\frac{1}{2}zJm_A m_B + \frac{1}{2}\gamma_A m_A + \frac{1}{2}\gamma_B m_B, \end{aligned} \quad (4)$$

where $g=\frac{\Phi}{N},\beta=\frac{1}{k_BT},N$ is the total number of sites of the lattice and z is the coordination number.

The sublattice magnetization per site m_A and m_B are defined by $m_A=\langle S_i^A\rangle_0$ and $m_B=\langle S_j^B\rangle_0$, thus

$$m_A=\frac{2\sinh(2\beta\gamma_A)+\exp(-3\beta D_A)\sinh(\beta\gamma_A)}{\cosh(2\beta\gamma_A)+\exp(-3\beta D_A)\cosh(\beta\gamma_A)+0.5\exp(-4\beta D_A)} \tag{5}$$

and

$$m_B=\frac{1}{2}\left[\frac{5\sinh\left(\frac{5}{2}\beta\gamma_B\right)+3\exp(-4\beta D_B)\sinh\left(\frac{3}{2}\beta\gamma_B\right)+\exp(-6\beta\gamma_B)\sinh\left(\frac{1}{2}\beta\gamma_B\right)}{\cosh\left(\frac{5}{2}\beta\gamma_B\right)+\exp(-4\beta D_B)\cosh\left(\frac{3}{2}\beta\gamma_B\right)+\exp(-6\beta D_B)\cosh\left(\frac{1}{2}\beta\gamma_B\right)}\right]. \tag{6}$$

Now, by minimizing the free energy (4) with respect to γ_A and γ_B, we obtain

$$\gamma_A=zJm_B,\ \gamma_B=zJm_A. \tag{7}$$

The mean-field properties of the present model are then given by Equations (4)-(7). Since the Equations (5)-(7) have in general several solutions for the pair (m_A,m_B), the stable phase will be the one which minimizes the free energy. When the system undergoes the second-order transition from an ordered state $(m_A\neq 0,m_B\neq 0)$, to the paramagnetic state $(m_A=0,m_B=0)$, this part of the phase diagram can be determined analytically.

Because the magnetizations m_A and m_B are very small in the neighborhood of second-order transition point, we can expand Equations (4)-(6) to obtain a Landau-like expansion.

$$g=g_0+am_A^2+bm_A^4+cm_A^6+O\left(m_A^8\right), \tag{8}$$

where the expansion coefficients are given by

$$g_0=-\frac{1}{2\beta}\ln\left[(1+X_A+Y_A)(X_B+Y_B+Z_B)\right], \tag{9}$$

$$a=\frac{1}{2\beta}\left[\frac{t^2}{4}a_1-\frac{t^2}{8}a_2-\frac{t^4}{32}a_1^2b_1\right], \tag{10}$$

$$b=\frac{1}{2\beta}\left[\frac{t^4}{768}a_1^2c_1+\frac{t^3}{192}c_2a_1a_2+\frac{t^2}{96}c_3\right], \tag{11}$$

$$c=\frac{1}{2\beta}\left[\frac{t^4}{11520}\left(c_4+\frac{6}{t}c_5\right)+t^5\left(\frac{2a_2}{18423}c_2\left(3a_1^2-a_3\right)-\frac{c_5}{7680}a_2a_1\right)+\frac{t^7c_2}{18432}\left(a_4-3a_1^3a_2^2\right)-\frac{t^8}{245760}c_6\right] \tag{12}$$

with

$$a_1=\frac{9R_1+R_2+25}{R_2+R_1+1},\quad a_2=\frac{4X_A+Y_A}{X_A+Y_A+1},$$

$$a_3=\frac{81R_1+R_2+625}{R_1+R_2+1},\quad a_4=\frac{16X_A+Y_A}{X_A+Y_A+1},$$

$$a_5=\frac{729R_1+R_2+15625}{R_1+R_2+1},\quad a_6=\frac{64X_A+Y_A}{X_A+Y_A+1},$$

$$b_1=\frac{25X_B+9Y_B+Z_B}{X_B+Y_B+Z_B},\quad b_2=\frac{625X_B+81Y_B+Z_B}{X_B+Y_B+Z_B},$$

$$b_3=\frac{15625X_B+729Y_B+Z_B}{X_B+Y_B+Z_B},$$

$$c_2=\frac{t^3}{2}\left(3a_1^2-a_3\right),\quad c_3=\frac{t^2}{4}\left(3b_1^2+4a_3-12a_1^2-b_2\right),$$

$$c_4=\frac{t^2}{4}\left(15b_1b_2-b_1-b_2-30\right),$$

$$c_5=\frac{t^3}{4}\left(a_5+30a_1^3-15a_3a_1\right),$$

$$c_6=\frac{t^4}{12}\left(15a_1^6a_2a_4-a_1^6a_6-30\left(a_1^2a_2\right)^3\right),$$

where

$$X_A=2\exp(4\beta D_A),\quad X_B=2\exp(25\beta D_B/4),$$

$$Y_A=2\exp(\beta D_A),\quad Y_B=2\exp(9\beta D_B/4),$$

$$Z_B=2\exp(\beta D_B/4),\quad R_1=2\exp(-4\beta D_A),$$

$$R_2=2\exp(-6\beta D_A).$$

In this way, critical and tricritical points are determined according to the following routine;

1) Second-order transition lines when $a=0$ and $b>0$;

2) Tricritical points when $a=b=0$, and $c>0$;

3) The first-order transition lines are determined by comparing the corresponding Gibbs free energies of the various solutions of Equations (5) and (6) for the pair (m_A,m_B). Even so, we have also checked that $c>0$ in all T, D_A, D_B space. The critical behaviour is the same for both ferromagnetic ($J>0$) and ferrimagnetic ($J<0$) systems, because the coefficients a, b and c are even

functions of J. On the other hand, the total magnetization per site.

$$M = \frac{1}{2}\left(m_A + m_B\right) \tag{13}$$

and the signs of sublattice magnetizations m_A and m_B are different, therefore, a compensation temperature $T_k\left(T_k < T_c\right)$ at which the total magnetization is equal to zero may be exist in the system, although $m_A \neq 0$ and $m_B \neq 0$. In our paper we shall prove whether the present mixed-spin system can exhibit a compensation point or not.

3. Results and Discussions

3.1. Phase Diagrams

The ground-state phase diagram is easily determined from Hamiltonian (1) by comparing the ground-state energies of the different phases and is shown in **Figure 1**. At zero temperature, we find six phases with different values of $\left\{m_A, m_B, q_A, q_B\right\}$, namely the ordered ferrimagnetic phases

$$O_1 = \left\{-2, \frac{5}{2}, 4, \frac{25}{4}\right\},\ O_2 = \left\{-2, \frac{3}{2}, 4, \frac{9}{4}\right\},$$

$$O_3 = \left\{-2, \frac{1}{2}, 4, \frac{1}{4}\right\},\ O_4 = \left\{-1, \frac{5}{2}, 1, \frac{25}{4}\right\},$$

$$O_5 = \left\{-1, \frac{3}{2}, 1, \frac{9}{4}\right\},\ O_6 = \left\{-1, \frac{1}{2}, 1, \frac{1}{4}\right\},$$

and three disordered phases

$$D_1 = \left\{0, 0, 0, \frac{25}{4}\right\},\ D_2 = \left\{0, 0, 0, \frac{9}{4}\right\},\ D_3 = \left\{0, 0, 0, \frac{1}{4}\right\},$$

where the parameters q_A and q_B are defined by:

$$q_A = \left\langle S_i^A \right\rangle^2,\ q_B = \left\langle S_j^B \right\rangle^2$$

3.2. Temperature Phase Diagrams

In **Figures 2** and **3**, the phase diagrams of the mixed spin-2 and spin-5/2 Ising ferrimagnetic system are shown in the $\left(D_A/z|J|, k_B T_c/z|J|\right)$ and $\left(D_B/z|J|, k_B T_c/z|J|\right)$ planes for some selected values of $D_B/z|J|$ for spin-5/2 and $D_A/z|J|$ for spin-2, respectively. The solid and light dotted lines are used for the second and first-order transition, respectively, the heavy dashed curve represents the positions of tricritical points. The second-order phase transition lines are easily obtained from Equations (10) and (11) by setting $a = 0$ and $b > 0$.

The tricritical points (the critical points at which the phase transitions change from second to first order) are determined from Equations (10) and (11) by setting $a = b = 0$, however, the first-order phase transitions must be determined by comparing the corresponding Gibbs free energies of the various solutions of (5) and (6) for the pair $\left(m_A, m_B\right)$.

In **Figure 2**, we note that the value of the critical temperature increases when $D_B/z|J|$ and $D_A/z|J|$ increases. Above each second-order lines the system is in the paramagnetic state, while below them is in the ferrimagnetic state. We note that the system gives only second-order phase transitions (solid lines) for all the values of $D_A/z|J| > -0.4661$ and the phase diagram is topologically equivalent to that of the spin-5/2 Blume-Capel model which does not include any tricritical point.

For the values of $-2.3315 \le D_A/z|J| \le -0.4661$ the system includes second-order phase transition lines (solid lines) at higher temperatures, first-order phase transition lines (light dotted lines) at lower temperatures and a curve of tricritical (heavy dashed lines) points separates

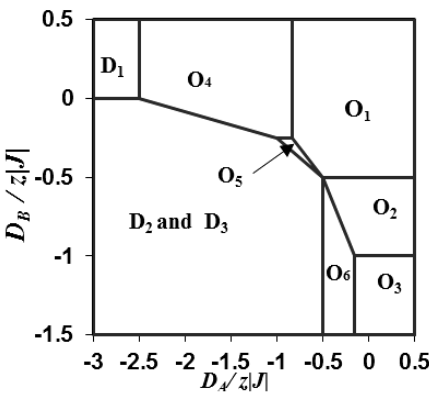

Figure 1. Ground-state phase diagram of mixed spin-2 and spin-5/2 Ising ferrimagnetic system with the coordination number z and different single-ion anisotropies D_A and D_B. The nine phases: ordered O_1, O_2, O_3, O_4, O_5, O_6 and disordered D_1, D_2, D_3 are separated by lines of first-order transitions.

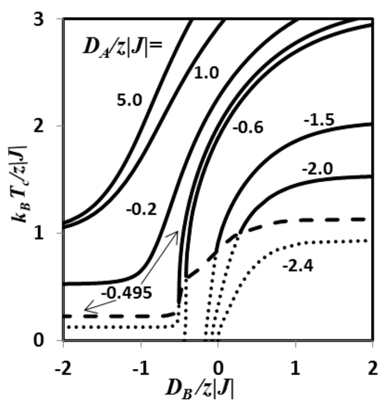

Figure 2. Phase diagram in the (D_B, T) plane for the mixed-spin Ising ferrimagnet with the coordination number z, when the value of $D_B/z|J|$ is changed. The solid and dotted lines, respectively, indicate second and first-order phase transitions, while the heavy dashed line represents the positions of tricritical points.

the second and the first-order critical lines.

When $-2.5 < D_A/z|J| < -2.3315$, the system gives only first-order phase transition lines.

In **Figure 3**, the phase diagrams of $(k_B T_c/z|J|)$ versus $D_A/z|J|$ are shown for selected values of $D_B/z|J|$ From this figure, it is clear that in regions of high temperatures, for all positive or negative values of, and for any value of $D_B/z|J|$, the phase diagram shows only second-order phase transitions.

When $D_B/z|J| \geq 1.4650$, all the second-order lines end in the same tricritical point given by $(D_A/z|J|, k_B T_{3c}/z|J|) = (-2.3315, 1.1360)$ and when $D_B/z|J| \leq -0.8450$, all the second-order lines end in the same tricritical point given by $(D_A/z|J|, k_B T_{3c}/z|J|) = (-0.4661, 0.2272)$. From this figure, we also note that for $D_B/z|J| \to +\infty$, the mixed spin Ising system behaves like a two-levels system since the spin-5/2 behaves like $S_j^B = \pm 5/2$ and the coordinates $(D_A/z|J|, k_B T_{3c}/z|J|)$ of the tricritical point are $(-2.3315, 1.1360)$.

On the other hand, for $D_B/z|J| \to -\infty$, the $S_j^B = \pm 5/2$ and $S_j^B = \pm 3/2$ states are suppressed and the system becomes equivalent to mixed spin-1/2 and spin-2 Ising model with tricritical point located at $(D_A/z|J|, k_B T_{3c}/z|J|) = (-0.4661, 0.2272)$. For this reason, the coordinates of the tricritical point in the limit of large positive $D_B/z|J|$ are five times higher than those for large negative $D_B/z|J|$.

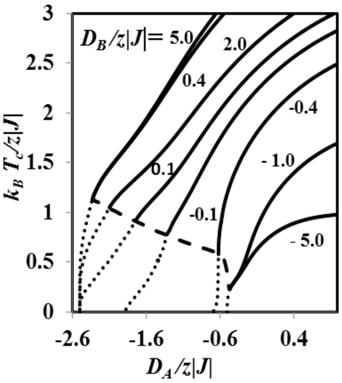

Figure 3. Phase diagram in the (D_A, T) plane for the mixed-spin Ising ferrimagnet with the coordination number z, when the value of $D_B/z|J|$ is changed. The solid and dotted lines, respectively, indicate second and first-order phase transitions, while the heavy dashed line represents the positions of tricritical points.

3.3. Magnetization Curves

Thermal behaviour of the sublattice magnetizations m_A and m_B are obtained by solving the coupled Equations. (5) and (6). The results are depicted in **Figure 4** for the system with $D_A/z|J| = 1.0$, when the value of $D_B/z|J|$ is changed from $D_B/z|J| = -0.45$ to −1.05. Notice that the selection of $D_B/z|J|$ corresponds to the crossover from the O_1 phase to the O_2 phase and from the O_2 to the O_3 phase (see the ground-state phase diagram in **Figure 1**). Therefore, the ground state is always ordered and **Figure 4** shows that the system undergoes only the second-order phase transition, because the sublattice magnetizations go to zero continuously as the temperature increases.

As shown in **Figure 4**, when $D_B/z|J| = -0.45$ (close to the boundary between the ordered-phase O_1 and the ordered phase O_2 in the ground-state phase diagram), the temperature dependences of m_B may exhibit a rather rapid decrease from its saturation value at $T = 0$ K. The phenomena is further enhanced when the value of $D_B/z|J|$ approaches the boundary. At $D_A/z|J| = -0.5$ and for $T = 0\,\mathrm{K}$, the saturation value of m_B is $m_B = 2.0$, which indicates that in the ground state the spin configuration of S_j^B in the system consists of the mixed state; in this state half of the spins on sublattice B are equal to

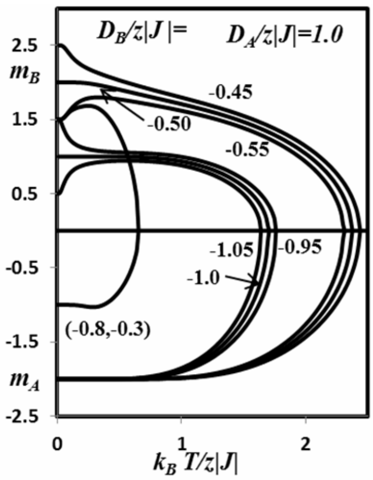

Figure 4. Thermal variations of sublattice magnetizations m_A, m_B for the mixed-spin Ising ferrimagnet with the coordination number z, when the value of $D_B/z|J|$ is changed for fixed $D_A/z|J|$ = 1.0. For one curve ($D_A/z|J|$, $D_B/z|J|$) = (−0.8, −0.3).

+5/2 (or −5/2) and the other half are equal to +3/2 (or −3/2). Note that this mixed state persists as long as $D_B/z|J| = -0.5$ and $D_A/z|J| > -0.5$.

In this case, the total magnetization for the ferrimagnetic system is $M = 0$ at $T = 0\,\mathrm{K}$, and hence, there is a compensation point at which the two sublattice magnetization cancel.

By further decreasing $D_B/z|J|$, the ground state becomes O_2, with $m_B = 1.5$ at $T = 0$ K. In this region, when $D_B/z|J| = -0.55$ (slightly below the boundary between the ordered phases O_1 and O_2) the thermal variation of m_B exhibits an interesting feature which is the initial rise of m_B with the increase of temperature before decreasing to zero at the critical point. On the other hand, for all values of $D_B/z|J|$, even though the sublattice magnetization m_A may show normal behaviour it is coupled to m_B.

When $D_B/z|J|$ has the values −0.95, −1.0 and −1.05 (close to the end at the boundary between the ordered-phases O_2 and O_3 in the ground-state phase diagram), it is clear from **Figure 4** that the temperature dependences of m_B and m_A exhibit similar behaviours to the temperature dependences of m_B and m_A in the previous case.

At the point $(D_A/z|J|, D_B/z|J|) = (-0.8, -0.3)$, the system will be in the ordered phase O_5 (see the ground-state phase diagram in **Figure 1**). In this case, the saturated values of (m_A, m_B) are $(-1, 3/2)$ at $T = 0$ K. Notice that the sublattice magnetization m_B has initial rise with temperature before decreasing to its zero value at the critical point, and the sublattice magnetization m_A may show a normal behaviour with temperature.

3.4. Compensation Temperature A

Compensation temperature T_k of the system can be evaluated by requiring the condition $M = 0$; in Equation (13).

Figures 5(a) and **(b)** show the behaviour of T_k (dotted lines) in the $(D_B/z|J|, k_BT/z|J|)$ plane for different values of $D_A/z|J|$. As seen from the figures, all T_k curves emerge from $D_B/z|J| = -0.5$ at $T = 0\,\text{K}$ and exhibit some characteristic behaviours when the value of $D_A/z|J|$ is changed.

In **Figure 5(a)**, all the curves increase monotonically with $D_B/z|J|$ and terminate at the corresponding phase boundaries (solid lines). This behaviour implies the occurrence of one compensation point only. As $D_A/z|J|$ is reduced, the range of $D_B/z|J|$ over which the compensation points occur gradually becomes small, but the compensation temperature still reaches the corresponding transition line. In the **Figure 5(b)**, and in a restricted region of $D_B/z|J|$, close to $D_B/z|J| = -0.5$, a new type of compensation curves appear and the compensation temperature lines exhibit an interesting features in their behaviours, which implies the occurrence of two, three, or four compensation points. In this figure, for $D_A/z|J|$, close to $D_A/z|J| = -0.5$, a new type of compensation curves appear: the T_k curves are extended to $D_B/z|J| \to -\infty$ below the corresponding transition lines. The curve labeled $D_A/z|J| = -0.498$ is an example of such behaviour of T_k. Finally, a total magnetization curve (which refers to the compensation temperatures presented in **Figure 5(b)**) when $D_A/z|J| = -0.498$ and $D_B/z|J| = -0.499222$ with four compensation points are shown in **Figure 6**. Furthermore, In **Figure 7(a)**, when $D_A/z|J| = -0.4999$ and $D_B/z|J| = -0.5$ (very close to the point $(D_A/z|J|, D_B/z|J|) = (-0.5, -0.5)$ which is in the boundary between five phases in the ground state phase diagram), the magnetization curves

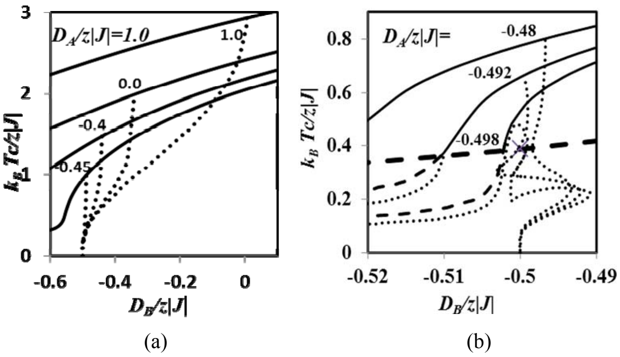

Figure 5. Dependence of the compensation temperature (dotted curves) on the single-ion anisotropy. $D_B/z|J|$ in a mixed-spin Ising ferrimagnet with coordination number z, when the value of $D_B/z|J|$ is changed. (a) The curves show the positions of one compensation points; (b) The curves show the positions of two, three and four compensation points. The solid and dashed curves represent the second and first-order transitions.

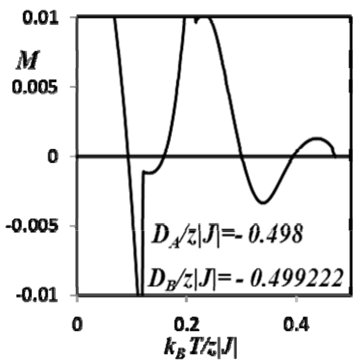

Figure 6. Thermal variations of the total magnetization M for the mixed-spin Ising ferrimagnet with the coordination number z, when the value of $D_A/z|J| = -0.498$ and the value of $D_B/z|J| = -0.499222$.

exhibit some outstanding features. At this point, as the temperature is increased from zero, the sublattice magnetizations m_A and m_B exhibit four jumps (discontinuity) before the magnetizations vanish, indicating the existence of four first order transitions at the temperature values $k_BT/z|J| = 0.0797$, 0.1583 and 0.2526 respectively. In the same time, as shown in **Figure 7(b)**, the total magnetization exhibits four first order transition points and four compensation temperatures.

4. Conclusion

In this paper, we have determined the global phase diagrams of the mixed spin-2 and spin-5/2 Ising ferrimagnetic system with different single-ion anisotropies acting on the spin-2 and spin-5/2 by using mean-field approximation. In the phase diagrams, the critical temperature lines versus single-ion anisotropies are shown. The system presents tricritical behaviour, *i.e.*, the second-order

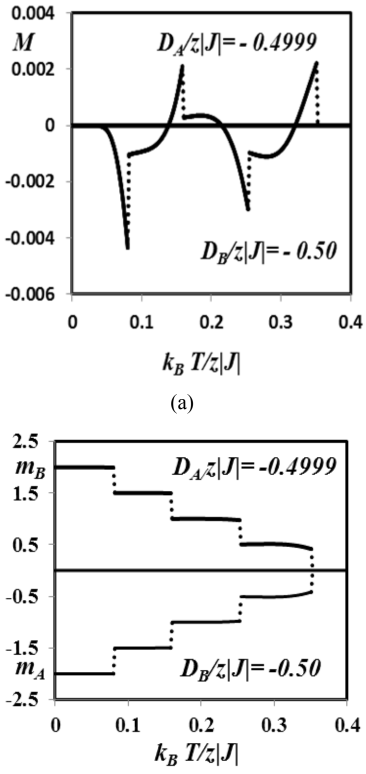

Figure 7. Thermal variations of (a) The total magnetization *M*; (b) The sublattice magnetizations m_A, m_B for the mixed-spin Ising ferrimagnet with the coordination number *z*, when the value of $D_A/z|J| = -0.4999$ and the value of $D_B/z|J| = -0.5$.

phase transition line is separated from the first-order transition line by a tricritical point. We also observed that this mixed-spin ferrimagnetic system may exhibit one, two, three or four compensation points. The theoretical prediction of the possibility of compensation points and the design and preparation of materials with such unusual behaviour will certainly open a new area of research on such materials.

REFERENCES

[1] O. Khan, "Molecular Magnetism," VCH publishers, New York, 1993.

[2] T. Kaneyoshi and Y. Nakamura, "A Theoretical Investigation for Low-Dimensional Molecular-Based Magnetic Materials," *Journal of Physics*: *Condensed Matter*, Vol. 10, No. 13, 1998, p. 3003. doi:10.1088/0953-8984/10/13/017

[3] T. Kaneyoshi, Y. Nakamura and S. Shin, "A Diluted Mixed Spin-2 and Spin-5/2 Ferrimagnetic Ising System; a Study of a Molecular-Based Magnet," *Journal of Physics*: *Condensed Matter*, Vol. 10, No. 31, 1998, p. 7025. doi:10.1088/0953-8984/10/31/018

[4] L. Néel, "Propriétées Magnétiques des Ferrites; Férrimagnétisme et Antiferromagnétisme," *Annales de Physique* (*Paris*), Vol. 3, 1948, pp. 137-198.

[5] M. Mansuripur, "Magnetization Reversal, Coercivity, and the Process of Thermomagnetic Recording in Thin Films of Amorphous Rare Earth Transition Metal Alloys," *Journal of Applied Physics*, Vol. 61, No. 4, 1987, pp. 1580-1587. doi:10.1063/1.338094

[6] F. Tanaka, S. Tanaka and N. Imamura, "Magneto-Optical Recording Characteristics of TbFeCo Media by Magnetic Field Modulation Method," *Japan Journal of Applied Physics*, Vol. 26, 1987, pp. 231-235. doi:10.1143/JJAP.26.231

[7] L. L. Goncaloves, "Uniaxial Anisotropy Effects in the Ising Model: An Exactly Soluble Model," *Physica Scripta*, Vol. 32, No. 3, 1985, p. 248. doi:10.1088/0031-8949/32/3/012

[8] L. L. Goncaloves, "Uniaxial Anisotropy Effects in the Ising Model: An Exactly Soluble Model," *Physica Scripta*, Vol. 33, No. 2, 1986, p. 192. doi:10.1088/0031-8949/33/2/018

[9] A. Dakhama and N. Benayad, "On the Existence of Compensation Temperature in 2d Mixed-Spin Ising Ferrimagnets: An Exactly Solvable Model," *Journal of Magnetism and Magnetic Materials*, Vol. 213, No. 1-2, 2000, pp. 117-125. doi:10.1016/S0304-8853(99)00606-X

[10] N. R. da Silva and S. R. Salinas, "Mixed-Spin Ising Model on Beth Lattice," *Physical Review*, Vol. 44, No. 2, 1991, pp. 852-855. doi:10.1103/PhysRevB.44.852

[11] J. W. Tucker, "The Ferrimagnetic Mixed Spin-1/2 and Spin-1 Sing System," *Journal of Magnetism and Magnetic Materials*, Vol. 195, No. 3, 1999, pp. 733-740. doi:10.1016/S0304-8853(99)00302-9

[12] T. Kaneyoshi and J. C. Chen, "Mean-Field Analysis of a Ferrimagnetic Mixed Spin System," *Journal of Magnetism and Magnetic Materials*, Vol. 98, No. 1-2, 1991, pp. 201-204. doi:10.1016/0304-8853(91)90444-F

[13] T. Kaneyoshi, "Curie Temperatures and Tricritical Points in Mixed Ising Ferromagnetic Systems," *The Physical Society of Japan*, Vol. 56, 1987, pp. 2675-2680. doi:10.1143/JPSJ.56.2675

[14] T. Kaneyoshi, "Phase Transition of the Mixed Spin System with a Random Crystal Field," *Physica A*, Vol. 153, No. 3, 1988, pp. 556-566. doi:10.1016/0378-4371(88)90240-3

[15] T. Kaneyoshi, M. Jascur and P. Tomczak, "The Ferrimagnetic Mixed Spin-1/2 and Spin-3/2 Ising System," *Journal of Physics*: *Condensed Matter*, Vol. 4, No. 49, 1992, pp. L653-L658. doi:10.1088/0953-8984/4/49/002

[16] T. Kaneyoshi, "Tricritical Behavior of a Mixed Spin-1/2 and Spin-2 Ising System," *Physica A*, Vol. 205, No. 4, 1994, pp. 677-686. doi:10.1016/0378-4371(94)90229-1

[17] A. Bobak and M. Jurcisin, "Discussion of Critical Behaviour in a Mixed-Spin Ising Model," *Physica A*, Vol. 240, No. 3-4, 1997, pp. 647-656. doi:10.1016/S0378-4371(97)00044-7

[18] S. G. A. Quadros and S. R. Salinas, "Renormalization-Group Calculations for a Mixed-Spin Ising Model," *Physica A*: *Statistical Mechanics and Its Applications*, Vol. 206, No. 3-4, 1994, pp. 479-496.

[19] G.-M. Zhang and C.-Z. Yang, "Monte Carlo Study of the

Two-Dimensional Quadratic Ising Ferromagnet with Spins S = 1/2 and S = 1 and with Crystal-Field Interactions," *Physical Review B*, Vol. 48, No. 13, 1993, pp. 9452-9455. doi:10.1103/PhysRevB.48.9452

[20] G. M. Buendia and M. A. Novotny, "Numerical Study of a Mixed Ising Ferrimagnetic System," *Journal of Physics*: *Condensed Matter*, Vol. 9, No. 27, 1997, pp. 5951-5964. doi:10.1088/0953-8984/9/27/021

[21] G. M. Buendia and J. A. Liendo, "Monte Carlo Simulation of a Mixed Spin-1/2 and Spin-3/2 Ising Ferrimagnetic System," *Journal of Physics*: *Condensed Matter*, Vol. 9, No. 25, 1997, pp. 5439-5448. doi:10.1088/0953-8984/9/25/011

[22] A. Bobak, "The Effect of Anisotropies on the Magnetic Properties of a Mixed Spin-1 and Spin-3/2 Ising Ferrimagnetic System," *Physica A*, Vol. 258, No. 1-2, 1998, pp. 140-156. doi:10.1016/S0378-4371(98)00233-7

[23] O. F. Bobak and D. H. Abubrig, "An Effective-Field Study of the Mixed Spin-1 and Spin-3/2 Ising Ferrimagnetic System," *Journal of Magnetism and Magnetic Materials*, Vol. 246, No. 1-2, 2002, pp. 177-183. doi:10.1016/S0304-8853(02)00048-3

[24] O. F. Abubrig, D. Horvath, A. Bobak and M. Jascur, "Mean-Field Solution of the Mixed Spin-1 and Spin-3/2 Ising System with Different Single-Ion Anisotropies," *Physica A*, Vol. 296, No. 3-4, 2001, pp. 437-450. doi:10.1016/S0378-4371(01)00176-5

[25] J. W. Tucker, "Mixed Spin-1 and Spin-3/2 Blume-Capel Ising Ferromagnet," *Journal of Magnetism and Mgnetic Materials*, Vol. 237, No. 2, 2001, pp. 215-224. doi:10.1016/S0304-8853(01)00691-6

[26] Y. Nakamura and J. W. Tucker, "Monte Carlo Study of a Mixed Spin-1 and Spin-3/2 Ising Ferromagnet," *IEEE Transactions on Magnetics*, Vol. 38, No. 5, 2002, pp. 2406-2408. doi:10.1109/TMAG.2002.803598

[27] A. Bobak and J. Dely, "Phase Transitions and Multicritical Points in the Mixed Spin-3/2 and Spin-2 Ising System with a Single-Ion Anisotropy," *Journal of Magnetism and Magnetic Materials*, Vol. 310, No. 2, 2007, pp. 1419-1421. doi:10.1016/j.jmmm.2006.10.427

[28] E. Albayrak, "The Critical and Compensation Temperatures of the Mixed Spin-3/2 and Spin-2 Ising Model," *Physica B*: *Condensed Matter*, Vol. 391, No. 1, 2007, pp. 47-53. doi:10.1016/j.physb.2006.08.045

[29] B. Deviren, E. Kantar and M. Keskin, "Magnetic Properties of a Mixed Spin-3/2 and Spin-2 Ising Ferrimagnetic System within the Effective-Field Theory," *Journal of the Korean Physical Society*, Vol. 56, No. 6, 2010, pp. 1738-1747. doi:10.3938/jkps.56.1738

[30] Y. Nakamura, S. Shin and T. Kaneyoshi, "The Effects of Transverse Field on the Magnetic Properties in a Diluted Mixed Spin-2 and Spin-5/2 Ising System," *Physica B*, Vol. 284-288, 2000, pp. 1479-1480. doi:10.1016/S0921-4526(99)02668-X

[31] Y. Nakamura, "Monte Carlo Study of a Mixed Spin-2 and Spin-5/2 Ising System on a Honeycomb Lattice," *Journal of Physics*: *Condensed Matter*, Vol. 12, No. 17, 2000, pp. 4067-4074. doi:10.1088/0953-8984/12/17/312

[32] Y. Nakamura, "Existence of a Compensation Temperature of a Mixed Spin-2 and Spin-5/2 Ising Ferrimagnetic System on a Layered Honeycomb Lattice," *Physical Review B*, Vol. 62, No. 17, 2000, pp. 11742-11746. doi:10.1103/PhysRevB.62.11742

[33] J. Li, A. Du and G. Z. Wei, "Green Function Study of a Mixed-Spin-2 and Spin-5/2 Heisenberg Ferrimagnetic System on a Honeycomb Lattice," *Physica Status Solidi* (*b*), Vol. 238, No. 1, 2003, pp. 191-197.

[34] J. Li, A. Du and G. Z. Wei, "The Compensation Behavior of a Mixed-Spin-2 and Spin-5/2 Heisenberg Ferrimagnetic System on a Honeycomb Lattice," *Physica B*, Vol. 348, No. 1-4, 2004, pp. 79-88. doi:10.1016/j.physb.2003.11.074

[35] G. Wei, Q. Zhang, Z. Xin and Y. Liang, "Internal Energy and Initial Susceptibility of Mixed Spin-2 and Spin-5/2 Ferrimagnetic Ising System with Interlayer Coupling," *Journal of Magnetism and Magnetic Materials*, Vol. 277, No 1-2, 2004, pp. 1-15. doi:10.1016/j.jmmm.2003.06.001

[36] E. Albayrak, "Mixed-Spin-2 and Spin-5/2 Blume-Emery-Griffiths Model," *Physica A*: *Statistical Mechanics and Its Applications*, Vol. 375, No. 1, 2007, pp. 174-184.

[37] M. Keskin and M. Ertas, "Existence of a Dynamic Compensation Temperature of a Mixed Spin-2 and Spin-5/2 Ising Ferrimagnetic System in an Oscillating Field," *Physical Review E*, Vol. 80, No. 6, 2009, Article ID: 061140. doi:10.1103/PhysRevE.80.061140

