Open Journal of Applied Sciences
Supplement: 2012 world Congress on Engineering and Technology

Cross-kink multi-soliton solutions for the (3+1)-D Jimbo-Miwa equation

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Abstract- In this paper, by using bilinear form and extended three-wave type of ans¨atz approach, we obtain new cross-kink multi-soliton solutions of the (3+1)-dimensional Jimbo-Miwa equation, including the periodic breather-type of kink three-soliton solutions, the cross-kink four-soliton solutions, the doubly periodic breathertype of soliton solutions and the doubly periodic breather-type of cross-kink two-soliton solutions. It is shown that the generalized three-wave method, with the help of symbolic computation, provides an effective and powerful mathematical tool for solving high dimensional nonlinear evolution equations in mathematical physics.

Keywords-Jimbo-Miwa equation; Extended three-wave method; Cross-kink multi-soliton.

1 Introduction

It is well known that many important phenomena in physics and other fields are described by nonlinear partial differential equations. As mathematical models of these phenomena, the investigation of exact solutions is important in mathematical physics. Many methods are available to look for exact solutions of nonlinear evolution equations, such as the inverse scattering method, the Lie group method, the mapping method, Exp-function method, ans¨atz technique, three-wave tape of ansatz approach and so on [1-3]. In this paper, we consider the following Jimbo-Miwa equation:

$$u_{xxxy}+3u_x u_{xy}+3u_{xx}u_y+2u_{yt}-3u_{xz}=0 \qquad (1)$$

which comes from the second member of a KP-hierarchy called Jimbo-Miwa equation firstly introduced by Jimbo-Miwa [4]. By means of the two-soliton method and bilinear methods, the the two-soliton solutions, three-wave solutions of the Jimbo-Miwa were found as well as [5-6].

In this paper, we discuss further the (3 + 1)-dimensional Jimbo-Miwa equation, by using bilinear form and extend three-wave type of ansatz approach, respectively[7-9], Some new cross-kink multi-soliton solutions are obtained.

2 The multi-soliton solutions

We assume $u=2(\ln f)_x$ (2)

Where $f=f(x,y,z,t)$ is unknown real function. Substituting Eq.(2) into Eq.(1), we can reduce Eq.(1) into the following Hirota bilinear equation

$$(D_x^3 D_y+2D_y D_t-3D_x D_z)f\cdot f=0 \qquad (3)$$

where the Hirota bilinear operator D is defined by $(m,n\geq 0)$

$$D_x^m D_t^n f(x,t)\cdot g(x,t) = \left(\frac{\partial}{\partial x}-\frac{\partial}{\partial x'}\right)^m\left(\frac{\partial}{\partial t}-\frac{\partial}{\partial t'}\right)^n[f(x,t)\cdot g(x',t')]\Big|_{x'=x,t'=t} \qquad (4)$$

Now we suppose the solution of Eq.(3) as

$$f=e^{-\xi}+\delta_1\cos(\eta)+\delta_2\sinh(\gamma)+\delta_3\cosh(\theta)+\delta_4 e^{\xi} \qquad (5)$$

Where $\xi=a_1x+b_1y+c_1z+d_1t$, $\eta=a_2x+b_2y+c_2z+d_2t$, $\gamma=a_3x+b_3y+c_3z+d_3t$ $\theta=a_4x+b_4y+c_4z+d_4t$

and $a_i,b_i,c_i,d_i (i=1,2,3,4)$ are some constants to be determined later. Substituting Eq.(5) into Eq.(3)and equating all the coefficients of different powers of $e^{\xi}, e^{-\xi}, \sin(\eta), \cos(\eta), \sinh(\gamma), \cosh(\gamma), \sinh(\theta), \cosh(\theta)$ and constant term to zero, we can obtain a set of algebraic equations for $a_i, b_i, c_i, d_i, \delta_j$ $(i=1,2,3,4;\cdot\ j=1,2,3,4)$,

Solving the system with the aid of Maple, we get the following results:

Case(I):

$$\begin{cases} a_1 = 0, a_3 = 0, b_2 = 0, b_4 = 0, \\ c_2 = 0, c_3 = \dfrac{b_3 c_1}{b_1}, c_4 = 0, d_1 = 0, d_3 = 0, \\ d_2 = \dfrac{a_2(a_2^2 b_1 - 3c_1)}{2b_1}, d_4 = -\dfrac{a_4(3c_1 + a_4^2 b_1)}{2b_1}, \\ \delta_1 = \delta_1, \delta_2 = \delta_2, \delta_3 = \delta_3, \delta_4 = \delta_4. \end{cases} \tag{6}$$

Where $a_2, a_4, b_1, b_3, c_1, \delta_1, \delta_2, \delta_3, \delta_4$ are some free real constants. Substituting Eq.(6) into Eq.(5) and taking $\delta_4 > 0$, we have

$$f_1 = 2\sqrt{\delta_4}\cosh(b_1 y + c_1 z + \frac{1}{2}\ln(\delta_4)) + \delta_1 \cos(a_2 x + K_1 t) + \delta_2 \sinh(b_3 y + L_1 z) + \delta_3 \cosh(a_4 x - H_1 t) \tag{7}$$

Where $K_1 = \dfrac{a_2(a_2^2 b_1 - 3c_1)}{2b_1}, L_1 = \dfrac{b_3 c_1}{b_1}, H_1 = \dfrac{a_4(3c_1 + a_4^2 b_1)}{2b_1}.$

Substituting Eq.(7) into Eq.(2) yields the periodic breather-type of kink three-soliton solutions for Jimbo-Miwa equation as follows:

$$u_1 = -\frac{2[a_2\delta_1 \sin(a_2 x + K_1 t) - a_4 \delta_3 \sinh(a_4 x - H_1 t)]}{2\sqrt{\delta_4}\cosh(b_1 y + c_1 z + \frac{1}{2}\ln(\delta_4)) + \delta_1 \cos(a_2 x + K_1 t) + \delta_2 \sinh(b_3 y + L_1 z) + \delta_3 \cosh(a_4 x - H_1 t)} \tag{8}$$

If taking $a_2 = iA_2$ in Eq.(7), then we have

$$f_2 = 2\sqrt{\delta_4}\cosh(b_1 y + c_1 z + \frac{1}{2}\ln(\delta_4)) + \delta_1 \cosh(A_2 x - K_2 t) + \delta_2 \sinh(b_3 y + L_1 z) + \delta_3 \cosh(a_4 x - H_1 t) \tag{9}$$

Where $\delta_4 > 0, K_2 = \dfrac{A_2(A_2^2 b_1 + 3c_1)}{2b_1}$. Substituting Eq.(9) into Eq.(2) yields the cross-kink four-soliton solutions of Jimbo-Miwa equation as follows:

$$u_2 = \frac{2[A_2\delta_1 \sinh(A_2 x - K_2 t) + a_4 \delta_3 \sinh(a_4 x - H_1 t)]}{2\sqrt{\delta_4}\cosh(b_1 y + c_1 z + \frac{1}{2}\ln(\delta_4)) + \delta_1 \cosh(A_2 x - K_2 t) + \delta_2 \sinh(b_3 y + L_1 z) + \delta_3 \cosh(a_4 x - H_1 t)} \tag{10}$$

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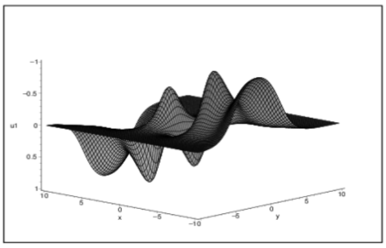
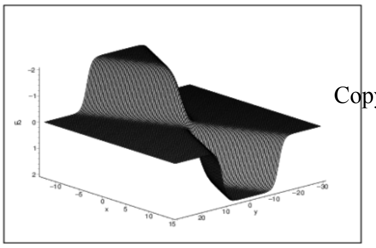


FIg(a).The figure of u_1 as $\delta_1 = \frac{1}{2}, \delta_2 = \frac{1}{4}, t = 0$



Fig(b).The figure of u_2 as $\delta_1 = \frac{1}{2}, \delta_2 = \frac{1}{5}, t = 1$

If taking $a_4 = iA_4$ in Eq.(7), then we have

$$f_3 = 2\sqrt{\delta_4}\cosh(b_1 y + c_1 z + \frac{1}{2}\ln(\delta_4)) + \delta_1 \cos(a_2 x + K_1 t) + \delta_2 \sinh(b_3 y + L_1 z) + \delta_3 \cos(A_4 x + H_2 t) \tag{11}$$

where $\delta_4 > 0, H_2 = \dfrac{A_4(A_4^2 b_1 - 3c_1)}{2b_1}$. Substituting Eq.(11) into Eq.(2) yields the doubly periodic breather-type of soliton solutions for Jimbo-Miwa equation as follows:

$$u_3 = -\frac{2[a_2\delta_1 \sin(a_2 x + K_1 t) + A_4 \delta_3 \sin(A_4 x + H_2 t)]}{2\sqrt{\delta_4}\cosh(b_1 y + c_1 z + \frac{1}{2}\ln(\delta_4)) + \delta_1 \cosh(a_2 x + K_1 t) + \delta_2 \sinh(b_3 y + L_1 z) + \delta_3 \cos(A_4 x - H_2 t)} \tag{12}$$

If taking $a_4 = iA_4, b_3 = iB_3, \delta_2 = iQ_2$ in Eq.(7), then we have

$$f_4 = 2\sqrt{\delta_4}\cosh(b_1 y + c_1 z + \frac{1}{2}\ln(\delta_4)) + \delta_1 \cos(a_2 x + K_1 t) - Q_2 \sin(B_3 y + L_2 z) + \delta_3 \cos(A_4 x + H_2 t) \tag{13}$$

where A_4, B_3, Q_2 are some free real constants , $L_2 = \dfrac{B_3 c_1}{b_1}$ and $\delta_3 > 0$. Substituting Eq.(13) into Eq.(2) yields the doubly periodic breather-type of soliton solutions for Jimbo-Miwa equation as follows:

$$u_4 = -\frac{2[a_2\delta_1 \sin(a_2 x + K_1 t) + A_4 \delta_3 \sin(A_4 x + H_2 t)]}{2\sqrt{\delta_4}\cosh(b_1 y + c_1 z + \frac{1}{2}\ln(\delta_4)) + \delta_1 \cosh(a_2 x + K_1 t) - Q_2 \sin(B_3 y + L_2 z) + \delta_3 \cos(A_4 x - H_2 t)} \tag{14}$$

This work was supported by Chinese Natural Science Foundation Grant No. 10971169. Sichuan Educationalscience Foundation Grant No.09zc008.

Case(II):

$$
\begin{cases}
a_1 = 0, a_2 = \dfrac{\sqrt{4-a_4^4}}{a_4}, a_3 = \dfrac{2i}{a_4}, b_1 = 1, b_2 = 0, b_3 = \dfrac{a_4^2 i}{\sqrt{4-a_4^4}}, b_4 = -\dfrac{2}{\sqrt{4-a_4^4}}, c_2 = 1, \\
c_3 = \dfrac{(a_4^2+2)i}{\sqrt{4-a_4^4}}, c_4 = \dfrac{a_4^2-2c_1}{\sqrt{4-a_4^4}}, d_1 = \dfrac{3\sqrt{4-a_4^4}}{2a_4}, d_2 = -\dfrac{(3a_4^2c_1+a_4^4-4)\sqrt{4-a_4^4}}{2a_4^3}, \\
d_3 = -\dfrac{(3a_4^4+6a_4^2c_1-8)i}{2a_4^3}, d_4 = -\dfrac{3a_4^2c_1+a_4^4-6}{2a_4^3}, \delta_4 = \dfrac{a_4^4(\delta_2^2+\delta_3^2-\delta_1^2)+4(\delta_1^2+\delta_2^2+\delta_3^2)}{4(4-a_4^4)}
\end{cases} \tag{15}
$$

where $a_4, c_1, \delta_1, \delta_2, \delta_3$ are some free real constants. Substituting Eq.(15) into Eq.(5) and taking $M > 0$, we have

$$
f_5 = 2\sqrt{M}\cosh(\xi + \frac{1}{2}\ln(M)) + \delta_1\cos(\eta) + \delta_2\sin(\gamma) + \delta_3\cosh(\theta) \tag{16}
$$

when $M > 0$. where $\xi = a_1 x + y + c_1 z + \dfrac{3\sqrt{4-a_4^4}}{2a_4}t$,

$$
\eta = \frac{\sqrt{4-a_4^4}}{a_4}x + z - \frac{(3a_4^2c_1+a_4^4-4)\sqrt{4-a_4^4}}{2a_4^3}t,
$$

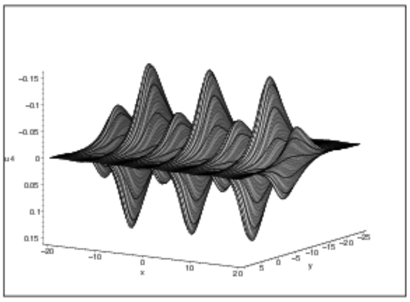
$$
\gamma = \frac{2}{a_4}x + \frac{a_4^2}{\sqrt{4-a_4^4}}y + \frac{a_4^2+2}{\sqrt{4-a_4^4}}z + \frac{3a_4^4+6a_4^2c_1-8}{2a_4^3}t,
$$

$$
\theta = a_4 x - \frac{2}{\sqrt{4-a_4^4}}y + \frac{a_4^2-2c_1}{\sqrt{4-a_4^4}}z + \frac{3a_4^2c_1+a_4^4-6}{2a_4^3}t,
$$

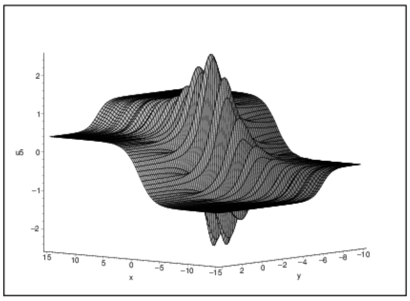
$$
M = \frac{a_4^4(\delta_2^2+\delta_3^2-\delta_1^2)+4(\delta_1^2+\delta_2^2+\delta_3^2)}{4(4-a_4^4)}.
$$

Substituting Eq.(16) into Eq.(2), we obtain the doubly periodic breather-type of cross-kink two-soliton solutions for Jimbo-Miwa equation as follows:

$$
u_5 = \frac{2[2a_1\sqrt{M}\sinh(\xi+\frac{1}{2}\ln(M)) + \frac{\delta_1\sqrt{4-a_4^4}}{a_4}\sin(\eta) - \frac{2\delta_2}{a_4}\cos(\gamma) - a_4\delta_3\sinh(\theta)]}{2\sqrt{M}\cosh(\xi+\frac{1}{2}\ln(M)) + \delta_1\cos(\eta) + \delta_2\sin(\gamma) + \delta_3\cosh(\theta)} \tag{17}
$$



Fig(c).The figure of u_3 as $\delta_1 = \frac{1}{5}, \delta_2 = \frac{1}{2}, t = 0$



Fig(b).The figure of u_4 as $\delta_1 = 1, \delta_2 = \frac{1}{3}, t = 0$

3 Conclusion

By using bilinear form and extended three-wave type of ans ¨ atz approach, we discuss further the (3 +1)-dimensional Jimbo-Miwa equation and find some new cross-kink multi-soliton solutions. The results show that the extended three-wave tape of ans ¨ atz approach may provide us with a straight-forward and effective mathematical tool for seeking multi-wave solutions of higher dimensional inear evolution equations.

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