Open Journal of Applied Sciences, **2012, 2, 188-192**
doi:10.4236/ojapps.2012.23028 Published Online September 2012 (http://www.SciRP.org/journal/ojapps)

Stability of Production Planning Problem with Fuzzy Parameters

Samir Abdou Abass
Department of Mathematics and Theoretical Physics, Nuclear Research Center, Atomic Energy Authority, Cairo, Egypt
Email: samir.abdou@gmail.com

Received June 3, 2012; revised July 5, 2012; accepted July 16, 2012

ABSTRACT

The traditional production planning model based upon the famous linear programming formulation has been well known in the literature. The consideration of uncertainty in manufacturing systems supposes a great advance. Models for production planning which do not recognize the uncertainty can be expected to generate inferior planning decisions as compared to models that explicitly account the uncertainty. This paper deals with production planning problem with fuzzy parameters in both of the objective function and constraints. We have a planning problem to maximize revenues net of the production inventory and lost sales cost. The existing results concerning the qualitative and quantitative analysis of basic notions in parametric production planning problem with fuzzy parameters. These notions are the set of feasible parameters, the solvability set and the stability set of the first kind.

Keywords: Production Planning; Stability; Linear Programming; Fuzzy Parameters

1. Introduction

The classical linear programming (LP) models for production planning have been around for many years. A typical formulation of the LP planning models has the objective minimizing the total production-related costs, such as variable production costs, inventory costs, and shortage costs, over the fixed planning horizon [1,2]. The usual constraints employed are: 1) inventory balance equations for making the inventory and/or shortages balanced with those from the previous period, production quantity, and the demand quantity; 2) capacity constraints which ensure the total workload for each resource not exceed the capacity in each period [3].

The LP model considers the limited availability of the resources (labor, machine, etc.) through the capacity constraints. In a real production system, such capacity constraints may not correct. Galbraith [4] defined uncertainty as the difference between the amount of information required to perform a task and the amount of information already possessed. Mula *et al.* [5] presented an exhaustive literature survey about models for production planning under uncertainty. Abouzar Jamalnia and M. Ali Soukhakian [6] introduced a hybrid fuzzy multi objective nonlinear programming model with different goal priorities. Zrinka *et al*. [7] introduced the production planning problem as a bilevel programming problem. In the real world, there are many forms of uncertainty that affect production process. Ho [8] categorizes them into two groups: 1) environmental uncertainty and 2) system uncertainty. Environmental uncertainty includes uncertainties beyond the production process, such as demand uncertainty and supply uncertainty. System uncertainty is related to uncertainties within the production process, such as operation yield uncertainty, production lead time uncertainty, quality uncertainty, failure of production system and changes to product structure, to mention some. In this paper, we will use the first category of uncertainty. The literature in production planning under uncertainty is vast. Different approaches have been proposed to cope with different forms of uncertainty (see, for example, [8-10]).

This paper is organized as follows. In next section, a model of production planning problem to maximize revenues net with fuzzy parameters is formulated. Section 3 presents a qualitative analysis of some basic notions for the problem of concern. An illustrative numerical example is provided in Section 4. Finally, Section 5 contains the concluding remarks.

2. Problem Formulation

For some production planning problems we have the option of not meeting all demand in each time period. Indeed, there might not be sufficient resources to meet all demand. In this case, the optimization problem is to decide what demand to meet and how. We assume that demand that cannot be met in a period is lost, thus reducing revenue. First we give the notion:

T: number of time periods;

I: number of items;

K: number of resources;

$\tilde{b}_{kt}$: fuzzy parameters represents the amount of resource k available in time period t;

$\tilde{d}_{it}$: fuzzy parameters represent the demand for item i in time period t;

r_{it} : unit revenue for item i in time period t;

cp_{it} : unit variable cost of production for item i in time period t;

cu_{it} : unit cost of not meeting demand for item i in time period t;

cq_{it} : unit inventory holding cost for item i in time period t.

The Decision Variables

p_{it} : production of item i during time period t;

q_{it} : inventory of item i at end of time period t;

u_{it} : unmet demand of item i during time period t.

The optimization model of production planning problem to maximize revenues net with fuzzy parameters is as follows:

$$\max \sum_{t=1}^{T}\sum_{i=1}^{I}\left[r_{it}\left(\tilde{d}_{it}-u_{it}\right)-cp_{it}p_{it}-cq_{it}q_{it}-cu_{it}u_{it}\right] \quad (1)$$

subject to

$$\sum_{i=1}^{I} a_{ik}p_{it} \le \tilde{b}_{kt} \quad \forall k,t \quad (2)$$

$$q_{i,t-1}+p_{it}-q_{it}+u_{it}=\tilde{d}_{it} \quad \forall i,t \quad (3)$$

$$p_{it},q_{it},u_{it} \ge 0 \quad \forall i,t \quad (4)$$

The α-level set of the fuzzy numbers $\tilde{b}_{kt}$ and $\tilde{d}_{it}$ are defined as the ordinary set $L_\alpha(\tilde{b})$ and $L_\alpha(\tilde{d})$ respectively for which the degree of their membership function exceeds the level $\alpha \in [0,1]$. This definition is introduced by Dubois and Prade [11,12]. For a certain degree α, problems (1)-(4) can be understood as the following nonfuzzy production planning problem:

$$\max \sum_{t=1}^{T}\sum_{i=1}^{I}\left[r_{it}\left(d_{it}-u_{it}\right)-cp_{it}p_{it}-cq_{it}q_{it}-cu_{it}u_{it}\right] \quad (5)$$

subject to

$$\sum_{i=1}^{I} a_{ik}p_{it} \le b_{kt} \quad \forall k,t \quad (6)$$

$$q_{i,t-1}+p_{it}-q_{it}+u_{it}=d_{it} \quad \forall i,t \quad (7)$$

$$d_{it}\in L_\alpha\left(\tilde{d}_{it}\right) \quad \forall\ i,t \quad (8)$$

$$b_{kt}\in L_\alpha\left(\tilde{b}_{kt}\right) \quad \forall\ k,t \quad (9)$$

$$p_{it},q_{it},u_{it} \ge 0 \quad \forall i,t \quad (10)$$

The nonfuzzy production planning problem can be rewritten in the following equivalent form:

$$\max \sum_{t=1}^{T}\sum_{i=1}^{I}\left[r_{it}\left(d_{it}-u_{it}\right)-cp_{it}p_{it}-cq_{it}q_{it}-cu_{it}u_{it}\right] \quad (11)$$

subject to

$$\sum_{i=1}^{I} a_{ik}p_{it} \le b_{kt} \quad \forall k,t \quad (12)$$

$$q_{i,t-1}+p_{it}-q_{it}+u_{it}=d_{it} \quad \forall i,t \quad (13)$$

$$h_{it}\le d_{it}\le H_{it} \quad \forall\ i,t \quad (14)$$

$$l_{kt}\le b_{kt}\le L_{kt} \quad \forall\ k,t \quad (15)$$

$$p_{it},q_{it},u_{it} \ge 0 \quad \forall i,t \quad (16)$$

where h_{it} and H_{it} are lower and upper bounds on d_{it} respectively and l_{it} and L_{it} are lower and upper bounds on b_{it} respectively.

3. Qualitative Analysis of Basic Notions for the Problems (11)-(16)

Let h_{it},H_{it},l_{kt} and L_{kt} $\forall i,k,t$ are assumed to be parameters rather than constants. The decision space of problem (11)-(16) can be defined as follows:

$$X(h,H,l,L) = \left\{ p_{it},q_{it},u_{it}\in R^{3IT}\forall i,t \,\middle|\, \text{satisfies the constraints (12)-(16)}\right\}$$

In what follows we are give the definitions of some basic notions for the problem (11)-(16). Such notions are the set of feasible parameters, the solvability set and the stability set of the first kind (see [13,l4]).

3.1. The Set of Feasible Parameters

The set of feasible parameters of the problems (11)-(16), which is denoted by U, is defined by:

$$U=\left\{(h,H,l,L)\in R^{2T(I+K)} \,\middle|\, X(h,H,l,L) \text{ is not empty}\right\}$$

3.2. The Solvability Set

The solvability set of problems (11)-(16), which is denoted by V, is defined by

$$V=\left\{(h,H,l,L) \in U \,\middle|\, \text{problem (11)-(16) has } \alpha\text{-optimal solution}\right\}.$$

3.3. The Stability Set of the First Kind

Suppose that $h^*,H^*,l^*,L^*\in V$ with a corresponding α-optimal solution $\left(p^*_{it},q^*_{it},u^*_{it}\right)$ for problems (11)-(16) together with the α-level optimal parameters $\left(b^*_{kt},d^*_{it}\right)$.

The stability set of the first kind of problems (11)-(16) that is denoted by $S\left(p_{it}^{*},q_{it}^{*},u_{it}^{*}\right)$ is defined by

$$S\left(p_{it}^{*},q_{it}^{*},u_{it}^{*}\right)=\left\{\begin{array}{l}(h,H,l,L)\in V\left|\left(p_{it}^{*},q_{it}^{*},u_{it}^{*}\right)\text{ is }\alpha\text{-optimal solution of problems (11)-(16)}\right.\\ \text{with corresponding }\alpha\text{-level optimal parameters }\left(d_{it}^{*},b_{kt}^{*}\right),\forall i,k,t\end{array}\right\}$$

3.4. Determination of the Stability Set of the First Kind

The Lagrange function of problems (11)-(16) can be written as follows:

$$\begin{aligned}LF=Z&+\lambda_{kt}\left(\left(\sum_{i=1}^{I}a_{ik}p_{it}\right)-b_{kt}\right)\\&+\beta_{it}\left(q_{i,t-1}+p_{it}-q_{it}+u_{it}-d_{it}\right)\\&+\theta_{kt}\left(b_{kt}-H_{kt}\right)+\rho_{kt}\left(h_{kt}-b_{kt}\right)\\&+\sigma_{it}\left(d_{it}-L_{it}\right)+\delta_{it}\left(l_{it}-d_{it}\right)\\&-\nu_{it}p_{it}-\gamma_{it}q_{it}-\varphi_{it}u_{it}\end{aligned}$$

where

$$Z=\max\sum_{t=1}^{T}\sum_{i=1}^{I}\left[r_{it}\left(d_{it}-u_{it}\right)-cp_{it}p_{it}-cq_{it}q_{it}-cu_{it}u_{it}\right].$$

The Kuhn-Tucker necessary optimality conditions for problems (11)-(16) are as follows:

$$\frac{\partial LF}{\partial u_{it}}=0,\ \frac{\partial LF}{\partial p_{it}}=0,\ \frac{\partial LF}{\partial q_{it}}=0\ \ \forall i,t$$

$$\sum_{i=1}^{I}a_{ik}p_{it}-b_{kt}\le 0\ \ \forall k,t$$

$$q_{i,t-1}+p_{it}-q_{it}+u_{it}-d_{it}=0\ \ \forall i,t$$

$$h_{it}-d_{it}\le 0,\ d_{it}-H_{it}\le 0\ \ \forall\ i,t$$

$$l_{kt}-b_{kt}\le 0,\ b_{kt}-L_{kt}\le 0\ \ \forall\ k,t$$

$$p_{it},q_{it},u_{it}\ge 0\ \ \forall i,t$$

$$\lambda_{kt}\left(\left(\sum_{i=1}^{I}a_{ik}p_{it}\right)-b_{kt}\right)=0\ \ \forall k,t$$

$$\beta_{it}\left(q_{i,t-1}+p_{it}-q_{it}+u_{it}-d_{it}\right)=0\ \ \forall i,k,t$$

$$\theta_{kt}\left(b_{kt}-H_{kt}\right)=0\ \ \forall k,t$$

$$\rho_{kt}\left(h_{kt}-b_{kt}\right)=0\ \ \forall k,t$$

$$\sigma_{it}\left(d_{it}-L_{it}\right)=0\ \ \forall i,t$$

$$\delta_{it}\left(l_{it}-d_{it}\right)=0\ \ \forall i,t$$

$$\nu_{it}p_{it}=0,\ \ \gamma_{it}q_{it}=0,\ \ \varphi_{it}u_{it}=0\ \ \forall i,t$$

$$\lambda_{kt},\beta_{it},\theta_{kt},\rho_{kt},\sigma_{it},\delta_{it},\nu_{it},\gamma_{it},\varphi_{it}\le 0\ \ \forall i,k,t$$

where all the relations of the above system are evaluated at the α-optimal solution $\left(p_{it}^{*},q_{it}^{*},u_{it}^{*}\right)$ with the corresponding α-level optimal parameters $\left(d_{it}^{*},b_{kt}^{*}\right)$. $\lambda_{kt},\beta_{it},\theta_{kt},\rho_{kt},\sigma_{it},\delta_{it},\nu_{it},\gamma_{it}$ and φ_{it} $\forall i,t$ are the Lagrange multipliers.

4. Illustrative Example

Let us consider the following production planning problem to maximize net revenues with fuzzy parameters. Consider I = 3, K = 3 and T = 2. **Table 1** contains the values of triangle fuzzy parameters $\tilde{d}_{it}$ and $\tilde{b}_{kt}$, $i=1,2,3$, $t=1,2$, $k=1,2,3$. **Table 2** contains the values of a_{ik}, cp_{it},cq_{it},r_{it} and cu_{it} $i=1,2,3$, $t=1,2$, $k=1,2,3$.

We assume that the membership functions to the triangle fuzzy numbers $\tilde{d}_{it}$ and $\tilde{b}_{kt}$ is take the following form:

$$d_{\alpha}=\left[d_{1}+\alpha\left(d_{2}-d_{1}\right)+\alpha\left(d_{3}-d_{2}\right)\right]$$

and

$$b_{\alpha}=\left[b_{1}+\alpha\left(b_{2}-b_{1}\right)+\alpha\left(b_{3}-b_{2}\right)\right]$$

let α = 0.5, problems (11)-(16) can be written as follows:

$$\begin{aligned}\max Z=&\,2d_{11}-8u_{11}-4q_{11}+d_{21}-9u_{21}-8q_{21}\\&+3d_{31}-7u_{31}-8q_{31}+2d_{12}-7u_{12}-6q_{12}\\&+4d_{22}-11u_{22}-6q_{22}+9d_{32}-18u_{32}-9q_{32}\end{aligned}$$

Table 1. Values of triangle fuzzy parameters.

$\tilde{d}_{11}=(1,3,6)$	$\tilde{d}_{21}=(2,5,6)$	$\tilde{d}_{31}=(3,4,6)$
$\tilde{d}_{12}=(2,3,4)$	$\tilde{d}_{22}=(1,5,6)$	$\tilde{d}_{32}=(4,6,9)$
$\tilde{b}_{11}=(2,5,6)$	$\tilde{b}_{21}=(3,4,7)$	$\tilde{b}_{31}=(1,4,6)$
$\tilde{b}_{12}=(2,5,7)$	$\tilde{b}_{22}=(3,6,8)$	$\tilde{b}_{32}=(2,4,5)$

Table 2. Values of a_{ik}, cp_{it}, cq_{it}, r_{it} and cu_{it} $\forall$ i,t,k .

$a_{11}=3$	$a_{21}=1$	$a_{31}=2$
$a_{12}=4$	$a_{22}=6$	$a_{32}=5$
$a_{13}=4$	$a_{23}=5$	$a_{33}=6$
$cp_{11}=2$	$cp_{21}=5$	$cp_{31}=4$
$cp_{12}=3$	$cp_{22}=4$	$cp_{32}=6$
$cq_{11}=2$	$cq_{21}=3$	$cq_{31}=4$
$cq_{12}=3$	$cq_{22}=2$	$cq_{32}=3$
$cu_{11}=6$	$cu_{21}=8$	$cu_{31}=4$
$cu_{12}=5$	$cu_{22}=7$	$cu_{32}=9$
$r_{11}=2$	$r_{21}=1$	$r_{31}=3$
$r_{12}=2$	$r_{22}=4$	$r_{32}=2$

subject to

$$3p_{11}+p_{21}+2p_{31}\le b_{11},\quad 4p_{11}+6p_{21}+5p_{31}\le b_{21},$$

$$4p_{12}+5p_{21}+6p_{31}\le b_{31},$$

$$3p_{12}+p_{22}+2p_{32}\le b_{12},\quad 4p_{12}+6p_{22}+5p_{32}\le b_{22},$$

$$4p_{12}+5p_{22}+6p_{32}\le b_{32},$$

$$q_{1,0}+p_{11}-q_{11}+u_{11}-d_{11}=0,$$

$$q_{2,0}+p_{21}-q_{21}+u_{21}-d_{21}=0,$$

$$q_{3,0}+p_{31}-q_{31}+u_{31}-d_{31}=0,$$

$$q_{1,1}+p_{12}-q_{12}+u_{12}-d_{12}=0,$$

$$q_{2,1}+p_{22}-q_{22}+u_{22}-d_{22}=0,$$

$$q_{3,1}+p_{32}-q_{32}+u_{32}-d_{32}=0,$$

$$3.5\le b_{11}\le 5.5,\quad 3.5\le b_{21}\le 5.5,\quad 2.5\le b_{31}\le 5,$$

$$3.5\le b_{12}\le 6,\quad 4.5\le b_{22}\le 7,\quad 3\le b_{32}\le 4.5,$$

$$2\le d_{11}\le 4.5,\quad 3.5\le d_{21}\le 5.5,\quad 3.5\le d_{31}\le 5,$$

$$2.5\le d_{12}\le 3.5,\quad 3\le d_{22}\le 5.5,\quad 5\le d_{32}\le 7.5$$

So, we get the following results by using any software package for solving linear programming problem:

$$p_{11}=1.375, p_{32}=0.5, q_{12}=1.5, q_{22}=0.5, u_{11}=0.625,$$
$$u_{21}=u_{31}=3.5, d_{11}=2, d_{21}=3.5,$$

$$d_{31}=d_{12}=3.5, d_{22}=5.5, d_{32}=7.5, b_{11}=4.125,$$
$$b_{21}=5.5, b_{31}=2.5, b_{12}=3.5, b_{22}=4.5.$$

$b_{32}=3$ and all other variables are equal to zero. Objective function value is equal to 41.5. The sets of feasible parameters, solvability set and the stability set of the first kind are calculated as follows:

$$U=\{h_{it},H_{it},l_{kt},L_{kt}>0,i,k=1,2,3,t=1,$$
$$2|h_1\le H_1,h_2\le H_2,h_3\le H_3,l_1\le L_1,l_2\le L_2,l_3\le L_3\}$$

5. Conclusion

In this paper, we have discussed the concept of stability for the planning problem to maximize revenues net of the production inventory and lost sales cost. We used fuzzy parameters to represent both of amount of available resources and demand for item in period. We have defined and characterized some basic notions for the problem of concern. These notions are the set of feasible parameters, the solvability set and the stability set of the first kind. Although an extensive literature on models for production planning under uncertainty was reviewed, a need for further research is identified as the development of new models that contain additional sources and types of uncertainty, such as supply lead times, transport times, quality uncertainty, failure of production system and changes to product structure. Also as a point of further research, an investigation of incorporation all types of uncertainty is needed.

REFERENCES

[1] P. J. Billington, J. O. McClain and L. J. Thomas, "Mathematical Programming Approaches to Capacity Constrained MRP Systems: Review, Formulation and Problem Reduction," *Management Science*, Vol. 29, No. 15, 1983, pp. 1126-1141.

[2] A. C. Hax and D. Candea, "Production and Inventory Management," Prentice-Hall, Englewood, 1984.

[3] B. Kim and S. Kim, "Extended Model for a Hybrid Production Planning Approach," *International Journal of Production Economics*, Vol. 73, No. 2, 2001, pp. 165-173. doi:10.1016/S0925-5273(00)00172-9

[4] J. Galbraith, "Designing Complex Organizations," Addison-Wesley, Boston, 1973.

[5] J. Mula, R. Poler, J. P. Garcia-Sabater and F. C. Lario, "Models for Production Planning under Uncertainty: A Review," *International Journal of Production Economics*, Vol. 103, No. 1, 2006, pp. 271-285.

[6] A. Jamalnia and M. Ali Aoukhakian, "A Hybrid Fuzzy Goal Programming Approach with Different Goal Priorities to Aggregate Production Planning," *Computers & Industrial Engineering*, Vol. 56, No. 4, 2009, pp. 1474-1486.

[7] L. Zrinka, K. Soric and V. V. Rosenzweig, "Production Planning Problem with Sequence Dependent Setups as a Bilevel Programming Problem," *European Journal of Operational Research*, Vol. 187, No. 3, 2008, pp. 1504-1512.

[8] C. Ho, "Evaluating the Impact of Operating Environments on MRP System Nervousness," *International Journal of Production Research*, Vol. 27, No. 7, 1989, pp. 1115-1135.

[9] S. P. Sethi, H. Yan and Q. Zang, "Optimal and Hierarchical Controls in Dynamic Stochastic Manufacturing Systems: A Survey," *Manufacturing and Service Operations Management Journal*, Vol. 4, No. 2, 2002, pp. 133-170.

[10] C. A. Yano and H. L. Lee, "Lot Sizing with Random Yields: A Review," *Operations Research*, Vol. 43, No. 2, 1995, pp. 311-334.

[11] D. Dubois and H. Prade, "Fuzzy Sets and Systems, Theory and Applications," Academic Press, New York, 1980.

[12] D. Dubois and H. Prade, "Operations on Fuzzy Numbers," *International Journal of System Science*, Vol. 9, No. 6, 1978, pp. 613-626.

[13] M. Osman, "Qualitative Analysis of Basic Notions in Parametric Programming, II (Parameters in the Objective

Function)," *Applied Mathematics*, Vol. 22, No. 5, 1977, pp. 333-348.

[14] M. osman and A. H. El-Banna, "Stability of Multiobjective Nonlinear Programming Problems with Fuzzy Parameters," *Mathematics and Computers in Simulation*, Vol. 35, No. 4, 1993, pp. 321-326.