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Weakly Nonlinear Quantum Dust Ion-Acoustic Waves

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ABSTRACT

The one-dimensional quantum hydrodynamic (QHD) model for a three-specie quantum plasma is used to study the quantum counterpart of the well known dust ion-acoustic wave (DIAW). It is found that owing to the quantum effects, the dynamics of small but finite amplitude quantum dust ion-acoustic waves (QDIA) is governed by a deformed Korteweg-de Vries equation (dK-dV). The latter admits compressive as well as rarefactive stationary QDIA solitary wave solution. In the fully quantum case, the QDIA soliton experiences a spreading which becomes more significant as electron depletion is enhanced.

Keywords: Dusty Plasmas; Dust Ion-Acoustic Waves; Quantum Plasmas; QHD Model; Solitary Waves; Deformed K-dV Equation

1. Introduction

Linear as well as nonlinear collective processes in dusty or complex plasmas have received special attention in the past decade mainly due to the realization of their occurrence in both the laboratory and space environments [1-3]. A dusty plasma is a normal electron-ion plasma with an additional highly charged component of small micron or sub-micron sized extremely massive charged particulates (dust grains). Wave propagation in such complex systems is therefore expected to be substantially different from the ordinary two component plasmas and the presence of charged dust can have a strong influence on the characteristics of the usual plasma wave modes, even at frequencies where the dust grains do not participate in the wave motion. It has been found that the presence of static charged dust grains modifies the existing plasma wave spectra. On the other hand, it has been shown that the dust dynamics introduces new eigenmodes, such as, dust-acoustic (DA) mode [4] (weak coupling regime), dust-lattice (DL) mode [5] (strong coupling regime), dust Bernstein-Greene-Kruskal (DBGK) modes [6-8], etc. Among the host of modified dusty modes discussed in the literature, the dust ion-acoustic wave (DIAW) has received wide attention as well as experimental confirmation in several low-temperature dusty plasma devices [9-14]. Shukla and Silin [15] have first theoretically shown that due to the conservation of equi-

librium charge density and the strong electron depletion, a dusty plasma (with negatively charged static dust grains) supports low-frequency DIA waves. The latter is the usual ion-acoustic mode modified by the presence of dust particles. In contrast to the DAWs, the frequency of the DIAWs is much larger (smaller) than the dust (ion) plasma frequency, whereas their phase velocity is much larger (smaller) than the ion and dust (the electron) thermal velocities (velocity). Hence, in the DIAWs, the restoring force comes from the pressure of the inertialess electrons, while the ion mass provides the inertia to maintain the DIAWs. On a time scale much longer (shorter) than the ion (dust) plasma period, the dust grains remain almost immobile. Recent technological advances on miniaturized semiconductor devices and nanoscal objects have made it possible to envisage practical applications of plasma physics where the quantum nature of the particles plays a crucial role. One reason is that the great degree of miniaturization of today's electronic components is such that the De Broglie wavelength of the charge carriers becomes comparable to the system length and the tunneling effects are therefore no longer negligible. The topic of quantum plasmas has then attracted considerable attention [16-35]. For instance, quantum plasma echoes [19], the expansion of a quantum electron gas into vacuum [20], the quantum two and three stream instabilities [21-23], the self-consistent dy-

namics of Fermi gases [24], the quantum ion-acoustic waves [25], the quantum corrected electron holes [27], the modified quantum Zakharov equations [28], and the quantum dust modes have been the subject of intense investigations [29-33]. Recently [34], we have reported, for the first time, the nonlinear features of the quantum counterpart of the well known DIAW. The aim of the present paper is therefore to show the existence, formation and possible realization of weakly nonlinear quantum dust-ion acoustic (QDIA) solitary waves. The derivation goes parallel to that done in standard quantum hydrodynamic analysis [25]. The manuscript is organized as follows. The one-dimensional quantum hydrodynamic (QHD) model for three species quantum dusty plasma describing the dynamics of low phase velocity dust ion-acoustic oscillations is given in Section 2. A weakly nonlinear analysis is carried out in Section 3. A summary of our results and findings is given in Section 4.

2. Theoretical Model

We consider a system consisting of electrons, singly charged positive ions, equiradius spherical dust grains carrying identical charge and mass. The nonlinear dynamics of low phase velocity QDIA oscillations is governed by the one-dimensional quantum hydrodynamic (QHD) model

$$\frac{\partial n_e}{\partial t} + \frac{\partial (n_e u_e)}{\partial x} = 0 \tag{1}$$

$$\frac{\partial n_i}{\partial t} + \frac{\partial (n_i u_i)}{\partial x} = 0 \tag{2}$$

$$\frac{\partial u_e}{\partial t} + u_e \frac{\partial u_e}{\partial x} = \frac{e}{m_e} \frac{\partial \phi}{\partial x} - \frac{1}{m_e n_e} \frac{\partial p_e}{\partial x}$$
$$+ \frac{\hbar^2}{2m_e^2} \frac{\partial}{\partial x} \left(\frac{\partial^2 \sqrt{n_e}/\partial x^2}{\sqrt{n_e}} \right) \tag{3}$$

$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x}$$
$$= -\frac{e}{m_i} \frac{\partial \phi}{\partial x} + \frac{\hbar^2}{2m_i^2} \frac{\partial}{\partial x} \left(\frac{\partial^2 \sqrt{n_i}/\partial x^2}{\sqrt{n_i}} \right) \tag{4}$$

$$\frac{\partial^2 \phi}{\partial x^2} = 4\pi e \left(n_e - n_i + Z_d n_{d0} \right) \tag{5}$$

The dust grains are usually much heavier than the ions and electrons and their dynamics is on a much longer time. They are taken to be immobile and negatively charged, $q_d = -Z_d e$, where Z_d is the number of charges residing on the dust grain, $u_{e,i}$ is the electron (ion) fluid velocity, ϕ is the electrostatic potential, n_j refers to the charged particles number density, n_{j0} indicates the corresponding equilibrium values with obvious labels

e, i and d, m_j are the mass, while $\hbar$ represents the scaled Planck's constant. We assume that the electrons obey the following pressure law in a one-dimensional zero-temperature Fermi gas [24,25,35].

$$p_e = \frac{m_e V_{Fe}^2}{3 n_{e0}^3} n_e^3 \tag{6}$$

where $V_{Fe} = \left(2k_B T_{Fe}/m_e \right)^{1/2}$ is the electron Fermi speed, k_B the Boltzmann constant, and T_{Fe} the electron Fermi temperature. For the sake of simplicity, pressure effects are disregarded for ions. Notice that the quantum corrections (quantum diffraction and quantum statistics) appear through the terms proportional to $\hbar^2$ in (3) and (4) and via the equation of state (6). Adopting the following normalization

$$N_j \rightarrow n_j/n_{j0}, U_j \rightarrow u_j/C_s, \Phi \rightarrow e\phi/\left(2k_B T_{Fe} \right),$$
$$T \rightarrow \omega_{pi} t, X \rightarrow \omega_{pi} x/C_s ,$$

where $\omega_{pi} = \left(4\pi n_{i0} e^2/m_i \right)^{1/2}$ is the corresponding ion plasma frequency, and $C_s = \left(2k_B T_{Fe}/m_i \right)^{1/2}$ is a quantum ion-acoustic velocity, Equations (1)-(5) can be re-written as

$$\frac{\partial N_e}{\partial T} + \frac{\partial (N_e U_e)}{\partial X} = 0 \tag{7}$$

$$\frac{m_e}{m_i} \left(\frac{\partial U_e}{\partial T} + U_e \frac{\partial U_e}{\partial X} \right)$$
$$= \frac{\partial \Phi}{\partial X} - N_e \frac{\partial N_e}{\partial X} + \mu \frac{H_e^2}{2} \frac{\partial}{\partial X} \left(\frac{\partial^2 \sqrt{N_e}/\partial X^2}{\sqrt{N_e}} \right) \tag{8}$$

$$\frac{\partial N_i}{\partial T} + \frac{\partial (N_i U_i)}{\partial X} = 0 \tag{9}$$

$$\frac{\partial U_i}{\partial T} + U_i \frac{\partial U_i}{\partial X}$$
$$= -\frac{\partial \Phi}{\partial X} + \mu \frac{m_e}{m_i} \frac{H_e^2}{2} \frac{\partial}{\partial X} \left(\frac{\partial^2 \sqrt{N_i}/\partial X^2}{\sqrt{N_i}} \right) \tag{10}$$

$$\frac{\partial^2 \Phi}{\partial X^2} = N_e/\mu - N_i + \left(1 - 1/\mu \right) \tag{11}$$

Here, $\mu = n_{i0}/n_{e0}$ measures the unperturbed ion and electron number density imbalance, and $H_e = \hbar \omega_{pe}/\left(2k_B T_{Fe} \right)$ is a nondimensional quantum parameter determining the ratio between the electron plasmon energy and the electron Fermi energy, where

$\omega_{pe} = \left(4\pi n_{e0} e^2/m_e \right)^{1/2}$ is the corresponding electron plasma frequency. Neglecting the left-hand side of Equation (8) due to $m_e/m_i \ll 1$, integrating once and discarding terms proportional to $m_e/m_i \ll 1$ in Equation (10), we obtain the following reduced model

$$\Phi = -\frac{1}{2} + \frac{N_e^2}{2} - \frac{\mu H_e^2}{2\sqrt{N_e}}\frac{\partial^2 \sqrt{N_e}}{\partial X^2} \qquad (12)$$

$$\frac{\partial N_i}{\partial T} + \frac{\partial(N_i U_i)}{\partial X} = 0 \qquad (13)$$

$$\frac{\partial U_i}{\partial T} + U_i \frac{\partial U_i}{\partial X} = -\frac{\partial \Phi}{\partial X} \qquad (14)$$

$$\frac{\partial^2 \Phi}{\partial X^2} = N_e/\mu - N_i + (1 - 1/\mu) \qquad (15)$$

It may be useful to note that in the linear limit, the system (12)-(13) gives

$$\omega^2 = \frac{\mu k^2 \left(1 + \mu H_e^2 k^2/4\right)}{1 + \mu k^2 \left(1 + \mu H_e^2 k^2/4\right)} \qquad (16)$$

where ω and k represent, respectively, the normalized wave frequency and the normalized wave number.

3. Weakly Nonlinear QDIA Waves

To study small but finite amplitude QDIA solitary waves, we follow the well known reductive perturbation technique [36]. We first introduce the stretched variables $\xi = \varepsilon^{1/2}(x - v_0 t)$ and $\tau = \varepsilon^{3/2} t$, where ε is a smallness expansion parameter, measuring the amplitude of the wave or the strength of nonlinearity, and v_0 is the unknown soliton velocity normalized to C_d, to be determined later. Substituting power series expansions of $N_e, N_i,$ and U_i

$$N_e = 1 + \varepsilon N_e^{(1)} + \varepsilon^2 N_e^{(2)} + \cdots \qquad (17)$$

$$N_i = 1 + \varepsilon N_i^{(1)} + \varepsilon^2 N_i^{(2)} + \cdots \qquad (18)$$

$$U_i = \varepsilon U_i^{(1)} + \varepsilon^2 U_i^{(2)} + \cdots \qquad (19)$$

into Equations (12)-(15) gives to lowest order in ε, $N_e^{(1)} = \mu N_i^{(1)} = v_0 U_i^1 = U(\xi, \tau)$ and $v_0 = \sqrt{\mu}$. Considering the next higher order in ε, we obtain the following set of equations

$$\frac{\partial U_i^{(2)}}{\partial \xi} + \frac{2}{v_0^3} U \frac{\partial U}{\partial \xi} - v_0 \frac{\partial N_i^{(2)}}{\partial \xi} + \frac{1}{v_0^2}\frac{\partial U}{\partial \tau} = 0 \qquad (20)$$

$$\frac{1}{v_0}\frac{\partial U}{\partial \tau} + \frac{1}{v_0^2} U \frac{\partial U}{\partial \xi} - v_0 \frac{\partial U_i^{(2)}}{\partial \xi}$$

$$+ \frac{\partial N_e^{(2)}}{\partial \xi} + U \frac{\partial U}{\partial \xi} - \frac{\mu H_e^2}{4}\frac{\partial^3 U}{\partial \xi^3} = 0 \qquad (21)$$

$$\frac{N_e^{(2)}}{\mu} - N_i^{(2)} - \frac{\partial^2 U}{\partial \xi^2} = 0 \qquad (22)$$

from which we derive the following equation

$$\frac{\partial U}{\partial \tau} + \frac{\sqrt{\mu}}{2}\left(1 + \frac{3}{\mu}\right)U\frac{\partial U}{\partial \xi}$$

$$+ \frac{\mu^{3/2}}{2}\left(1 - \frac{H_e^2}{4}\right)\frac{\partial^3 U}{\partial \xi^3} = 0 \qquad (23)$$

Equation (23) is a deformed Korteweg-de Vries equation (dK-dV) in which quantum diffraction is responsible for the term proportional to H_e^2. For $H_e \neq 2$, we can transform the independent variables ξ and τ to $\eta = \xi - u_0 \tau$ and $\tau = \tau$, where u_0 is a normalized constant speed and impose appropriate boundary conditions for localized perturbations, namely $U \to 0, \partial U/\partial \eta \to 0, \partial^2 U/\partial \eta^2 \to 0$ as $|\eta| \to \infty$. Performing the last step in deriving soliton solutions, one gets

$$U(\xi, \tau) = U_m \operatorname{Sech}^2\left(\frac{\xi - u_0 t}{\delta}\right) \qquad (24)$$

where $U_m = 6u_0 / \left[\sqrt{\mu}(1 + 3/\mu)\right]$ and

$\delta = \sqrt{2\mu^{3/2}\left(1 - \dfrac{H_e^2}{4}\right)/u_0}$ represent the amplitude and

the width of the solitary wave, respectively. Note that U_m does no longer depend on H_e. Despite the fact that quantum effects leave the absolute amplitude of the QDIA soliton unaffected, for H_e smaller or greater than 2 the soliton may exhibit compression (with a phase spped $u_0 > 0$) or rarefaction (with a phase speed $u_0 < 0$). As a result of quantum effects, the QDIA soliton experiences either a compression for $H_e < 2$ (**Figure 1**) or a spreading in the fully quantum case (**Figure 2**). This spreading is more significant as the unperturbed ion and electron number density imbalance (or electron depletion) is enhanced (**Figure 3**) (it is worth to note that certain values
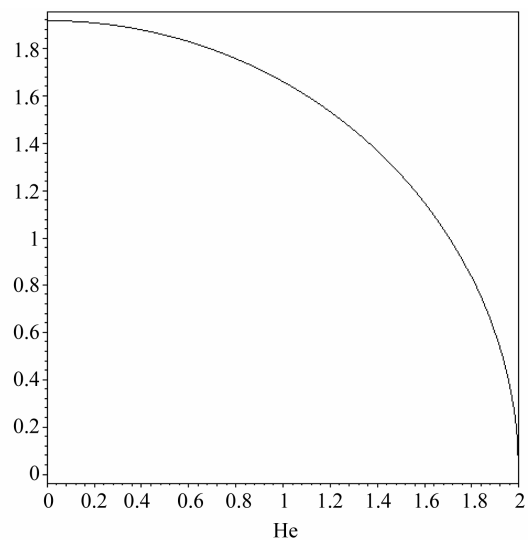

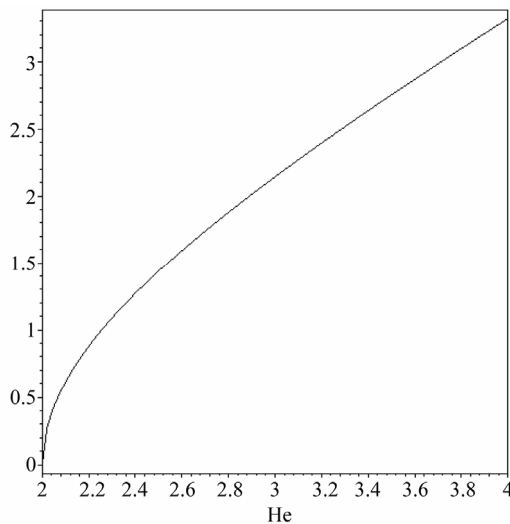**Figure 1. Plot of the QDIA soliton width δ versus H_e < 2 with u_0 = 1 and μ = 1.2.**

Figure 2. Plot of the QDIA soliton width δ versus $H_e > 2$ with $u_0 = -1$ and $\mu = 1.2$.
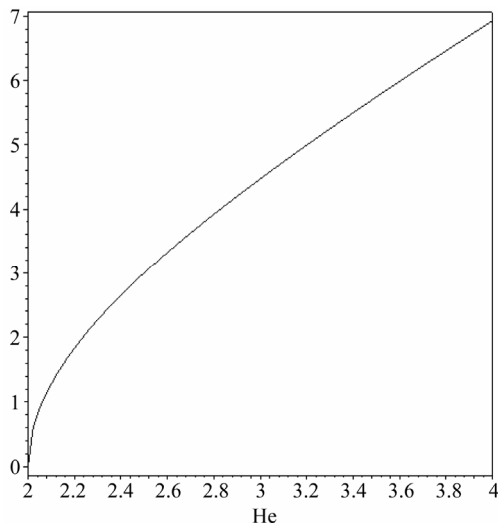

Figure 3. Plot of the QDIA soliton width δ versus $H_e > 2$ with $u_0 = -1$ and $\mu = 4$.

of δ and H_e which make U_m large enough to break the validity of the weakly nonlinear analysis have to be discarded).

4. Conclusion

To conclude, we have addressed the problem of quantum dust ion-acoustic solitary waves. The dynamics of small but finite amplitude QDIA waves is governed by a deformed Korteweg-de Vries equation. The latter admits compressive as well as rarefactive stationary solitary wave solution. For $H_e < 2$, the quantum effects tend to lower the soliton width. In the fully quantum case, the QDIA soliton experiences a spreading which becomes more significant as the unperturbed ion and electron number density imbalance is enhanced. Our results

should help for diagnostics of charged impurities in micloelectronics and to understand the salient features of coherent nonlinear structures that may occur in space quantum dusty plasmas.

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