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Surface Wave Characteristics at the Interface of Welded Elastic Halfspaces

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The present article concentrates on the propagation of generalized surface acoustic waves in a composite structure consisting of piezoelectric and non-piezoelectric semiconductor media. The mathematical model of the problem is depicted by a set of partial differential equations of motion, Gauss equation in piezoelectric and electron diffusion equation in semiconductor along with boundary conditions to be satisfied at the interface. The secular equation that governs the propagation of surface waves has been derived in compact form after obtaining the formal solution. The analytic expressions for displacements, stresses, piezoelectric potential and electron concentration during the surface wave propagation at the interface have also been obtained. The numerical solution of the secular equation is carried out for the cadmium selenide and silicon composite by employing fixed point functional iteration numerical method along with irreducible Cardano method. The computer simulated results with the help of MATLAB software in respect of dispersion curves, attenuation coefficient, displacements, stresses, carrier concentration and piezoelectric potential are presented graphically. This work may be useful in surface acoustic wave (SAW) devices and electronic industry.

Keywords: Piezoelectrics, Life Time, Silicon, Dispersive Waves, Attenuation

Introduction

The piezoelectric effect in certain noncentro-symmetric crystalline materials was discovered by Curie and Curie (1880). Parmenter (1953) regarded the appearance of DC electric field along the direction of propagation of a acoustic wave in a medium containing mobile charges as acoustoelectric effect. Weinreich *et al.* (1959) termed acoustoelectric effect as wave particle drag phenomenon. Hutson and White (1962) found that the field produced along the traveling acoustic wave produces current and space charges which results in acoustic dispersion and loss. According to White (1962) an acoustic wave traveling in a piezoelectric semiconductor can be amplified or attenuated by the application of a DC electric field. Collins *et al.* (1968) found the strong interaction between the wave on the surface of piezoelectric crystal and the wave on the drifting carriers in a nearby semiconductor. Bluestein (1968) and Gulyaev (1969) studied surface acoustic waves in piezoelectric materials. Fischler (1970) proposed that acoustoelectric amplification can be better obtained in composite structure of semiconductor and piezoelectric materials. Dietz *et al.* (1988) explored that the acoustoelectric amplification of acoustic waves can also be achieved through composite of a piezoelectric dielectric and non-piezoelectric semiconductor.

de Lorenzi and Tierten (1975), and Maugin and Dehar (1986) developed nonlinear theories for deformable semiconductors. Ingebrigtsen (1970) studied linear and non-linear attenuation of acoustic surface waves in a piezoelectric coated with a semiconductor film. Tien (1968) presented the nonlinear theory of ultrasonic amplification and current saturation in piezoelectric semiconductors. Kagan (1997) investigated the surface wave propagation in a piezoelectric crystal underlying a two dimensional conducting layer. Jin *et al*. (2002) studied the Lamb wave propagation in a metallic semi-infinite medium covered with piezoelectric layer. Wang (2002) investigated wave propagation in the piezoelectric solid medium. Yang and Zhou (2005) investigated amplification of acoustic waves in piezoelectric semiconductor plates. Yang and Zhou (2005) also studied the propagation and amplification of gap waves between a piezoelectric halfspace and a semiconductor film. Maruszewski (1989) considered the interactions between elastic, thermal and charge carrier's fields in semiconductors and predicted the existence of two kinds of waves namely, polarized and dispersive waves. Kleinert *et al.* (2005) studied the surface-acoustic-wave-induced space-charge waves in electron-hole systems.

Sharma and Pal (2004) investigated the propagation of Lamb waves in homogeneous, transversely isotropic, piezothermoelastic plate. Sharma *et al.* (2005) studied the propagation characteristics of Rayleigh waves in transversely isotropic piezothermoelastic materials. The phase velocity profiles are found to be dispersive at small values of wave number and these become asymptotically linear at higher values of wave numbers. Sharma and Walia (2007) carried out further investigations on the propagation of Rayleigh waves in a homogeneous, transversely isotropic, piezothermoelastic semi-space. Sharma and Thakur (2006) studied the plane harmonic elasto-thermodiffusive waves in semiconductor materials. Sharma *et al.* (2007, 2009) also investigated the characteristics of elasto-thermodiffusive wave propagation on semiconductor materials and observed that life time of charge carriers and thermal relaxation time affects the wave characteristics significantly at long wavelengths as compared to that at short wavelengths. Sharma *et al.* (2008) investigated the elasto-thermodiffusive surface waves in a semiconductor halfspace underlying a fluid with varying temperature. Recently, Sharma *et al.* (2010) studied the surface waves at the interface of semiconductor layer over a piezoelectric halfspace and found that phase velocity as well as attenuation decreases with the decreasing life time of the carrier field.

Keeping in view the above work, the present article is devoted to give detailed information of generalized surface acoustic waves at the interface of the piezoelectric and semiconductor halfspaces. The behavior of displacement components, stresses, electron concentration and piezoelectric potential at the interface of considered structure has been discussed. The effect of life time of the carrier field on phase velocity and attenuation coefficient is also taken into consideration so as to understand the interaction of acoustic wave in the piezoelectric halfspace with the carriers in the semiconductor halfspace.

Formulation of Problem

We consider a composite structure consisting of a homogeneous transversely isotropic piezoelectric halfspace and a homogeneous isotropic, non-piezoelectric elastic semiconductor halfspace which are in welded contact with each other as shown in Figure 1. We take the origin of coordinate system $oxyz$ at any point on the plane surface (interface) and z-axis pointing vertically downward into the piezoelectric halfspace along the poling direction. Thus, the piezoelectric halfspace and the semiconductor medium are represented by $z \ge 0$ and $z \le 0$ respectively. We choose *x*-axis along the direction of wave propagation in such a way that all particles on a line parallel to the *y*-axis are equally displaced. Therefore, all field quantities are independent of y-coordinate.

Further, the disturbance is assumed to be confined in the neighborhood of the interface $(z=0)$ and hence vanishes as $z \to \infty$. The basic governing equations of motion and electron diffusion for the composite structure under study, in the absence of body forces and electric sources, are given below:

1) Homogeneous isotropic, *n*-type semiconductor elastic halfspace [Maruszewski (1989), Sharma *et al.* (2007)]:

$$\mu\nabla^2\boldsymbol{u}^s + (\lambda+\mu)\nabla\nabla\cdot\boldsymbol{u}^s - \lambda^n\nabla N = \rho^s\ddot{\boldsymbol{u}}^s \tag{1}$$

$$\rho^s D^n\nabla^2 N - \rho^s\left(1+t^n\frac{\partial}{\partial t}\right)\dot{N} - a_2^n T_0\lambda^T\nabla\cdot\dot{\boldsymbol{u}}^s = -\left(1+t^n\frac{\partial}{\partial t}\right)\left(\frac{\rho^s}{t_n^+}\right)N \tag{2}$$

2) Homogeneous, transversely isotropic, piezoelectric (6 mm class) medium [Sharma and Pal (2004)]:

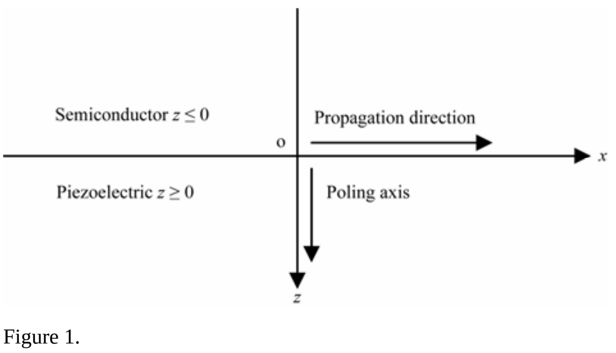

Figure 1.
Geometry of the problem.

$$c_{11}u^p_{,xx} + c_{44}u^p_{,zz} + (c_{13}+c_{44})w^p_{,xz} + (e_{15}+e_{31})\phi^p_{,xz} = \rho^p\ddot{u}^p \tag{3}$$

$$(c_{13}+c_{44})u^p_{,xz} + c_{44}w^p_{,xx} + c_{33}w^p_{,zz} + e_{15}\phi^p_{,xx} + e_{33}\phi^p_{,zz} = \rho^p\ddot{w}^p \tag{4}$$

$$(e_{15}+e_{31})u^p_{,xz} + e_{15}w^p_{,xx} + e_{33}w^p_{,zz} - \varepsilon_{11}\phi^p_{,xx} - \varepsilon_{33}\phi^p_{,zz} = 0 \tag{5}$$

where the notations $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}$, $N = n - n_0$, $a_2^n = \frac{a^{Qp}}{a^Q}$, $\lambda^T = (3\lambda+2\mu)\alpha_T$ have been used. In the above equations the superposed dots on various quantities denote time differentiation and comma nota- tion is used for spatial derivatives. Here λ, μ are Lamè's parameters; ρ^s is the density; λ^n is the elastodiffusive con- stants of electrons; D^n is the diffusion coefficient of electron; t_n^+ and t^n are the life time and relaxation time of the carriers fields; n_0 and n are the equilibrium and non-equilibrium values of electrons concentration; α_T is the coefficient of linear thermal expansion of the semiconductor material. The quantities a^{Qp}, a^Q are flux-like constants and T_0 is the uniform temperature; $\boldsymbol{u}^s = (u^s, 0, w^s)$ and $\boldsymbol{u}^p = (u^p, 0, w^p)$ are displacement vectors for semiconductor and of piezoelectric materials respectively. The quantities ϕ^p, ρ^p, c_{ij} and e_{ij} are the electric potential, density, elastic parameters and piezoelectric constants; ε_{11} and ε_{33} are the electric permittivity perpendicular and along the axis of symmetry of piezoelectric material, respectively. Throughout this paper the superscripts p, s on the field quantities and material parameters refers to piezoelectric and semiconductor materials respectively.

The non-vanishing components of stresses, current density and electric displacement in both the media are:

$$\begin{aligned}
\tau^s_{zz} &= (\lambda+2\mu)\frac{\partial w^s}{\partial z} + \lambda\frac{\partial u^s}{\partial x} - \lambda^n N,\\
\tau^s_{xz} &= \mu\left(\frac{\partial u^s}{\partial z} + \frac{\partial w^s}{\partial x}\right),\\
J^s_z &= -eD^n N_{,z}
\end{aligned} \tag{6}$$

$$\begin{aligned}
\tau^p_{zz} &= c_{13}\frac{\partial u^p}{\partial x} + c_{33}\frac{\partial w^p}{\partial z} + e_{33}\frac{\partial \phi^p}{\partial z},\\
\tau^p_{xz} &= \frac{c_{44}}{2}\left(\frac{\partial u^p}{\partial z} + \frac{\partial w^p}{\partial x}\right) + e_{15}\frac{\partial \phi^p}{\partial x},\\
D^p_z &= e_{31}\frac{\partial u^p}{\partial x} + e_{33}\frac{\partial w^p}{\partial z} - \varepsilon_{33}\frac{\partial \phi^p}{\partial z}
\end{aligned} \tag{7}$$

Here τ^s_{ij} and τ^p_{ij} are the stress tensors. The quantities J^s_z and $N_{,z}$ respectively denote the current density and carrier density gradient in semiconductor; D^p_z is the electric displacement vector of piezoelectric material and e is the electronic charge. The above model consisting of partial differential equations of motion, Gauss equation and equation for electron diffusion is also subjected to the continuity of stresses, displacements, electric fields and current density at the interface $(z=0)$ of two media. Mathematically, this requirement leads to the following interfacial boundary conditions:

$$\tau^p_{zz} = \tau^s_{zz},\ \tau^p_{xz} = \tau^s_{xz},\ u^p = u^s,\ w^p = w^s,\ \phi^p_z = N,\ D^p_z = J^s_z \tag{8}$$

We define the following quantities

$$x' = \frac{\omega^* x}{v_l},\ z' = \frac{\omega^* z}{v_l},\ t' = \omega^* t,\ t^{n'} = \omega^* t^n,\ t_n^{+'} = \omega^* t_n^+,$$
$$N' = \frac{N}{n_0},\ D_z^{p'} = \frac{\rho^s v_l^2}{\lambda^n n_0 e_{33}} D_z^p,\ u^{s'} = \frac{\rho^s \omega^* v_l}{\lambda^n n_0} u^s,$$
$$w^{s'} = \frac{\rho^s \omega^* v_l}{\lambda^n n_0} w^s,\ u^{p'} = \frac{\rho^s \omega^* v_l}{\lambda^n n_0} u^p,\ w^{p'} = \frac{\rho^s \omega^* v_l}{\lambda^n n_0} w^p,$$
$$\tau_{ij}^{p'} = \frac{\tau_{ij}^p}{\lambda^n n_0},\ \tau_{ij}^{s'} = \frac{\tau_{ij}^s}{\lambda^n n_0},\ J_z^{s'} = \frac{J_z^s}{e n_0 v_l},\ c_1 = \frac{c_{33}}{c_{11}},$$
$$c_2 = \frac{c_{44}}{c_{11}},\ c_3 = \frac{c_{13}+c_{44}}{c_{11}},\ e_1 = \frac{e_{15}+e_{31}}{e_{33}},\ v_l^2 = \frac{\lambda + 2\mu}{\rho^s}, \tag{9}$$
$$e_2 = \frac{e_{15}}{e_{33}},\ \bar{\varepsilon} = \frac{\varepsilon_{11}}{\varepsilon_{33}},\ \eta_3 = \frac{\varepsilon_{33} c_{11}}{e_{33}^2},\ \omega' = \frac{\omega}{\omega^*},\ c' = \frac{c}{v_l}$$
$$v_p = \sqrt{\frac{c_{11}}{\rho^p}},\ \bar{\rho} = \frac{\rho^p}{\rho^s},\ \bar{\lambda}_n = \frac{\lambda^n n_0}{\lambda^T T_0},\ \delta_1^2 = \frac{v_l^2}{v_p^2}$$
$$\phi^{p'} = \varepsilon_p \phi^p,\ \varepsilon_p = \frac{e_{33}\omega^* \rho^s v_l}{c_{11}\lambda^n n_0},\ \varepsilon_n = \frac{a_2^n \lambda^{T^2} T_0^2 \bar{\lambda}_n}{\rho^s (\lambda + 2\mu) n_0}$$
$$\omega^* = \frac{v_l^2}{D^n},\ \delta^2 = \frac{v_t^2}{v_l^2},\ v_t^2 = \frac{\mu}{\rho^s}$$

where ω^* is the characteristic frequency, and v_l, v_t are respectively, the longitudinal and shear wave velocities. Upon introducing the quantities (9) in Equations (1) to (5) we obtain

$$\delta^2 \nabla^2 \boldsymbol{u}^s + (1-\delta^2)\nabla\nabla\cdot \boldsymbol{u}^s - \nabla N = \ddot{\boldsymbol{u}}^s \tag{10}$$

$$\nabla^2 N - \left[-\frac{1}{t_n^+} + \left(1 - \frac{t^n}{t_n^+}\right)\frac{\partial}{\partial t} + t^n \frac{\partial^2}{\partial t^2} \right] N - \varepsilon_n \nabla \cdot \dot{\boldsymbol{u}}^s = 0 \tag{11}$$

$$u_{,xx}^p + c_2 u_{,zz}^p + c_3 w_{,xz}^p + e_1 \phi_{,xz}^p = \delta_1^2 \ddot{u}^p \tag{12}$$

$$c_3 u_{,xz}^p + c_2 w_{,xx}^p + c_1 w_{,zz}^p + e_2 \phi_{,xx}^p + \phi_{,zz}^p = \delta_1^2 \ddot{w}^p \tag{13}$$

$$e_1 u_{,xz}^p + e_2 w_{,xx}^p + w_{,zz}^p - \eta_3 \bar{\varepsilon} \phi_{,xx}^p - \eta_3 \phi_{,zz}^p = 0 \tag{14}$$

Formal Solution of the Problem

In order to facilitate solution in semiconductor medium, we introduce the scalar and vector point potential functions ϕ^s and ψ^s through the relations

$$u^s = \frac{\partial \phi^s}{\partial x} + \frac{\partial \psi^s}{\partial z},\ w^s = \frac{\partial \varphi^s}{\partial z} - \frac{\partial \psi^s}{\partial x} \tag{15}$$

Upon using relations (15) in Equations (10)-(11), we obtain

$$\nabla^2 \phi^s - N - \ddot{\phi}^s = 0 \tag{16}$$

$$\nabla^2 \psi^s = \frac{\ddot{\psi}^s}{\delta^2} \tag{17}$$

$$\nabla^2 N - \left[-\frac{1}{t_n^+} + \left(1 + \frac{t^n}{t_n^+}\right)\frac{\partial}{\partial t} + t^n \frac{\partial^2}{\partial t^2} \right] N - \varepsilon_n \nabla^2 \dot{\phi}^s = 0 \tag{18}$$

The Equation (17) corresponds to purely transverse wave in the semiconductor which get decoupled from rest of the motion and not affected by the charge carrier fields.

We consider the case of time harmonic plane waves and assume wave solution of the form

$$\begin{aligned}&\left(\psi^s, \phi^s, N, u^p, w^p, \phi^p\right) \\ &= \left(\bar{\psi}^s, \bar{\phi}^s, \bar{N}, \bar{u}^p, \bar{w}^p, \bar{\phi}^p\right)\exp\{ik(x - ct)\}\end{aligned} \tag{19}$$

where $c = \dfrac{\omega}{k}$ is the phase velocity, k and ω are the wave number and angular frequency of the waves respectively. Upon using solution (19) in Equations (16) to (18) and (12) to (14), the straightforward algebraic simplification leads to the following formal solution which satisfies the radiation condition in both the media:

1) Semiconductor halfspace $z \le 0$:

$$\psi^s = A_3^s e^{\beta z} \exp\{ik(x - ct)\} \tag{20}$$

$$\left(\phi^s, N\right) = \sum_{i=1}^{2} (1, S_i) A_i^s e^{n_i z} \exp\{ik(x - ct)\} \tag{21}$$

2) Piezoelectric halfspace $z \ge 0$:

$$\left(u^p, w^p, \phi^p\right) = \sum_{i=1}^{3} (1, M_i, P_i) A_i^p \exp\{-m_i z + ik(x - ct)\} \tag{22}$$

where

$$\begin{aligned}
&S_i = n_i^2 - \alpha^2 \\
&M_i = \frac{ikm_i \left\{c_3\eta_3\left(m_i^2 - k^2\bar{\varepsilon}\right) + e_1\left(m_i^2 - k^2 e_2\right)\right\}}{\left(c_1 m_i^2 - k^2 c_2 + \delta_1^2 k^2 c^2\right)\left(m_i^2 - k^2\bar{\varepsilon}\right)\eta_3 + \left(m_i^2 - k^2 e_2\right)^2} \\
&P_i = \frac{-ike_1 m_i}{\eta_3\left(m_i^2 - k^2\bar{\varepsilon}\right)} + \frac{\left(m_i^2 - k^2 e_2\right)}{\eta_3\left(m_i^2 - k^2\bar{\varepsilon}\right)} M_i
\end{aligned} \tag{23}$$

$$\begin{aligned}&\alpha^2 = k^2(1 - c^2),\ \beta^2 = k^2\left(1 - \frac{c^2}{\delta^2}\right),\ n_i^2 = k^2\left(1 - c^2 a_i^2\right), \\ &i = 1, 2\end{aligned} \tag{24}$$

Here the quantities $a_i^2\ (i = 1,2)$ and $m_i^2\ (i = 1,2,3)$ are given by:

$$\begin{aligned}
&a_1^2 + a_2^2 = 1 + t^n + i\omega^{-1}\left(1 + \varepsilon_n - \frac{t^n}{t_n^+}\right) + \frac{1}{\omega^2 t_n^+}, \\
&a_1^2 a_2^2 = t^n + i\omega^{-1}\left(1 - \frac{t^n}{t_n^+}\right) + \frac{1}{\omega^2 t_n^+}
\end{aligned} \tag{25}$$

$$\begin{aligned}
&\sum m_1^2 = k^2 \frac{c_2 A + \xi(1 + \eta_3 c_1) - c_3(c_3\eta_3 + 2e_1) + c_1 e_1^2}{c_2(1 + \eta_3 c_1)}, \\
&\sum m_1^2 m_2^2 = k^4 \frac{c_2 B + \xi A - c_3\left(c_2\eta_3\bar{\varepsilon} + 2e_1 e_2\right) + e_1^2\left(c_2 - \delta_1^2 c^2\right)}{c_2(1 + \eta_3 c_1)} \\
&m_1^2 m_2^2 m_3^2 = k^6 \frac{\xi B}{c_2(1 + \eta_3 c_1)}
\end{aligned} \tag{26}$$

$$\begin{aligned}
&\xi = \left(1 - \delta_1^2 c^2\right), \\
&A = \left(c_1\bar{\varepsilon} + c_2 - \delta_1^2 c^2\right)\eta_3 + 2e_2, \\
&B = \left(c_2 - \delta_1^2 c^2\right)\eta_3\bar{\varepsilon} + e_2^2
\end{aligned} \tag{27}$$

Upon using the solution (20) to (22) in Equations (6) and (7) via quantities (9) and Equation (15), the normal stresses, shear stresses, current density and displacements for the semiconductor and piezoelectric solid are obtained as:

$$\tau_{zz}^s = \left\{ p\sum_{i=1}^{2} A_i^s \mathrm{e}^{n_i z} - q\beta A_3^s \mathrm{e}^{\beta z} \right\} \exp\left\{ ik\left(x - ct\right) \right\} \tag{28}$$

$$\tau_{xz}^s = \left\{ q\sum_{i=1}^{2} n_i A_i^s \mathrm{e}^{n_i z} + pA_3^s \mathrm{e}^{\beta z} \right\} \exp\left\{ ik\left(x - ct\right) \right\} \tag{29}$$

$$J_z^s = -\sum_{i=1}^{2} S_i n_i A_i^s \mathrm{e}^{n_i z} \exp\left\{ ik\left(x - ct\right) \right\} \tag{30}$$

$$\left(\tau_{zz}^p, \tau_{xz}^p, D_z^p\right) = \sum_{i=1}^{3} \left(y_i, d_i, b_i\right) A_i^p \exp\left\{ -m_i z + ik\left(x - ct\right) \right\} \tag{31}$$

$$u^s = \left\{ \frac{q}{2}\sum_{i=1}^{2} A_i^s \mathrm{e}^{n_i z} + \beta A_3^s \mathrm{e}^{\beta z} \right\} \exp\left\{ ik\left(x - ct\right) \right\} \tag{32}$$

$$w^s = \left\{ \sum_{i=1}^{2} n_i A_i^s \mathrm{e}^{n_i z} - \frac{q}{2} A_3^s \mathrm{e}^{\beta z} \right\} \exp\left\{ ik\left(x - ct\right) \right\} \tag{33}$$

where

$$\begin{aligned} &p = \left(k^2 + \beta^2\right),\ q = 2ik, \\ &y_i = \frac{\bar{\rho}}{\delta^2 \delta_1^2} \left\{ ik\left(c_3 - c_2\right) - \left(c_1 M_i + P_i\right) m_i \right\}, \\ &d_i = \frac{\bar{\rho}}{\delta^2 \delta_1^2} \left\{ \frac{c_2}{2}\left(ikM_i - m_i\right) + ike_2 P_i \right\}, \\ &b_i = ik\left(e_1 - e_2\right) - m_i\left(M_i - \eta_3 P_i\right),\ i = 1,2,3 \end{aligned} \tag{34}$$

and A_i^s, $A_i^p\left(i = 1,2,3\right)$ are the unknowns to be determined.

Secular Equation

We obtain a system of six homogeneous algebraic equations in the six unknowns A_i^s and $A_i^p\left(i = 1,2,3\right)$ upon using the formal solution obtained in the previous section in the boundary conditions (8) which has a non-trivial solution if the determinant of the coefficient of $A_i^s, A_i^p, \left(1,2,3\right)$ vanishes and this require lengthy algebraic reductions and simplifications which leads to the following secular equation for the propagation of guided waves in the considered composite structure

$$\det\left(a_{ij}\right) = 0,\ \left(i, j = 1,2,3,\cdots,6\right) \tag{35}$$

where the non-zero elements a_{ij} are given below

$$\begin{aligned} &a_{12} = p,\ a_{13} = -q\beta,\ a_{1k} = y_k\left(k = 4,5,6\right),\ a_{21} = q, \\ &a_{23} = p,\ a_{2k} = d_k\left(k = 4,5,6\right),\ a_{32} = \frac{q}{2},\ a_{33} = \beta, \\ &a_{34} = a_{35} = a_{36} = 1,\ a_{41} = 1,\ a_{43} = -\frac{q}{2}, \\ &a_{4k} = M_k\left(k = 4,5,6\right),\ a_{51} = n_1 + n_2,\ a_{52} = -\left(n_1 n_2 + \alpha^2\right), \\ &a_{5k} = P_k\left(k = 4,5,6\right),\ a_{61} = \frac{S_1 n_1 - S_2 n_2}{n_1 - n_2}, \\ &a_{52} = -n_1 n_2\left(n_1 + n_2\right),\ a_{6k} = -b_k\left(k = 4,5,6\right) \end{aligned} \tag{36}$$

The complex secular Equation (35) contains complete information about the characteristics of the waves traveling at the interface.

Solution of Secular Equation

In general, wave number and hence the phase velocities of the waves are complex quantities, therefore the waves are attenuated in space. In order to solve the secular equations, we take

$$c^{-1} = V^{-1} + i\omega^{-1}Q \tag{37}$$

where $k = R + iQ$, $R = \frac{\omega}{V}$ and R, Q are real numbers. Here, it may be noted that V and Q respectively, represent the phase velocity and attenuation coefficient of the waves. Upon using representation (37) in various relevant relations, the complex roots $m_i^2\left(i = 1,2,3\right)$ can be computed from (26) with the help of Cardano's method. The roots $m_i^2\left(i = 1,2,3\right)$ are further used to solve secular Equation (35) to obtain phase velocity $\left(V\right)$ and attenuation coefficient $\left(Q\right)$ of the surface waves by using function iteration numerical technique whose procedure is outlined by Sharma *et al.* (2010).

For initial value of $c = c_0 = \left(V_0, Q_0\right)$, the roots $m_i\left(i = 1,2,3\right)$ are computed from Equations (26) by using Cardano's method for each value of non-dimensional wave number $\left(R\right)$ for assigned frequency. The values of $m_i\left(i = 1,2,3\right)$ so obtained are then used in secular Equation (35) to obtain the current values of V and Q. The process is terminated as and when the condition $\left|V_{n+1} - V_n\right| < \varepsilon$, ε being arbitrarily small number to be selected at random to achieve the accuracy level, is satisfied. The procedure is continuously repeated for different values of R to obtain corresponding values of the V and Q. Thus, the real phase velocity and attenuation coefficient of Rayleigh type surface waves in the composite structure under study can be computed from dispersion relation (35).

Amplitudes of Field Functions

The amplitudes of various field functions at the surface $\left(z = 0\right)$ are obtained as:

$$\left(u^s, w^s, N, \tau_{zz}^s, \tau_{xz}^s\right) = \left(U^s, W^s, N^*, \sigma_{zz}^s, \sigma_{xz}^s\right) A_1^s \exp\left\{ iR\left(x - Vt\right) \right\}$$

$$\left(u^p, w^p, \phi^p, \tau_{zz}^p, \tau_{xz}^p\right) = \left(U^p, W^p, \Phi^p, \sigma_{zz}^p, \sigma_{xz}^p\right) A_1^s \exp\left\{ iR\left(x - Vt\right) \right\}$$

where

$$U^s = \left\{ \frac{q}{2}\left(1 + L_2^s\right) + \beta L_3^s \right\} \exp\left(-Qx\right)$$

$$W^s = \left(n_1 + n_2 L_2^s - \frac{q}{2} L_3^s \right) \exp\left(-Qx\right)$$

$$N^* = \left(S_1 + S_2 L_2^s\right) \exp\left(-Qx\right)$$

$$U^p = \left(L_1^p + L_2^p + L_3^p\right) \exp\left(-Qx\right)$$

$$W^p = \left(M_1 L_1^p + M_2 L_2^p + M_3 L_3^p\right) \exp\left(-Qx\right)$$

$$\Phi^p = \left(P_1 L_1^p + P_2 L_2^p + P_3 L_3^p\right) \exp\left(-Qx\right)$$

$$\sigma_{zz}^{s}=\left\{p\left(1+L_2^s\right)-q\beta L_3^s\right\}\exp\left(-Qx\right)$$

$$\sigma_{xz}^{s}=\left\{q\left(n_1+n_2L_2^s\right)+pL_3^s\right\}\exp\left(-Qx\right)$$

$$\sigma_{zz}^{p}=\left(y_1L_1^p+y_2L_2^p+y_3L_3^p\right)\exp\left(-Qx\right)$$

$$\sigma_{xz}^{p}=\left(d_1L_1^p+d_2L_2^p+d_3L_3^p\right)\exp\left(-Qx\right)$$

Here

$$L_2^s=-\frac{\Delta_{ij}^{2s}}{\Delta_{ij}^{0}},\ L_3^s=\frac{\left(y_2-y_3\right)\Delta_{ij}^{3s}}{\Delta_{ij}^{0}},\ L_1^p=-\frac{\left(y_2-y_3\right)\Delta_{ij}^{1p}}{\Delta_{ij}^{0}},$$

$$L_2^p=\frac{\left(y_2-y_3\right)^2\Delta_{ij}^{2p}}{\left(y_1-y_3\right)\Delta_{ij}^{0}},\ L_3^p=\frac{\left(y_2-y_3\right)^2\Delta_{ij}^{3p}}{\left(y_2-y_1\right)\Delta_{ij}^{0}}$$

The elements of matrices Δ_{ij}^{0}, Δ_{ij}^{2s}, Δ_{ij}^{3s}, Δ_{ij}^{1p}, Δ_{ij}^{2p} and Δ_{ij}^{3p} are defined in appendix.

Numerical Results and Discussion

In order to illustrate the analytical developments in the previous section, we now perform some numerical computations and simulations. The composite material chosen for the purpose of numerical calculations is composed of 6 mm class cadmium selenide (CdSe) piezoelectric material and n-type silicon (Si) semiconductor. The physical data for piezoelectric and semiconductor half spaces are given as under:

1) Piezoelectric half space [Sharma and Pal (2004)]:

$$\begin{aligned}
&c_{11}=7.41\times10^{10}\ \text{nm}^{-2},\ c_{13}=3.93\times10^{10}\ \text{nm}^{-2},\\
&c_{33}=8.36\times10^{10}\ \text{nm}^{-2},\ c_{44}=1.32\times10^{10}\ \text{nm}^{-2},\\
&e_{31}=-0.160\ \text{cm}^{-2},\ e_{33}=0.347\ \text{cm}^{-2},\\
&e_{15}=-0.138\ \text{cm}^{-2},\ \varepsilon_{11}=8.26\times10^{11}\ \text{C2N-1m}^{-2},\\
&\varepsilon_{33}=9.03\times10^{11}\ \text{C2N-1m}^{-2},\ \rho^{p}=5504\ \text{kgm}^{-3}.
\end{aligned}$$

2) Semiconductor halfspace [Sharma *et al.* (2007)]:

$$\begin{aligned}
&\lambda=0.64\times10^{11}\ \text{nm}^{-2},\ \mu=0.65.0\ \text{nm}^{-2},\\
&D^{n}=0.35\times10^{-2}\ \text{m}^{-2}\text{s}^{-1},\ n_0=10^{20}\ \text{m}^{-3},\\
&\alpha_T=2.6\times10^{-6}\ \text{K}^{-1},\ \rho^{s}=2300\ \text{kg}\cdot\text{m}^{-3}.
\end{aligned}$$

Here we present the effect of different interacting fields and corresponding parameters on the surface wave at the interface of considered structure. The profiles are plotted with respect to non-dimensional wave number $\left(R\right)$ on linear-log scales. The corresponding results in the physical domain can be obtained with the help of quantities defined in Equation (9) from the instant non-dimensional one. The numerical computations have been performed; correct upto four decimal places here, by employing the procedure outlined in section (Solution of Secular Equation) by using MATLAB programming. The computer simulated results have been presented graphically in **Figures 2** to **9**.

Figure 2 represents the variations of longitudinal and transverse displacements versus distance (x) for semiconductor halfspace in the considered composite. The profiles show that as we move along the direction of wave propagation, the displacements of the particles of the medium decreases and ultimately vanish at some distance. Moreover the magnitude of the longitudinal displacement is higher than that of transverse displacement. The magnitude decreases because of the resistance offered by the medium to the wave propagation due to the anelastic properties of the materials, in which the energy of the elastic wave is lost to heat the material by causing permanent deformations.

Figure 3 presents the variations of the longitudinal and transverse displacements for the piezoelectric halfspace versus distance in the composite structure. Here we have found the similar profiles as in case of silicon halfspace in the considered composite, which justify the boundary conditions which require that the respective displacements in both the materials must balance the effect of each other at the interface in order to stabilize the welded contact at $z=0$, otherwise such structure is impossible to exist.

The **Figure 4** displays the variations of the carrier concentration at the interface of composite with the distance. It is found

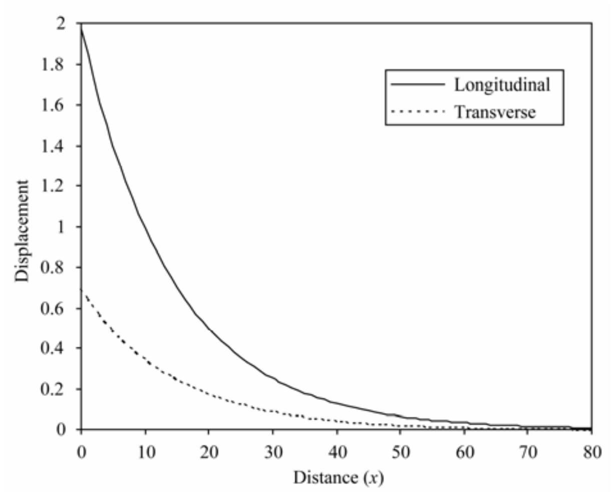

Figure 2.
Variations of displacements for semiconductor halfspace versus distance.

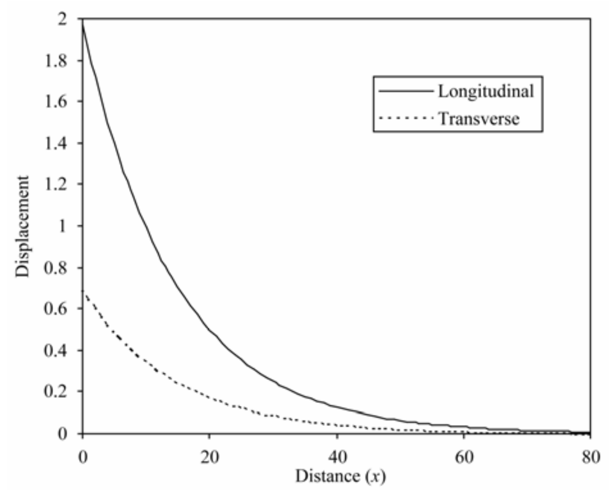

Figure 3.
Variations of displacements for piezoelectric halfspace versus distance.

that the change in the carrier concentration also decreases with distance, before it ultimately vanishes after some distance along the direction of wave propagation.

In **Figure 5** the variations of piezoelectric potential are plotted with the distance, which also follow the similar trend as that of the carrier concentration. It also justifies the boundary condition that change in carrier concentration balance the change in piezoelectric potential at the interface. The disturbance causes a surface acoustic at piezoelectric halfspace which is associated with an electric field. This electric field changes the carrier concentration at the interface as the negatively charged electrons interact with it. In this process the carriers follow the electric field associated with the surface acoustic wave and acquire energy from this electric field.

Figure 6 shows the variations of normal and shear stresses for the semiconductor halfspace versus distance in the composite structure. It is observed that both the stresses decrease with the increase in the distance along the direction of wave propagation. The shear stress possesses higher magnitude than that of the normal stress. In **Figure 7**, the variations of normal and shear stresses for piezoelectric halfspace in the same composite are plotted. We found the similar profiles with equal magnitudes and same vanishing distance along the direction of wave propagation as in case of semiconductor halfspace. The shear stress possesses larger magnitude in comparison to the normal stress in both the material components of the composite. The results show that at the interface the stresses balance the effect of each other.

Figure 8 displays the variations of phase velocity with the wave number at the interface of the composite. The profiles are noticed to be clearly dispersive, hence showing that phase velocity is dependant on the wavelength of the wave. Phase velocity possesses large magnitude at long wavelengths in comparison to small wavelengths. This is due to the reason that long wavelengths penetrate the medium to a greater extent thereby brings the various coupling field in to play which con-

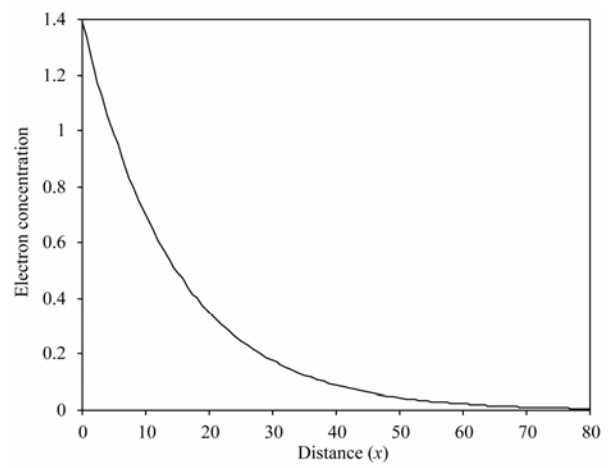

Figure 4.
Variations of electron concentration for semiconductor halfspace versus distance.

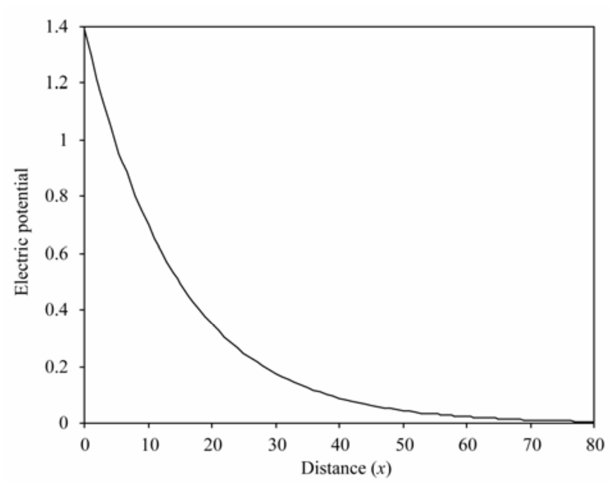

Figure 5.
Variations of electric potential for piezoelectric halfspace versus distance.

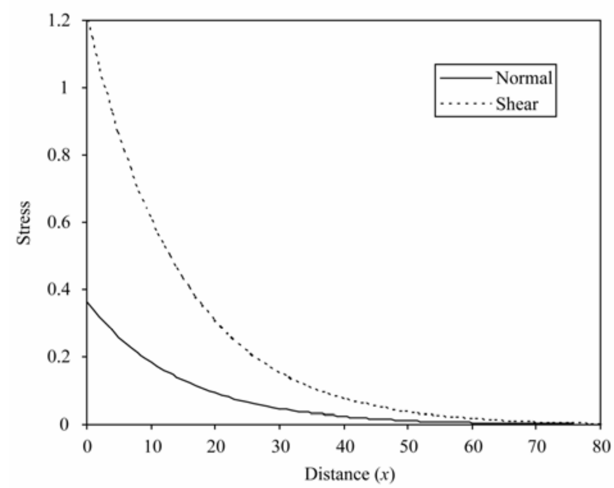

Figure 6.
Variations of stresses for semiconductor halfspace versus distance.

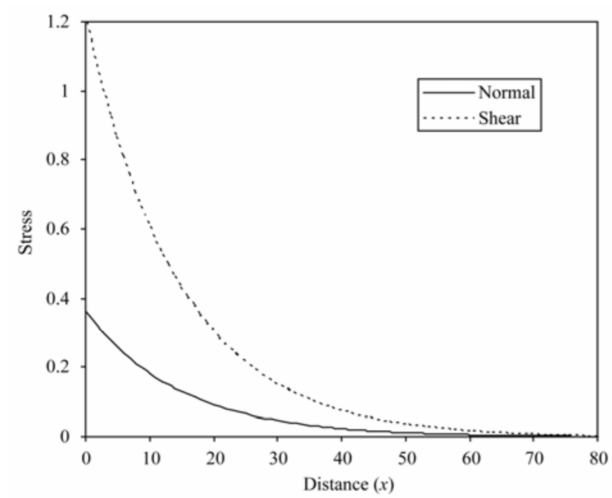

Figure 7.
Variations of stresses for piezoelectric halfspace versus distance.

tribute to increase the phase velocity. The magnitude of phase velocity decreases with decrease in the life time of the carrier field.

Figure 9 shows the variations of attenuation coefficient with the wave number at the interface of the composite. The attenuation increases with decreasing wavelength. It is also noticed that it decreases with decreasing life time of the carrier field.

Concluding Remarks

1) The functional iteration method along with the Cardano method has been successfully employed to solve complex characteristic equations to obtain the surface waves characteristics at the interface of composite.

2) At the interface of the considered composite the displacements, stresses, electron concentration, electric potential decrease along the direction of wave propagation and then vanish after some distance.

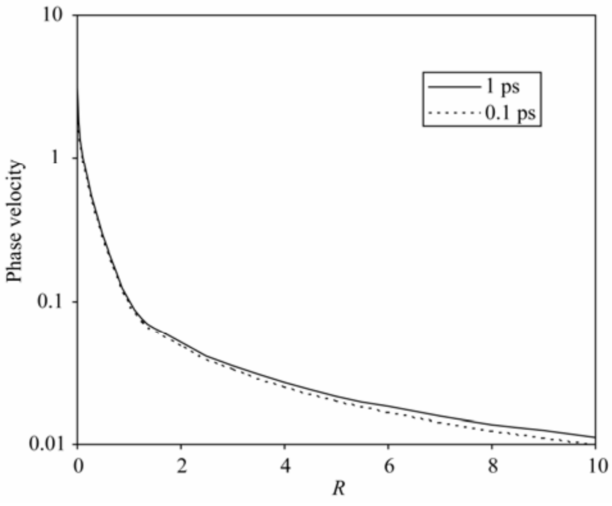

Figure 8.
Variations of phase velocity versus wave number.

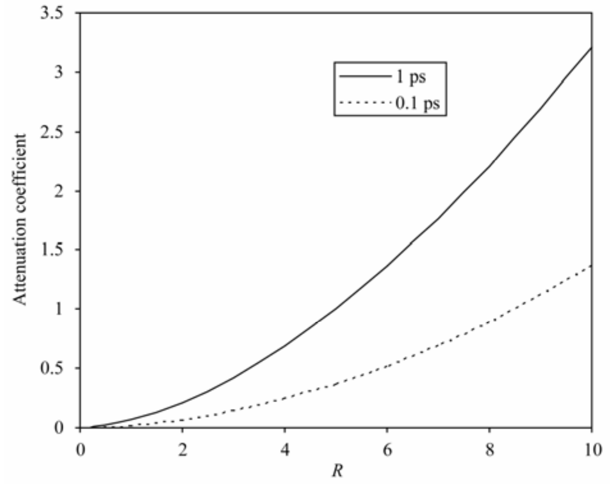

Figure 9.
Variations of attenuation coefficient versus wave number.

3) The phase velocity possesses large magnitude at long wavelengths which goes on decreasing with the decreasing wavelength hence showing a dispersive character.

4) The attenuation increases with the decreasing wavelength in the considered composite structure.

5) The phase velocity as well as attenuation decreases with decreasing life time of the carrier field.

6) The study may find applications in fabrication of micro-electromechanical surface acoustic wave devices.

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Appendix

The elements of Δ_{ij}^{0}, Δ_{ij}^{2s}, Δ_{ij}^{3s}, Δ_{ij}^{1p}, Δ_{ij}^{2p} and Δ_{ij}^{3p} are given by

$$\Delta_{11}^{0}=q\left(n_2-\frac{d_3}{2}\right)(y_2-y_3)-\left(p-\frac{qy_3}{2}\right)(d_2-d_3),$$

$$\Delta_{12}^{0}=(\beta d_3-p)(y_2-y_3)-q(\beta+y_3)(d_2-d_3),$$

$$\Delta_{13}^{0}=(d_1-d_3)(y_2-y_3)-(y_1-y_3)(d_2-d_3),$$

$$\Delta_{21}^{0}=\left(n_2-\frac{q}{2}M_3\right)(y_2-y_3)-\left(p-\frac{q}{2}y_3\right)(M_2-M_3),$$

$$\Delta_{22}^{0}=\left(\frac{q}{2}+\beta M_3\right)(y_2-y_3)-\beta(q+y_3)(M_2-M_3),$$

$$\Delta_{23}^{0}=(M_1-M_3)(y_2-y_3)-(y_1-y_3)(M_2-M_3),$$

$$\Delta_{31}^{0}=\left(S_2-\frac{q}{2}P_3\right)(y_2-y_3)-\left(p-\frac{q}{2}y_3\right)(P_2-P_3),$$

$$\Delta_{32}^{0}=P_3(y_2-y_3)-\beta(q+y_3)(P_2-P_3),$$

$$\Delta_{33}^{0}=(P_1-P_3)(y_2-y_3)-(y_1-y_3)(P_2-P_3),$$

The quantities Δ_{ij}^{2s} can be obtained from Δ_{ij}^{0} by changing n_2 by n_1, n_1 by n_2 and S_2 by S_1.

$\Delta_{11}^{3s}=2ik\delta^2$, $\Delta_{12}^{3s}=\Delta_{11}^{2s}$, $\Delta_{13}^{3s}=\Delta_{13}^{2s}$, $\Delta_{21}^{3s}=1$, $\Delta_{22}^{3s}=\Delta_{21}^{2s}$, $\Delta_{23}^{3s}=\Delta_{23}^{2s}$, $\Delta_{31}^{3s}=n_1+n_2$, $\Delta_{32}^{3s}=\Delta_{31}^{2s}$, $\Delta_{33}^{3s}=\Delta_{33}^{2s}$, $\Delta_{11}^{1p}=\Delta_{31}^{3s}$,

$$\Delta_{12}^{1p}=\left(p-\frac{q}{2}y_2\right)(d_2-d_3)-q\left(n_1-\frac{q}{2}d_2\right)(y_2-y_3),$$

$$\Delta_{13}^{1p}=q(\beta+y_2)(d_2-d_3)-(\beta d_2-p)(y_2-y_3),$$

$$\Delta_{21}^{1p}=\Delta_{21}^{3s},$$

$$\Delta_{22}^{1p}=\left(p-\frac{q}{2}y_2\right)(M_2-M_3)-\left(n_1-\frac{q}{2}M_2\right)(y_2-y_3),$$

$$\Delta_{23}^{1p}=\beta(q+y_2)(M_2-M_3)-\left(\frac{q}{2}+\beta M_2\right)(y_2-y_3),$$

$$\Delta_{31}^{1p}=\Delta_{31}^{3s},\quad \Delta_{32}^{1p}=\left(p-\frac{q}{2}y_2\right)(P_2-P_3)-\left(S_1-\frac{q}{2}P_2\right)(y_2-y_3),$$

$$\Delta_{33}^{1p}=\beta(q+y_2)(P_2-P_3)-P_2(y_2-y_3),$$

The expressions for Δ_{ij}^{2p} and Δ_{ij}^{3p} can be written from those of Δ_{ij}^{1p} by cyclic permuting the suffixes of quantities y_i, d_i, P_i, and M_i cyclically.