Open Access Library Journal
2019, Volume 6, e5736
ISSN Online: 2333-9721
ISSN Print: 2333-9705

Quasi-Quadrature Solution of Integral Equations Fredholm of the Second Kind in the Class of Integrable Functions

Igor Petrovich Dobrovolsky

Institute of Physics of the Earth, Russian Academy of Sciences, Moscow, Russia
Email: dipedip@gmail.com

How to cite this paper: Dobrovolsky, I.P. (2019) Quasi-Quadrature Solution of Integral Equations Fredholm of the Second Kind in the Class of Integrable Functions. *Open Access Library Journal*, **6**: e5736.
https://doi.org/10.4236/oalib.1105736

Received: August 26, 2019
Accepted: September 9, 2019
Published: September 12, 2019

Copyright © 2019 by author(s) and Open Access Library Inc.
This work is licensed under the Creative Commons Attribution International License (CC BY 4.0).
http://creativecommons.org/licenses/by/4.0/
Open Access

Abstract

The analog of the quadrature solution of the equation of Fredholm of the second kind is considered. Fundamental difference from classical quadrature formulas is as follows. On segments of the chosen grid not values of functions, but their integral average values are used. Computing examples show expediency of such approach in appropriate cases.

Subject Areas

Integral Equation

Keywords

Integral Average Value

1. Introduction

Numerical methods of the solution of the equations are universal. This is their important quality. Quadrature formulas are quite often used in the solution of the integral equations. Quadrature formulas are applicable only to continuous functions. In other words, the integrable function has to have a certain value at each point. This requirement cannot be fulfilled for the class of integrable functions. In such situation the alternative version of the quadrature solution of the integral equation is possible. This work is further work [1].

2. Algorithm

The essence of the offered algorithm is as follows. On the main segment of the integral equation the system of segments is formed and on each such segment the integral average value of function is defined.

The equation is considered

$$y(x)-\int_a^b K(x,t)y(t)\mathrm{d}t = f(x) \tag{2.1}$$

which has the unique solution in a class of the integrable functions.

We form on a segment [*a*, *b*] a uniform grid of *N* segments with a step

$$h=\frac{b-a}{N} \tag{2.2}$$

On this grid there is $N+1$ of points. We build segments δ_i in the vicinity of each point

$$\begin{aligned}&\delta_0=\left[a,a+\frac{h}{2}\right],\\&\delta_i=\left[a+\frac{h}{2}(2i-1),a+\frac{h}{2}(2i+1)\right],\ i=1,\cdots,N-1,\\&\delta_N=\left[b-\frac{h}{2},b\right]\end{aligned} \tag{2.3}$$

We will enter concept of integral average values on these segments. They have the form for functions $y(x)$ and $f(x)$

$$w_i=\frac{1}{\delta_i}\int_{\delta_i} y(x)\mathrm{d}x,\quad f_j=\frac{1}{\delta_j}\int_{\delta_j} f(x)\mathrm{d}x \tag{2.4}$$

Let's provide the function $y(t)$ under integral in (2.1) in the form

$$y(t)=\sum_{i=0}^{N} w_i\delta_i \tag{2.5}$$

where δ_i is characteristic function of the corresponding segment.

Then (2.1) receives the form

$$y(x)-\sum_{i=0}^{N} K_i(x)w_i = f(x) \tag{2.6}$$

where $K_i(x)=\int_{\delta_i} K(x,t)\mathrm{d}t$.

Let's carry out to (2.6) integral averaging on segments δ_j

$$w_j-\sum_{i=1}^{N} K_{ji}w_i = f_j,\ \ j=0,\cdots,N \tag{2.7}$$

where $K_{ji}=\frac{1}{\delta_j}\int_{\delta_j}\int_{\delta_i} K(x,t)\mathrm{d}t\mathrm{d}x$, $f_j=\frac{1}{\delta_j}\int_{\delta_j} f(x)\mathrm{d}x$, *j* is number of the equation, *i* is number of item in the equation.

(2.7) is system of the equations for definition of integral average values of unknown function on system of segments (2.3).

3. Example 1

The equation is considered

$$y(x)-\int_0^1\frac{y(t)\mathrm{d}t}{\sqrt{|t-x|}}=\sqrt{x}-\frac{x\pi}{2}+\frac{x\ln x}{2}-\sqrt{1-x}-\frac{1}{2}x\ln\left(2-x+2\sqrt{1-x}\right) \tag{3.1}$$

which has the exact solution $y = \sqrt{x}$.

Results of the quasi-quadrature solution of this equation are presented in Table 1.

In Table 1 y_0 and y_N are the integral average values calculated according to the exact solution. It is possible to claim that the received solution is satisfactory.

4. Example 2

The equation is considered

$$y(x) - \int_0^1 \sqrt{|t-x|}\, y(t)\,\mathrm{d}t = \frac{1}{\sqrt{x}} - \frac{x\pi}{2} - \frac{x\ln x}{2} - \sqrt{1-x} + \frac{1}{2}x\ln\left(2 - x + 2\sqrt{1-x}\right) \quad (4.1)$$

which has the exact solution $y = \frac{1}{\sqrt{x}}$.

Results of the quasiquadrature solution of this equation are presented in Table 2.

Let's make some analysis of the received solution. We see that the function $y(x)$ grows at approach to a point $x = 0$. We will assume that this growth has power character. Specifically: we will assume that in the vicinity of a point $x = 0$ function has the form $\bar{y} = p + qx^n$ with unknown p, q and n. Then

$$\bar{y}_0 = \frac{2}{h}\int_0^{h/2} \bar{y}\,\mathrm{d}x = p + \frac{q}{2^n (n+1) N^n} \quad (4.2)$$

Table 1. Quasiquadrature solution of Equation (3.1).

N	segment δ_0		segment δ_N	
	w_0	y_0	w_N	y_N
20	0.11104	0.10541	0.99347	0.99372
40	0.07689	0.07454	0.99647	0.99686
80	0.05372	0.05270	0.99825	0.99843
160	0.03773	0.03727	0.99914	0.99921
320	0.02657	0.02635	0.99958	0.99960
640	0.01871	0.01863	0.99976	0.99980

Table 2. Quasiquadrature solution of Equation (4.1).

N	segment δ_0		segment δ_N	
	w_0	y_0	w_N	y_N
20	12.647499	12.649110	1.00249	1.00632
40	17.888020	17.888543	1.00166	1.00314
80	25.298077	25.298221	1.00100	1.00156
160	35.777063	35.777088	1.00057	1.00078
240	43.817803	43.817805	1.00041	1.00052
320	50.596448	50.596443	1.00031	1.00039

For definition of unknown *p, q* and *n* we equate the received expression to three values from Table 2

$$
\begin{aligned}
p+\frac{q}{2^n(n+1)160^n} &= w_0(160) = 35.777063 \\
p+\frac{q}{2^n(n+1)240^n} &= w_0(240) = 43.817803 \\
p+\frac{q}{2^n(n+1)320^n} &= w_0(320) = 50.596448
\end{aligned}
\tag{4.3}
$$

The solution of system (4.3) gives (after rounding) $n=-0.5$, $p=0$, $q=1$. This solution correctly defines behavior of function $y(x)$.

5. Conclusion

The conducted research shows that the method of the quasi-quadrature solution can take its place among other methods of approximate solution of integral equations.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Dobrovolsky, I.P. (2015) The Estimation of the Error at Richardson's Extrapolation and the Numerical Solution of Integral Equations of the Second Kind. *Open Access Library Journal*, **2**, e2051. http://dx.doi.org/10.4236/oalib.1102051