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A Note on Momentum Conservation in the Non-Relativistic Lorentz Theory of Radiation Reaction

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Abstract

The Lorentz theory of radiation reaction in the non relativistic limit is critically examined from the principles of symmetry. In Newtonian motion the applied force and the consequent momentum change generally always have a direction. In the case of a charged body this momentum change is accompanied by the radiation of photons. The question that needs to be answered is: does this radiated field carry any momentum? The following result of investigations seems to indicate this fact contrary to the prediction of Lorentz theory which states that the field carries no net momentum.

Subject Areas

Theoretical Physics

Keywords

Radiation Reaction, Classical Electrodynamics, Momentum Conservation

1. Introduction

Electromagnetic radiation reaction entails both the conservation of energy and momentum. When a charged particle moves in a straight line under the action of an external force directed along its line of motion it is expected to radiate electromagnetic energy in the shape of photons. Now these photons also carry away with them momentum and it cannot be assumed that the net momentum carried out by the electromagnetic field will be zero purely from considerations of symmetry for this highly asymmetrical problem. Suppose that the line of accelerated motion of the charged particle is along the z axis then from symmetry consideration one can say that there will be no net momentum carried out by the photons which will have x and y components but certainly that will not be true for the z component of momentum. In fact we have tried to show in our earlier publication [1] that there will be a z component of momentum which will be carried out by the electromagnetic field for such a motion of charge. In this note we show that the prediction of Lorentz theory (or rather the Lorentz, Abraham and Dirac-LAD theory) in the non-relativistic limit under the action of a constant force within a finite time interval as was worked out by us earlier [2] will yield zero net momentum transfer from the charge to the photons and this is likely to be incorrect. A discussion and elaboration on momentum and energy loss in the case of a new radiation reaction formula as was worked out by us in [1] is included at the end of this note.

2. The Loss Rate of Momentum in Non-Relativistic LAD Theory

The following equation for the acceleration $a(T)$ of a charged particle as a function of time T was derived by us in [2] which is acted on by an external force of the form $f_{ext} = f_0\left[u(T)-u(T-t)\right]$ and can be written as (see Equation (4) of [2])

$$a(T)=\frac{f_0}{m}\left[\mathrm{e}^{T/\tau_0}\left(1-u(T)\right)+u(T)\right]-\frac{f_0}{m}\left[\mathrm{e}^{(T-t)/\tau_0}\left(1-u(T-t)\right)+u(T-t)\right] \quad (1)$$

where $u(T)$ is the unit step function. The equation of motion $ma = f_{ext} + m\tau_0 \frac{\mathrm{d}a}{\mathrm{d}t}$ can be written in the integrated form as

$$mv(T)\Big|_{-\infty}^{t} = \int_0^t f_{ext}\mathrm{d}T + m\tau_0 a(T)\Big|_{-\infty}^{t}, \quad (2)$$

where $v(T)=\int a(T)\mathrm{d}T$. The net momentum carried out by the radiated photons is given by the term $m\tau_0 a(T)\Big|_{-\infty}^{t}$ and it is easily seen to be zero as both $a(t)=0$ and $a(-\infty)=0$. This means that the photons carry $+z$ and $-z$ components of momentum in equal amounts although the particle gains $+z$ component of momentum only in the case of motion along the positive z direction. In our opinion this is incorrect from the considerations of the symmetry of the problem itself. Even in an inertial frame of reference in which the charged particle is momentarily at rest there is a direction in which it is accelerated and so some z directed momentum is transferred to the particle by the force which is acting on it and hence is it not to be accepted that the radiated electromagnetic field will be imparted with a z directed net momentum too? We think that the answer to this question will be in the affirmative but then there will be a distinction between the natures of the momentum transfer by the force to the charged particle on one hand and by the force to the electromagnetic field on the other.

3. The Loss Rate of Momentum in the Non-Relativistic Theory of Radiation Reaction Proposed by Us

The loss rate of momentum $\boldsymbol{P}_L$, was worked out by us for a charged particle having a velocity v and acceleration a in the z direction and this is expressed as (see Equation (5) of [1])

$$\boldsymbol{P}_L = \hat{u}_z \frac{q^2 a^2 v}{5\pi c^5 \varepsilon_0} \tag{3}$$

where $\hat{u}_z$ is the unit vector in the z direction; q is the charge carried by the particle. In addition we have the fundamental constants of nature c the velocity of light and ε_0 the permittivity of free space. We had stated that this is consistent with the energy loss rate in a certain frame of reference which for the moment we just call a privileged inertial frame. A return to the discussion of the nature of this frame will be made shortly. For the moment we would like to remind the reader that the laws of Newtonian mechanics are unchanged under a Galilean transformation that is the momentum and energy conservation are both observed in different inertial frames. The momentum conservation follows directly from $m\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = \boldsymbol{f}_{ext}$ in some inertial frame which becomes in another inertial frame having relative velocity $\boldsymbol{V}$ with respect to the first frame $m\frac{\mathrm{d}(\boldsymbol{v}-\boldsymbol{V})}{\mathrm{d}t} = \boldsymbol{f}_{ext}$ after the transformation is applied. The two are the same equations of motion as $\frac{\mathrm{d}\boldsymbol{V}}{\mathrm{d}t} = 0$. Similarly for energy we have the two equations $m\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t}\cdot\boldsymbol{v} = \boldsymbol{f}_{ext}\cdot\boldsymbol{v}$ and $m\frac{\mathrm{d}(\boldsymbol{v}-\boldsymbol{V})}{\mathrm{d}t}\cdot(\boldsymbol{v}-\boldsymbol{V}) = \boldsymbol{f}_{ext}\cdot(\boldsymbol{v}-\boldsymbol{V})$ which are also consistent with each other. The situation changes when dissipation is present for example in the case of a viscous drag in a fluid and this was mentioned by the author earlier [3]. We usually model it as a drag force proportional to the velocity of the body in a fluid that is of the form $-k\boldsymbol{v}$. It must be emphasized that this is the form of the force only in a reference frame in which the fluid is at rest since in other frames moving with velocity $\boldsymbol{V}$ with respect to this rest frame the force is not of the form $-k(\boldsymbol{v}-\boldsymbol{V})$ but rather $-k\boldsymbol{v}$. Thus there is a special frame in which both energy and momentum conservation equations take a simple form but in other frames they need modification. With this in mind we have a look at Equation (3) which being the loss rate of momentum to photons is the negative of radiation reaction force and has a form resembling in some respects the drag force in a fluid. However there is as yet no guideline as to how to determine the correct frame of reference as unlike the case of viscous drag there is no fluid whose rest frame can be taken to be the privileged frame. Our answer to this question is that there is a privileged frame in which the loss rate of energy and that of momentum will give an identical result as far as the radiation reaction force is concerned. This was the basis of our radiation reaction formula that is Equations (8) and (9) of [1]. The only problem is that the magnitude of the velocity $\boldsymbol{v}$ is quite high $\sqrt{\frac{5}{6}}c$ for the rate of energy loss and momentum loss to be consistent with each other and this velocity cannot be considered as nonrelativistic. We have taken it to be the starting point for an approximate expression for the radiation reaction force in the non-relativistic limit but we feel better formulas of both $\boldsymbol{P}_L$ compared to Equation (3) above and the Larmor's formula itself containing terms of the higher orders in $\frac{\boldsymbol{v}}{c}$ are possible.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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