



# Interacting Dark Fluids in LRS Bianchi Type-II Universe

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## Abstract

At the present paper, Locally Rotationally Symmetric (LRS) Bianchi type-II cosmological model with interacting dark matter (DM) and holographic dark energy (DE) have been discussed. In order to obtain solutions of the field equations, it is assumed that the shear scalar ( $\sigma$ ) is proportional to expansion scalar ( $\theta$ ). To have a general description of holographic dark energy and dark matter, a phenomenological parameterization of dark energy in terms of its equation of state (EoS) has been taken. Statefinder diagnostic pair *i.e.*  $\{r, s\}$  is adopted to separate other existing dark energy models from this model. Here we discuss two models: when  $n = 1/2$ , we obtain acyclic universe and the model converges into phantom region whereas when  $n = 3/2$ , we get an expanding universe and the model converges into quintessence region. Some important geometrical and physical features regarding to this model have also been studied.

## Subject Areas

Particle Physics

## Keywords

LRS Bianchi Type-II Space-Time, Interacting Dark Fluids, Statefinder Parameters

## 1. Introduction

The phase of accelerating expansion of the universe is indicated by Type Ia Supernovae (SNeIa) [1], [2], the Sloan Digital Sky Survey (SDSS) (Seljack *et al.* [3]) and Wilkinson Microwave Anisotropy Probe (WMAP) [4]. The astrophysical

observation, also indicates some evidence that cosmic media not a perfect fluid [5]. The mysterious term, known as dark energy, occupying about 73% of dark energy of the universe, is considered to be responsible for its acceleration. The remaining configuration of energy of the universe is composed of 23% of dark matter and 4% of baryonic matter. The easiest candidate for DE is cosmological constant ( $\Lambda$ ) [6], which is used to explain the observed cosmological investigations.

During the evolution of the universe, EoS parameter can be used to differentiate several dark energy models in standard cosmological model. To fit different properties of dark energy many candidates have been proposed such as tachyon [7]; k-essence [8]; phantom [9], [10]; quintom [11]; ghost condensate [12]; Brane world models [13]; interacting dark energy models [14]; Chaplygin gas models [15]; holographic dark energy models [16]; phantom scalar field [17]. Cosmological models like coupled dark energy, holographic dark energy,  $F(R, T)$  gravity,  $F(T)$  gravity,  $F(R)$  gravity, scalar field theory and  $\Lambda$ CDM cosmological models represent the increasing expansion with phantom/quintessence nature with cosmography tests have been studied by Bamba *et al.* [18]. In the framework of different matrices, exact general relativistic cosmological models in context of DE in presence of anisotropic fluid have been investigated (Yadav and Yadav [19]; Kumar and Yadav [20]; Reddy *et al.* [21]; Kumar and Akarsu [22]; Naidu *et al.* [23]; Akarsu and Kiling, [24]; Adhav *et al.* [25]; Katore *et al.* [26]). In the literature, anisotropic Bianchi type-I, III & V dark energy models in presence of usual perfect fluid have also been investigated (Yadav *et al.* [27]; Akarsu and Kiling [28]; Singh and Chaubey [29]; Pradhan [30]; Yadav and Shah [31]; Saha and Yadav [32]).

According to some basic quantum gravitational principle, nature of dark energy known as holographic dark energy can be studied. This principle [33] also states that the degree of freedom in a bounded system must be finite and doesn't scale by its volume but with its boundary era. Here  $\rho_\Lambda$  means vacuum energy density. With the help of cosmology, we consider  $\rho_\Lambda$  as DE density. Hooft [34] consider this holographic principle in context of black hole physics, which is another alternative to the solution of DE problem. In purview of dark energy problem, however, holographic principle put forwards a relation between Hubble parameter and Holographic dark energy density as  $\rho_\Lambda = H^2$ . Present accelerated expansion of the universe doesn't given by this value. The form of holographic density is given as  $\rho_\Lambda \approx \alpha H^2 + \beta \dot{H}$  in which  $H$  denote Hubble parameter,  $\alpha$  and  $\beta$  are constant. This was imposed by Granda and Olivers [35], which must satisfy current observational data. For the purpose of reconstruction of holographic scalar field models of the dark energy, numbers of works have been done.

An important role in the early stage of the universe was played by anisotropy thereby making it necessary to study the homogeneous as well as the anisotropic cosmological models. Therefore the models based on Bianchi types universe are assumed to be spatially homogeneous models of cosmology which are in general

anisotropic. The correspondences between the holographic dark energy and each one of phantom, tachyon, chaplygin gas and generalized chaplygin gas in FRW universe have been investigated by Setare [36] [37] [38] [39]. Cosmological dynamics of interacting holographic dark energy model have been pointed out by Setare and Venegus [40]. Bianchi type space-time having holographic dark energy's evolution with the help of constant deceleration parameter has been shown by Sarkar and Mahanta [41]. For instant, Sarkar [42] [43] [44] have studied holographic dark energy model in Bianchi typespace-time with linearly varying deceleration parameter in general relativity. Also Adhav *et al.* [45] [46], Kiran *et al.* [47] and Reddy *et al.* [48] have investigated holographic DE in Bianchi models.

The geometric parameters Hubble parameter ( $H$ ) and deceleration parameter ( $q$ ) were duly defined for characterizing the expansion history of the universe. From the observation of the cosmic acceleration  $q < 0$ , it can be stated that DE with negative equation of state (EoS)  $\omega < -1/3$  and likely  $\omega \sim -1$  exist. Leading to the need of higher time derivative of scale factor  $a$ , new term  $\ddot{a}$  is introduced for the deduction of the informational context regarding the dynamical evolution of  $\omega$ . Sahni *et al.* [49] defined a new parameter called as statefinder parameter as

$$r = \frac{\ddot{a}}{aH^3}, \quad s = \frac{r-1}{3(q-1/2)}$$

Note that  $r$  also known as cosmic jerk. The geometry describe by the set of quantities extended to include  $\{h, q, r, s\}$ . Different cosmological models corresponding to trajectories in the  $s-r$  plane shows quantitatively different behaviors. Fixed point  $\{s, r\}|_{\Lambda\text{CDM}} = \{1, 0\}$  corresponds to a spatially flat  $\Lambda$ CDM (cosmological constant  $\Lambda$  with cold dark matter) scenario. A better way of establishing the “distance” from a given point of departure of dark energy to the fixed point from spatially flat  $\Lambda$ CDM was provide in this model, can be demonstrated from [49] [50]. For differentiation of variety of dark energy involvingcosmological constant, interacting dark energy model, Brane world model, the chaplygin gas, quintessence, quintom, phantom, use of statefinder parameters may be successful (Sahni *et al.* [49]; Alam *et al.* [50]; Zimdahl and Pavon [51]; Zhang [52]). With the help of  $r(s)$  evolution diagram, it is very easy to identify “distance” the flat  $\Lambda$ CDM scenario to the given dark energy model.

At the present paper, the LRS Bianchi type-II cosmological modelwith interacting dark matter (DM) and holographic dark energy (DE) are investigated. Statefinder diagnostic pair *i.e.*  $\{r, s\}$  is adopted to separate other existing dark energy models from this model. Some important geometrical and physical features of the models have also been discussed.

The physical parameters of cosmological importance for LRS Bianchi type-II space-time are:

The volume scale factor:

$$V = a_1^2 a_2 \tag{1.1}$$

The average scale factor:

$$a = (a_1^2 a_2)^{\frac{1}{3}} \quad (1.2)$$

The mean Hubble parameter:

$$H = \frac{1}{3} \frac{\dot{V}}{V} \quad (1.3)$$

The deceleration parameter:

$$q = \frac{d}{dt} \left( \frac{1}{H} \right) - 1 \quad (1.4)$$

The mean anisotropy parameter:

$$\Delta = \frac{1}{3} \sum_{i=1}^3 \left( \frac{H_i - H}{H} \right)^2 \quad (1.5)$$

where  $H_1 = H_3 = \frac{\dot{a}_1}{a_1}$ ,  $H_2 = \frac{\dot{a}_2}{a_2}$  be the directional Hubble parameters along  $x, y, z$  axes respectively.

## 2. Metric and Field Equations

The spatially homogeneous and anisotropic LRS Bianchi type-II metric can be written as

$$ds^2 = -dt^2 + a_1^2 dx^2 + a_2^2 (dy - x dz)^2 + a_1^2 dz^2 \quad (2.1)$$

where  $a_1$  and  $a_2$  are the scale factors and functions of the cosmic time  $t$  only.

The Einstein's field equations in GR given as ( $8\pi G = 1$  and  $c = 1$ )

$$R_{ij} - \frac{1}{2} g_{ij} R = - ({}^m T_{ij} + {}^\Lambda T_{ij}) \quad (2.2)$$

Also, matter tensor for cold dark matter (pressureless *i.e.*  $w_m = 0$ ) and holographic dark energy are given as

$${}^m T_{ij} = \rho_m u_i u_j \quad (2.3)$$

$${}^\Lambda T_{ij} = (\rho_\Lambda + p_\Lambda) u_i u_j + g_{ij} p_\Lambda \quad (2.4)$$

where  $\rho_m$  denote the energy density of dark matter,  $\rho_\Lambda$  represent the energy density and  $p_\Lambda$  is the pressure of holographic dark energy.

The Einstein's field Equation (2.2) for LRS Bianchi type-II model (2.1) with the help of matter tensor (2.3) to (2.4) can be written as

$$\frac{2\ddot{a}_1}{a_1} + \frac{\dot{a}_1^2}{a_1^2} - \frac{3a_2^2}{4a_1^4} = -p_\Lambda \quad (2.5)$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\dot{a}_1 \dot{a}_2}{a_1 a_2} + \frac{a_2^2}{4a_1^4} = -p_\Lambda \quad (2.6)$$

$$2 \frac{\dot{a}_1 \dot{a}_2}{a_1 a_2} + \frac{\dot{a}_1^2}{a_1^2} - \frac{a_2^2}{4a_1^4} = \rho_\Lambda + \rho_m \quad (2.7)$$

where overhead dot denotes derivative with respect to cosmic time  $t$ .

Conservation equations stating the interaction between the DM and DE as,

$$\dot{\rho}_m + \left(\frac{\dot{V}}{V}\right)\rho_m = Q \quad (2.8)$$

$$\dot{\rho}_\Lambda + \left(\frac{\dot{V}}{V}\right)(1+w_\Lambda)\rho_\Lambda = -Q \quad (2.9)$$

$w_\Lambda$  represents the state parameter for holographic dark energy and is given by  $w_\Lambda = \frac{P_\Lambda}{\rho_\Lambda}$ .

The quantity  $Q$  indicates for the interacting term. The direction of transfer of energy depends upon the sign of  $Q$ . We consider that  $Q$  be positive, which stands for energy transfer from dark energy to dark matter. If  $Q$  is negative, stands for energy transfer from dark matter to dark energy. Overhead dot denotes the derivative with respect to commoving time. These continuity equations state that interacting term should be a function of a quantity with unit inverse of time *i.e.*  $Q \propto \frac{1}{t}$  (Hubble parameter is the first and natural choice) multiplied with energy density. The conservation equation and other models stating the interaction between DE and DM were established by Wetterich [53] [54] (see also Billyard and Coley [55]) and were used side by side the holographic DE by Horvat [56]. Expression for interaction between dark energy (DE) and dark matter (DM) can be written phenomenological as (Cai and Wang [57]; Amendola *et al.* [58]; Guo *et al.* [59] [60]).

$$Q = 3b^2H\rho_m = b^2\frac{\dot{V}}{V}\rho_m \quad (2.10)$$

where  $b^2$  represents a coupling constant.

We obtain the energy density of DM from equations (2.8) and (2.9) as

$$\rho_m = \rho_0 V^{(b^2-1)} \quad (2.11)$$

where  $\rho_0 > 0$  is a real constant of integration.

Using Equations (2.10) and (2.11), we get the interacting term as

$$Q = 3\rho_0 b^2 H V^{(b^2-1)} \quad (2.12)$$

### 3. Cosmological Solutions

It may be simply observed that the system have three equations as Equations (2.5)-(2.7) having five unknowns  $a_1$ ,  $a_2$ ,  $\rho_m$ ,  $\rho_\Lambda$  and  $p_m$ . Therefore to get set of solutions, one extra probable relation among these parameters needed. For this we considered that expansion scalar ( $\theta$ ) proportional to the shear scalar ( $\sigma$ ). This gives

$$a_1 = ma_2^n \quad (3.1)$$

where  $a_1$  and  $a_2$  are the metric potentials and  $n > 0$ ,  $m > 0$  are constants. Thorne [61] explained the intension behind considering this condition. The universe's Hubble parameter is isotropy in present day within  $\approx 30$  percent (Kan-

towski and Sachs [62]; Kristian and Sachs [63]). This has been suggested by the reviews of the velocity-red-shift relation for extra lactic sources. To place accurately, red-shift studies space the limit

$$\frac{\sigma}{H} \leq 0.3$$

on the proportion of shear ( $\sigma$ ) to Hubble parameter ( $H$ ) in neighborhood of galaxy at present day. Usual congruence to the homogeneous expansion persuades that the relation  $\frac{\sigma}{\theta}$  remain invariant, this has been noted by Collins *et al.* [64].

From Equations (2.5) and (2.6), we get,

$$\frac{\dot{a}_1 \dot{a}_2}{a_1 a_2} - \frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{a_2^2}{a_1^4} - \frac{\dot{a}_1^2}{a_1^2} = 0 \tag{3.2}$$

Equation (3.2) with the help of by Equation (3.1) leads to

$$\frac{\ddot{a}_2}{a_2} + 2n \left( \frac{\dot{a}_2^2}{a_2^2} \right) + \frac{a_2^{2-4n}}{(1-n)m^4} = 0, \quad n \neq 1,$$

which reduces to

$$2\ddot{a}_2 + 4n \left( \frac{\dot{a}_2^2}{a_2} \right) = \frac{2a_2^{3-4n}}{(n-1)m^4}, \quad n \neq 1 \tag{3.3}$$

On simplification Equation (3.3) convert to

$$\frac{d}{da_2} (\phi^2) + \frac{4n}{a_2} \phi^2 = \frac{2a_2^{3-4n}}{(n-1)m^4} \tag{3.4}$$

where  $\dot{a}_2^2 = \phi(a_2)$ ,  $\ddot{a}_2 = \phi\phi'$ ,  $\phi' = \frac{d\phi}{da_2}$ .

From Equation (3.4), we get

$$\phi^2 = \dot{a}_2^2 = \frac{a_2^{4-4n}}{2(n-1)m^4} + ka_2^{-4n} \tag{3.5}$$

where  $k > 0$  be real integrating constant.

Equation (3.5) leads to

$$\frac{a_2^{2n} dS}{\sqrt{\frac{a_2^4}{2m^4(n-1)} + k}} = dt \tag{3.6}$$

The solution of (3.6) is not tenable for  $n = 1$ . One can choose the value of  $n$  such that the above relation can be integrable. We consider the following two cases.

**First Model for  $n = \frac{1}{2}$**

When  $n = \frac{1}{2}$ , Equation (3.6) reduces to

$$\frac{a_2 da_2}{\sqrt{k - \frac{a_2^4}{m^4}}} = dt \tag{3.7}$$

which after integration and  $k = 1$ , leads to

$$a_2 = m \sqrt{\sin\left(\frac{2(t+c_1)}{m^2}\right)} \quad (3.8)$$

where  $c_1$  is constant of integration. Equation (3.13) for  $n = \frac{1}{2}$ , leads to

$$a_1 = m^{3/2} \sqrt[4]{\sin\left(\frac{2(t+c_1)}{m^2}\right)} \quad (3.9)$$

Therefore, the metric (3) reduces to

$$\begin{aligned} ds^2 = & -dt^2 + m^3 \sqrt{\sin\left(\frac{2(t+c_1)}{m^2}\right)} dx^2 + m^2 \sin\left(\frac{2(t+c_1)}{m^2}\right) (dy - xdz)^2 \\ & + m^3 \sqrt{\sin\left(\frac{2(t+c_1)}{m^2}\right)} dz^2 \end{aligned} \quad (3.10)$$

The volume scale factor, average scale factor, mean Hubble parameter, deceleration parameter, shear scalar and mean anisotropy parameter obtained as,

$$V = m^4 \sin\left(\frac{2(t+c_1)}{m^2}\right) \quad (3.11)$$

$$a = \left( m^4 \sin\left(\frac{2(t+c_1)}{m^2}\right) \right)^{1/3} \quad (3.12)$$

$$H = \frac{2}{3m^2} \cot\left(\frac{2(t+c_1)}{m^2}\right) \quad (3.13)$$

$$q = -1 + \frac{3}{m^2} \sec^2\left(\frac{2(t+c_1)}{m^2}\right) \quad (3.14)$$

$$\Delta = \frac{1}{8} \quad (3.15)$$

Using Equation (3.11) in Equations (2.11) and (2.12), we get energy density of dark matter and interacting term as,

$$\rho_m = \rho_0 m^{4(b^2-1)} \sin^{(b^2-1)}\left(\frac{2(t+c_1)}{m^2}\right) \quad (3.16)$$

$$Q = 2b^2 \rho_0 m^{2(2b^2-3)} \sin^{(b^2-2)}\left(\frac{2(t+c_1)}{m^2}\right) \cos\left(\frac{2(t+c_1)}{m^2}\right) \quad (3.17)$$

With the help of Equations (3.7), (3.8) and (2.18) in Equation (2.7), we get the energy density of holographic dark energy as,

$$\rho_\Lambda = \frac{5 \cot^2\left(\frac{2(t+c_1)}{m^2}\right) - 1}{4m^4} - \rho_0 m^{4(b^2-1)} \sin^{(b^2-1)}\left(\frac{2(t+c_1)}{m^2}\right) \quad (3.18)$$

With the help of Equations (3.7), (3.8) in Equation (2.6), we get the pressure of holographic dark energy as,

$$p_\Lambda = \frac{5 \cot^2 \left( \frac{2(t+c_1)}{m^2} \right) + 11}{4m^4} \tag{3.19}$$

Using Equations (3.18) and (3.19), the EoS parameter of holographic dark energy is obtain as,

$$w_\Lambda = \frac{\rho_\Lambda}{p_\Lambda} = \frac{5 \cot^2 \left[ \frac{2(t+c_1)}{m^2} \right] + 11}{4m^4} \tag{3.20}$$

$$w_\Lambda = \frac{5 \cot^2 \left[ \frac{2(t+c_1)}{m^2} \right] - 1}{4m^4 - \rho_0 m^{4(b^2-1)} \sin^{(b^2-1)} \left[ \frac{2(t+c_1)}{m^2} \right]}$$

Statefinder parameters is defined and given as,

$$r = \frac{\ddot{a}}{aH^3}, \quad s = \frac{r-1}{3(q-1/2)}, \quad s = \frac{2(r-1)}{r-11} \tag{3.21}$$

**Second Model for  $n = \frac{3}{2}$**

When  $n = \frac{3}{2}$ , Equation (3.6) reduces to

$$\frac{a_2^3 dS}{\sqrt{\frac{a_2^4}{m^4} + k}} = dt \tag{3.22}$$

which after integration leads to

$$a_2 = \left( \frac{4}{m^4} (t+k_1)^2 - m^4 k \right)^{\frac{1}{4}} \tag{3.23}$$

where  $c_1$  is constant of integration. Equation (3.1) for  $n = \frac{3}{2}$ , leads to

$$a_1 = m \left( \frac{4}{m^4} (t+k_1)^2 - m^4 k \right)^{\frac{3}{8}} \tag{3.24}$$

Therefore, the metric (2.1) reduces to

$$ds^2 = -dt^2 + m^2 \left( \frac{4}{m^4} (t+k_1)^2 - m^4 k \right)^{\frac{3}{4}} dx^2$$

$$+ \frac{16}{m^2} \left( \frac{4}{m^4} (t+k_1)^2 - m^4 k \right)^{\frac{1}{2}} (dy - xdz)^2 \tag{3.25}$$

$$+ m^2 \left( \frac{4}{m^4} (t+k_1)^2 - m^4 k \right)^{\frac{3}{4}} dz^2$$

The volume scale factor, average scale factor, mean Hubble parameter, deceleration parameter, shear scalar and mean anisotropy parameter are obtained as,



$$V = m^2 \left( \frac{4}{m^4} (t + k_1)^2 - m^4 k \right) \quad (3.26)$$

$$a = m^{2/3} \left( \frac{4}{m^4} (t + k_1)^2 - m^4 k \right)^{2/3} \quad (3.27)$$

$$H = \frac{8(t + k_1)}{3 \left[ 4(t + k_1)^2 - m^8 k \right]} \quad (3.28)$$

$$q = \frac{1}{2} + \frac{3m^8 k}{8(t + k_1)^2} \quad (3.29)$$

$$A_m = \frac{1}{32} \quad (3.30)$$

Using Equation (3.27) in Equations (2.11) and (2.12), we get energy density of dark matter and interacting term as,

$$\rho_m = \rho_0 m^{-2} \left[ 4(t + k_1)^2 - m^8 k \right]^{(b^2-1)} \quad (3.31)$$

$$Q = 8b^2 \rho_0 (t + k_1) m^{-2(b^2-1)} \left[ 4(t + k_1)^2 - m^8 k \right]^{(b^2-1)} \quad (3.32)$$

With the help of Equations (3.24), (3.25) and (3.32) in the Equation (3.2.7), the energy density of holographic dark energy obtain as,

$$\rho_\Lambda = \frac{21(t + k_1)^2}{\left[ 4(t + k_1)^2 - m^8 k \right]^2} - \frac{1}{4 \left[ 4(t + k_1)^2 - m^8 k \right]} - \rho_0 m^{-2} \left[ 4(t + k_1)^2 - m^8 k \right]^{(b^2-1)} \quad (3.33)$$

With the help of Equations (3.24), (3.25) in the linear combination of Equations (2.5)-(2.6), the pressure of holographic dark energy obtained as,

$$p_\Lambda = \frac{21(t + k_1)^2}{\left[ 4(t + k_1)^2 - m^8 k \right]^2} - \frac{21}{4 \left[ 4(t + k_1)^2 - m^8 k \right]} \quad (3.34)$$

Using Equations (3.34) and (3.35), the EoS parameter of holographic dark energy is obtain as,

$$w_\Lambda = \frac{p_\Lambda}{\rho_\Lambda} = \frac{\frac{21(t + k_1)^2}{\left[ 4(t + k_1)^2 - m^8 k \right]^2} - \frac{21}{4 \left[ 4(t + k_1)^2 - m^8 k \right]}}{\frac{21(t + k_1)^2}{\left[ 4(t + k_1)^2 - m^8 k \right]^2} - \frac{1}{4 \left[ 4(t + k_1)^2 - m^8 k \right]} - \rho_0 m^{-2} \left[ 4(t + k_1)^2 - m^8 k \right]^{(b^2-1)}} \quad (3.35)$$

Statefinder parameters is defined as

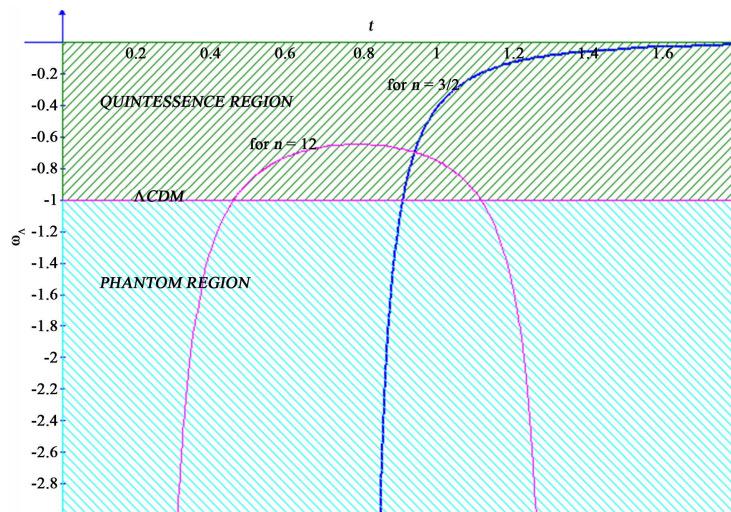
$$r = \frac{\ddot{a}}{aH^3} = 1 + \frac{9m^8 k}{4(t + k_1)^2} \quad (3.36)$$

$$s = \frac{r-1}{3(q-1/2)} = 2 \tag{3.37}$$

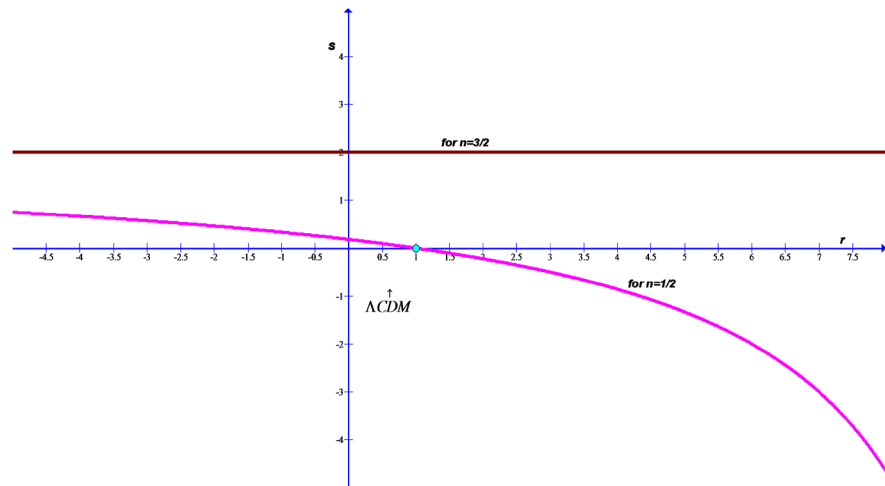
### 4. Discussion

Here we discussed two models for  $n=1/2$  and  $n=3/2$ . In the model  $n=1/2$ , as time increases spatial volume also increases. As  $t \rightarrow 0$ ,  $V \rightarrow m^4 \sin\left(\frac{2c_1}{m^2}\right)$  and as  $t \rightarrow \infty$ ,  $V \rightarrow \infty$ . Accelerating expansion of the model occurs for the situation  $\sec^2\left(\frac{2(t+c_1)}{m^2}\right) < \frac{m^2}{3}$ . Whereas it is simply observe that model is in state of deceleration is obtained for the situation when  $\sec^2\left(\frac{2(t+c_1)}{m^2}\right) > \frac{m^2}{3}$ . It is

important to note that the model (3.10) include periodic function which is responsible for cyclic universe. Also in this model  $\omega_\Lambda$  starts from phantom region and it reaches to  $\omega_\Lambda=1$  (*i.e.* cosmological constant). After some finite time it enter into quintessence region ( $-1 < \omega_\Lambda < -1/3$ ) and again come back into phantom region. Model for  $n=3/2$ , as  $t \rightarrow 0$ ,  $V \rightarrow \text{constant}$  and as  $t \rightarrow \infty$ ,  $V \rightarrow \infty$  *i.e.* universe expands as time increases. As  $t \rightarrow 0$ ,  $\omega_\Lambda \rightarrow 0$ , and as time increases  $\omega_\Lambda$ , starts from phantom region. For some finite time, it extended to  $\omega_\Lambda = -1$  (*i.e.* cosmological constant), which denotes that our model drive to  $\Lambda$ CDM. After some time, it reaches to quintessence region ( $-1 < \omega_\Lambda < -1/3$ ) for late time as shown in **Figure 1**. The available observation in cosmology, especially the three year WMAP data (Sprengel *et al.* [65]), the SNeIa data (Riess *et al.* [66]; Asteir *et al.* [67]) and the SDSS data (Eisenstein *et al.* [68]) pointed out that the  $\Lambda$ CDM model or the model reducible to  $\Lambda$ CDM are present as a standard model in cosmology, which is similar to present universe. **Figure 2** shows that the  $s-r$  plane having evolving trajectory for corresponding model is different from other dark energy models.



**Figure 1.** Evolution of EoS parameter  $\omega_\Lambda$  versus cosmic time  $t$ .



**Figure 2.** State finder parameters  $s$  versus  $r$ .

## 5. Conclusion

In this paper, we have studied LRS Bianchi Type-II universe filled with interacting dark matter and holographic dark energy. Here we have discussed two models for 1)  $n = 1/2$ ; 2)  $n = 3/2$ . In the model  $n = 1/2$ , we obtained the cyclic universe and in case of model for  $n = 3/2$ , we obtained the expanding universe. The cosmological parameter matches with the present day observations. The model for  $n = 1/2$  converges into phantom region whereas the model for  $n = 3/2$  converges into quintessence region. The statefinder parameter  $\{r, s\}$  has been calculated in order to differentiate our model of dark energy with all other models of dark energy. We aspire that the future accurate observation will be capable of determining these statefinder parameters and consequently explore the nature of DE.

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## Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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