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Compact-Open and Point Wise Convergence Topologies

Mohammed Nokhas Murad Kaki

Department of Mathematics, College of Science, University of Sulaimani, Sulaimani, Iraq
Email: Muradkakaee@yahoo.com

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Abstract

In this paper, we have investigated and introduced some new definitions of transitivity on the set of all continuous maps, denoted by $C(X,Y)$, called the point-wise convergence transitive, the compact-open transitive and point wise convergence topological transitive sets. Relationship between these new definitions is studied. Finally, we have introduced a number of very important topological concepts and shown that every compact-open convergence transitive map implies point wise transitive maps but the converse not necessarily true.

Subject Areas

Mathematical Analysis

Keywords

Compact-Open Topology, Transitive Set, Chaotic Sets, Point Wise Convergence Mixing

1. Introduction

Let (X,τ) and (Y,σ) be two topological spaces and $C(X,Y)$ be the set of all continuous maps from X into Y. Consider all possible sets of maps of the form

$$[K,U]=\{f\in C(X,Y): f(K)\subset U\},$$

where K is a compact set in X and U an open set in Y. The topology τ_3 generated by these sets $[K,U]$ as a subbase is called the compact-open topology on $C(X,Y)$. Note that any open set in τ_3 is called co-open set and $(C(X,Y),\tau_3)$ is called co-topological space. The compliment of co-open set is called co-closed set. We have introduced some new definitions of transitivity on $C(X,Y)$, called the point-wise convergence transitive set, the compact-open transitive and point wise convergence topological transitive sets in $C(X, Y)$. Relationship between these new definitions is studied. Finally, we have introduced a number of very important topological concepts and shown that every compact-open convergence transitive set implies point wise transitive set and that every compact-open-mixing system implies point wise convergence system but not conversely. Finally, we have shown that every strongly compact-open-mixing set implies strongly point wise convergence mixing set but the converse not necessarily true.

2. New Theorems of Point Wise-Convergence Topology

Definition 2.1. Consider in $C(X,Y)$ the sets

$$\{x_i,U_i\}_{i1}^{k}=\{f\in C(X,Y):f(x_i)\in U_i,i=1,\cdots,k\}$$

where $x_1,\cdots,x_k\in X$, $U_1,\cdots,U_k$ are open sets in *Y*.

The topology τ_2 generated by these sets in their capacity as a subset is called the topology of point-wise convergence on $C(X,Y)$.

Note that any open set in τ_2 is called pc-open set and $(C(X,Y),\tau_2)$ is called pc-topological space. The compliment of pc-open set is called pc-closed set.

Definition 2.2. A function $F:C(X,Y)\to C(X,Y)$ is called pc-irresolute if the inverse image of each pc-open set is a pc-open set in $C(X,Y)$.

Definition 2.3. A map $F:C(X,Y)\to C(X,Y)$ is *pcr*-homeomorphism if it is bijective and thus invertible and both F and F^{-1} are *pc*-irresolute.

The systems $F:C(X,X)\to C(X,X)$ and $G:C(Y,Y)\to C(Y,Y)$ are topologically pcr-conjugate if there is a pcr-homeomorphism $H:C(X,X)\to C(Y,Y)$ such that $H\circ F=G\circ H$.

Let $(C(X,Y),\tau_2)$ be a pc-topological space. The intersection of all pc-closed sets of $(C(X,Y),\tau_2)$ containing *A* is called the pc-closure of *A* and is denoted by $Cl_{pc}(A)$.

Definition 2.4. Let $(C(X,Y),\tau_2)$ be a point wise convergence-topological space, and $F:C(X,Y)\to C(X,Y)$ be a map. The map F is said to have pc-dense *orbit* if there exists $f\in C(X,Y)$ such that $Cl_{pc}(O_F(f))=C(X,Y)$.

Definition 2.5. Let $(C(X,Y),\tau_2)$ be a pc-topological space, and $F:C(X,Y)\to C(X,Y)$ be a pc-irresolute map, then F is said to be a point-wise-converge-transitive (shortly pc-transitive) map if for every pair of pc-open sets U and V in $(C(X,Y),\tau_2)$ there is a positive integer *n* such that $F^n(U)\cap V\neq\phi$.

Definition 2.6. Let $(C(X,Y),\tau_2)$ be a point wise convergence-topological space, and $F:C(X,Y)\to C(X,Y)$ be a pc-irresolute then the set $A\subseteq C(X,Y)$ is called pc-type transitive set if for every pair of non-empty pc-open sets U and V in $C(X,Y)$ with $A\cap U\neq\phi$ and $A\cap V\neq\phi$ there is a positive integer n such that $F^n(U)\cap V\neq\phi$.

Definition 2.7. 1) Let $(C(X,Y),\tau_1)$ be a point-wise convergence-topological space, and $F:C(X,Y)\to C(X,Y)$ be a pc-irresolute then the set $A\subseteq C(X,Y)$ is called is called topologically pc-mixing set if, given any nonempty pc-open subsets $U,V\subseteq C(X,Y)$ with $A\cap U\neq\phi$ and $A\cap V\neq\phi$ then $\exists N>0$ such that $F^n(U)\cap V\neq\phi$ for all $n>N$.

2) The set $A\subseteq C(X,Y)$ is called a weakly pc-mixing set of $(C(X,Y),F)$ if for any choice of nonempty pc-open subsets V_1,V_2 of *A* and nonempty pc-open subsets U_1,U_2 of $C(X,Y)$ with $A\cap U_1\neq\phi$ *and* $A\cap U_2\neq\phi$ there exists $n\in\mathrm{N}$ such that $F^n(V_1)\cap U_1\neq\phi$ and $F^n(V_1)\cap U_2\neq\phi$.

3) The set $A\subseteq C(X,Y)$ is *strongly pc-mixing* if for any pair of pc-open sets U and V with $U\cap A\neq\phi$ and $V\cap A\neq\phi$, there exist some $n\in\mathbf{N}$ such that $F^k(U)\cap V\neq\phi$ for any $k\geq n$.

4) Any element $f\in C(X,Y)$ such that its orbit $O_F(f)$ is pc-dense in *X*. is called hypercyclic element.

5) A system $(C(X,Y),F)$ is said to be topologically pc-mixing if, given pc-open sets U and V in $C(X,Y)$, there exists an integer *N*, such that, for all $n>N$, one has $F^n(U)\cap V\neq\phi$.

6) A system $(C(X,Y),F)$ is called *topologically* pc-*mixing if* for any *non-empty* pc-open set U, there exists $n\in\mathrm{N}$ such that $\bigcup_{n\geq N}F^n(U)$ is pc-dense in $C(X,Y)$.

3. Definitions and Theorems of Compact-Open Topology

The following definition supplies another version of a topology on the set $C(X,Y)$.

Definition 3.1. Consider all possible sets of maps of the form [1]

$$[K,U]=\{f\in C(X,Y):f(K)\subset U\}$$

where *K* is a compact set in *X* and *U* an open set in *Y*. The topology τ_3 generated by these sets $[K,U]$ as a subbase is called the compact-open topology on $C(X,Y)$.

Note that any open set in τ_3 is called co-open set and $(C(X,Y),\tau_3)$ is called co-topological space. The compliment of co-open set is called co-closed set.

Definition 3.2. Let $(C(X,Y),\tau_3)$ be a co-topological space. The map $F:C(X,Y)\to C(X,Y)$ is called co-irresolute if for every subset $A\in\tau_3$, $F^{-1}(A)\in\tau_3$. or, equivalently, *F* is *co*-irresolute if and only if for every co-closed set *A*, $F^{-1}(A)$ is co-closed set.

Definition 3.3. A map $F:C(X,Y)\to C(X,Y)$ is *cor*-homeomorphism if it is bijective and thus invertible and both *F* and F^{-1} are *co*-irresolute.

The systems $F:C(X,X)\to C(X,X)$ and $G:C(Y,Y)\to C(Y,Y)$ are topologically cor-conjugate if there is a cor-homeomorphism $H:C(X,X)\to C(Y,Y)$ such that $H\circ F=G\circ H$.

Let $(C(X,Y),\tau_3)$ be a co-topological space. The intersection of all co-closed sets of $\left(C(X,Y),\tau_3\right)$ containing A is called the co-closure of A and is denoted by $Cl_{co}(A)$.

Definition 3.4. Let $\left(C(X,Y),\tau_3\right)$ be a compact-open topological space, and $F: C(X,Y) \to C(X,Y)$ be a map. The map F is said to have co-dense *orbit* if there exists $f \in C(X,Y)$ such that $Cl_{co}\left(O_F(f)\right) = C(X,Y)$.

Definition 3.5. Let $\left(C(X,Y),\tau_3\right)$ be a co-topological space, and $F: C(X,Y) \to C(X,Y)$ be a co-irresolute map, then F is said to be a compact-open-transitive (shortly co-transitive) map if for every pair of co-open sets U and V in $\left(C(X,Y),\tau_3\right)$ there is a positive integer n such that $F^n(U) \cap V$ is not empty.

Definition 3.6. Let $\left(C(X,Y),\tau_3\right)$ be a co-topological space, and $F: C(X,Y) \to C(X,Y)$ be a co-irresolute then the set $A \subseteq C(X,Y)$ is called co-type transitive set if for every pair of non-empty co-open sets U and V in $C(X,Y)$ with $A \cap U \neq \phi$ and $A \cap V \neq \phi$ there is a positive integer n such that $F^n(U) \cap V \neq \phi$.

Definition 3.7. 1) Let $\left(C(X,Y),\tau_3\right)$ be a co-topological space, and $F: C(X,Y) \to C(X,Y)$ be a co-irresolute then the set $A \subseteq C(X,Y)$ is called is called topologically co-mixing set if, given any nonempty co-open subsets $U, V \subseteq C(X,Y)$ with $A \cap U \neq \phi$ and $A \cap V \neq \phi$ then $\exists N > 0$ such that $F^n(U) \cap V \neq \phi$ for all $n > N$.

2) *The set* $A \subseteq C(X,Y)$ is called a weakly co-mixing set of $\left(C(X,Y),F\right)$ if for any choice of nonempty co-open subsets V_1, V_2 of A and nonempty co-open subsets U_1, U_2 of $C(X,Y)$ with $A \cap U_1 \neq \phi$ *and* $A \cap U_2 \neq \phi$ there exists $n \in \mathrm{N}$ such that $F^n(V_1) \cap U_1 \neq \phi$ and $F^n(V_1) \cap U_2 \neq \phi$.

3) The set $A \subseteq C(X,Y)$ is *strongly co-mixing* if for any pair of co-open sets U and V with $U \cap A \neq \phi$ and $V \cap A \neq \phi$, there exist some $n \in \mathbf{N}$ such that $F^k(U) \cap V \neq \phi$ for any $k \geq n$.

4) A system $\left(C(X,Y),F\right)$ is said to be topologically co-mixing if, given co-open sets U and V in $C(X,Y)$, there exists an integer *N*, such that, for all $n > N$, one has $F^n(U) \cap V \neq \phi$. For related works about weakly mixing see [2], [3] and [4].

4. Conclusions

We have the following results:

1) Every compact-open-transitive set implies point wise convergence set but not conversely.

2) Every compact-open-mixing system implies point wise convergence system but not conversely.

3) Every strongly compact-open-mixing set implies strongly point wise convergence mixing set.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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