Spreading Dynamic of a *PLSGP* Giving up Smoking Model on Scale-Free Network

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Abstract

A new *PLSGP* (potential smokers-light smokers-persistent smokers-giving up smokers-potential smokers) model with birth and death rates on complex heterogeneous networks is presented. Using the mean-field theory, we obtain the basic reproduction number R_0 and find that basic reproduction number for constant contact is independent of the topology of the underlying networks. When $R_0 < 1$, the smoking-free equilibrium is globally asymptotically stable, then the smoking will disappear. When $R_0 > 1$, the smoking-present equilibrium is global attractivity, then the number of smoker will remain stable and smoking will become endemic. Numerical simulations illustrated theoretical results. Our result shows that the model is very important to control the spread of the smoking.

Subject Areas

Complex Network Models, Dynamical System, Network Modeling and Simulation, Numerical Mathematics, Numerical Methods

Keywords

Spreading Dynamic, Smoking, Basic Reproduction Number, Equilibrium, Scale-Free Network

1. Introduction

Smoking is closely related to health, and smoking ranks fourth among the top 10 risk factors for health, according to the World Health Organization report. Tobacco has been identified as a primary carcinogen around the world. Smokers are 10 to 30 times more likely to develop lung cancer than non-smokers. Smoking problem of people has become a significant public health concern. The beha-

vior of smoking often causes a range of negative consequences. Long-term smoking produces negative changes in the heart, such as heart rate and blood pressure rise. Smoking damages almost all parts of the human body and contributes to a number of human diseases including lung cancer, respiratory disease, heart disease, alimentary canal effect and eventually death. Due to the increasing in the number of smokers, tobacco use is also as a disease to be treated.

In recent years, many types of epidemic models are discussed, such as virus dynamics models [1] [2], tuberculosis models [3] [4], and HIV models [5] [6]. One of the famous representative works in this area was done by Pastor-Satorras and Vespignani [7]. They presented a detailed analytical and numerical study on an SIS epidemic model in the highly heterogeneous networks (*i.e.*, scale-free networks). The most striking result is that they found the absence of the epidemic threshold in these networks. That is, the threshold approaches zero in the limit of a large number of edges and nodes, and even a quite small infectious rate can produce a major epidemic outbreak. Liu XD *et al.* [8] presented an SIS (susceptible-infected-susceptible) epidemic model with infective medium and feedback mechanism on scale-free networks. They found that the epidemic is not only spread between individuals by direct contacts but also transmitted by medium, people's initial response when epidemic disease outbreaks have been also considered. Moreno *et al.* [9] also found these similar conclusions for an *SIR* (susceptible-infected-removed) epidemic model on scale-free networks. The epidemic model is constantly evolving, such as *SIRS* (susceptible-infected-removed-susceptible), *SEIR* (susceptible-infected-exposed-removed), *SIQRS* (susceptible-infected-quarantined-recovered-susceptible) [10] [11] [12] [13] [14].

Studying of smoking behavior has attracted the attention of many scholars and researchers recently. In order to explore the spread rule of smoking, some models are developed. Castillo-Garsow *et al.* [15] proposed a mathematical model for giving up smoking in the first time. In this model, a total constant population was divided into three states: potential smokers, that is, people who do not smoke yet but might become smokers in the future (*P*), smokers (*S*), and quit smokers (*Q*). Zaman [16] extended the work of Castillo-Garsow *et al.* [15] by adding the population of occasional smokers in the model, and presented qualitative behavior of the model. Zaman [17] presented the optimal campaigns in the smoking dynamics. They consider two possible control variables in the form of education and treatment campaigns oriented to decrease the attitude towards smoking and first showed the existence of an optimal control for the control problem.

However, in real life, some potential smokers may become light smoker since they contact with light smokers or persistent smokers. Some quit smokers may be only temporary quit smokers, so they will become potential smokers again. Enlightening by the previously mentioned cases, we present a *PLSGP* giving up smoking model on scale-free network. The paper is organized as follows: The model is formulated in Section 2. The basic reproduction number and existence

of smoking equilibriums are calculated in Section 3. In Section 4, we analyze the stability of the equilibria. In Section 5, sensitivity analysis and numerical simulations are illustrated. In Section 6, we give some conclusions and discussions.

2. The Model Formulation

In this paper, we establish the giving up smoking model as **Figure 1**. From **Figure 1**, the total population is divided into four compartments, namely, the potential smokers compartment (P), light or occasion smokers compartment (L), persistent smokers compartment (S), and quit smokers group (G). The total recruitment number into this homogeneous social mixing community is b. Transmission coefficient from the potential smokers compartment to the light smokers compartment is ρ_1, Transmission coefficient from the potential smokers compartment to the persistent smokers compartment is ρ_2, Transmission coefficient from the light smokers compartment to the persistent smokers compartment is α, The permanent quit smoking rate is β, The relapse rate of which temporary quit people to become potential smokers is δ. Naturally death rate is μ. The total population size is $N(t)$. Let $P_k(t)$, $L_k(t)$, $S_k(t)$, $G_k(t)$ be the relative densities of potential smokers, light smokers, persistent smokers and quit smokers nodes of degree k at time t, respectively.

With these assume, the dynamic mean-field equations of the $PLSGP$ model can be written as follows:

$$\begin{cases} \dfrac{dP_k(t)}{dt} = b + \delta G_k(t) - k(\rho_1\Theta_1 + \rho_2\Theta_2)P_k(t) - \mu P_k(t) \\[2mm] \dfrac{dL_k(t)}{dt} = k(\rho_1\Theta_1 + \rho_2\Theta_2)P_k(t) - (\alpha + \mu + \gamma)L_k(t) \\[2mm] \dfrac{dS_k(t)}{dt} = \alpha L_k(t) - (\mu + \beta)S_k(t) \\[2mm] \dfrac{dG_k(t)}{dt} = \beta S_k(t) + \gamma L_k(t) - (\mu + \delta)G_k(t) \end{cases} \tag{2.1}$$

where

$$\begin{aligned} \Theta_1(t) &= \sum_i P(i\,|\,k)L_i(t) = \langle k\rangle^{-1}\sum_i iP(i)L_i(t) \\ \Theta_2(t) &= \sum_i P(i\,|\,k)S_i(t) = \langle k\rangle^{-1}\sum_i iP(i)S_i(t) \end{aligned} \tag{2.2}$$
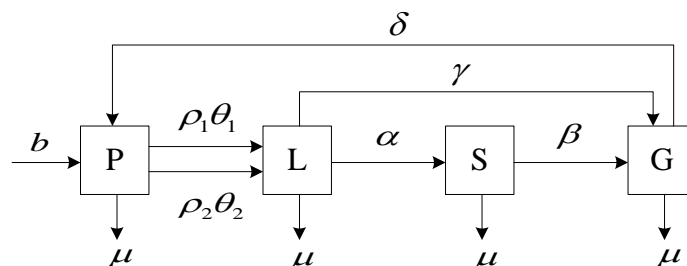

Figure 1. Transfer diagram for somking model.

where, $P(k) > 0$ is the probability that a node has degree k and thus $\sum_{k=1}^{n} P(k) = 1$, $\langle k \rangle = \sum_{k=1}^{n} kP(k)$ denotes the average degree. $\rho = \rho_1 \Theta_1 + \rho_2 \Theta_2$. Clearly, these variables obey the normalization condition:

$$P_k(t) + L_k(t) + S_k(t) + G_k(t) = 1. \tag{2.3}$$

The initial conditions for system can be given as follows

$S_k(0) = 1 - P_k(0) - L_k(0) - G_k(0) \geq 0$, $R_k(0) \geq 0$, $S_k(0) \geq 0$, $C_k(0) \geq 0$. In this model, we assumed μ equal to p.

$$\begin{cases} \dfrac{dL_k(t)}{dt} = k\rho \left(1 - L_k(t) - S_k(t) - G_k(t)\right) P_k(t) - (\alpha + \mu + \gamma) L_k(t) \\[2mm] \dfrac{dS_k(t)}{dt} = \alpha L_k(t) - (\mu + \beta) S_k(t) \\[2mm] \dfrac{dG_k(t)}{dt} = \beta S_k(t) + \gamma L_k(t) - (\mu + \delta) G_k(t) \end{cases} \tag{2.4}$$

3. The Basic Reproduction Number and Equilibrium

Theorem 1. Consider system (2.1). Define $R_0 = \dfrac{\langle k^2 \rangle (\rho_1(\beta + \mu) + \rho_2 \alpha)}{\langle k \rangle (\beta + \mu)(\alpha + \mu + \gamma))}$. There always exists the smoking-free equilibrium $E_0(1,0,0,0)$. When $R_0 > 1$, the system has an occasion smoking equilibrium $E^*\left(P_k^*, L_k^*, S_k^*, G_k^*\right)$.

Proof. To get the information-prevailing equilibrium solution $E^*\left(P_k^*, L_k^*, S_k^*, G_k^*\right)$, we need to make the right side of system equal to zero, it should satisfy

$$\begin{cases} b + \delta G_k^*(t) - k\rho^* P_k^*(t) - \mu P_k^*(t) = 0 \\[2mm] k\rho^* P_k^*(t) - (\alpha + \mu + \gamma) L_k^*(t) = 0 \\[2mm] \alpha L_k^*(t) - (\mu + \beta) S_k^*(t) = 0 \\[2mm] \beta S_k^*(t) + \gamma L_k^*(t) - (\mu + \delta) G_k^*(t) = 0 \end{cases} \tag{3.1}$$

where $\rho^* = \rho_1 \Theta_1^* + \rho_2 \Theta_2^*$, we follow from (3.1) that

$$S_k^*(t) = \frac{k\rho\alpha(\mu + \delta)}{(\mu + \delta)(\mu + \beta)(\mu + \alpha + \gamma) + k\rho\left[(\mu + \beta)(1 + \gamma) + \alpha(\beta + (\mu + \delta))\right]}$$

$$L_k^*(t) = \frac{k\rho(\mu + \delta)(\mu + \beta)}{(\mu + \delta)(\mu + \beta)(\mu + \alpha + \gamma) + k\rho\left[(\mu + \beta)(1 + \gamma) + \alpha(\beta + (\mu + \delta))\right]} \tag{3.2}$$

Obviously, $\rho^* = 0$ satisfies (3.1). Hence, $P_k = 1$ and $L_k = S_k = R_k = 0$ is an equilibrium of (2.1), which is called the smoking-free equilibrium.

Substituting L_k^* and S_k^* of (3. 2) into ρ^*

Let $\rho^* \cong f(\rho^*)$

Clearly, $\rho^* = 0$ is a solution of equation. To ensure the equation has a non-trivial solution, the following condition must should satisfied

$$\left.\frac{\mathrm{d}f\left(\rho^*\right)}{\mathrm{d}\rho^*}\right|_{\rho^*} = \frac{\partial f\left(\rho^*\right)}{\partial L_k^*} + \frac{\partial f\left(\rho^*\right)}{\partial S_k^*} > 1 \quad \text{and} \quad f\left(1\right) \le 1. \tag{3.3}$$

We can obtain the reproductive number

$$R_0 = \frac{\left\langle k^2 \right\rangle \left(\rho_1\left(\beta+\mu\right)+\rho_2\alpha\right)}{\left\langle k \right\rangle \left(\beta+\mu\right)\left(\alpha+\mu+\gamma\right)}.$$

4. Stability Analysis of the Equilibrium

Theorem 2. When $R_0 < 1$, the smoking-free equilibrium of system (2.1) is globally asymptotically stable.

Proof. The Jacobian matrix of the smoking-free equilibrium of system (2.1), which is a $3n \times 3n$ matrix, can be written as follows:

$$J = \begin{pmatrix} A_{11} & \cdots & A_{1n} \\ \vdots & \ddots & \vdots \\ A_{n1} & \cdots & A_{nn} \end{pmatrix}$$

where

$$A_{11} = \begin{pmatrix} -\delta-\mu & -\delta-\rho_1 P_1 & -\delta-\rho_2 P_1 \\ 0 & \rho_1 P_1 - \left(\alpha+\mu+\gamma\right) & \rho_2 P_1 \\ 0 & \alpha & -\mu-\beta \end{pmatrix}$$

$$A_{1n} = \begin{pmatrix} 0 & -\rho_1 P_n & -\rho_2 P_n \\ 0 & \rho_1 P_n & \rho_2 P_n \\ 0 & 0 & 0 \end{pmatrix}$$

$$A_{1n} = \begin{pmatrix} 0 & -n\rho_1 P_n & -n\rho_2 P_n \\ 0 & n\rho_1 P_n & n\rho_2 P_n \\ 0 & 0 & 0 \end{pmatrix}$$

$$A_{nn} = \begin{pmatrix} -\delta-\mu & -\delta-n\rho_1 P_n & -\delta-n\rho_2 P_n \\ 0 & n\rho_1 P_n - \left(\mu+\alpha+\gamma\right) & n\rho_2 P_n \\ 0 & \alpha & -\mu-\beta \end{pmatrix}$$

A direct calculation leads to the characteristic polynomial of the smoking-free equilibrium in the following from:

$$\left(\lambda+\left(\mu+\alpha+\gamma\right)\right)^{n-1}\left(\lambda+\mu+\beta\right)\left(\lambda^2+p\lambda+q\right)=0,$$

where $p = \left(\mu+\beta\right)+\left(\mu+\alpha+\gamma\right)-\rho_1\sum_{i=1}^{n} iP\left(i\right)$ and
$q = \left(\mu+\beta\right)\left(\mu+\alpha+\gamma+\rho_1\right)-\alpha\rho_2\sum_{i=1}^{n} iP\left(i\right)$.

Note that $R_0 < 1$ is equivalent to $q > 0$ and that $R_0 < 1$ also implies: $\left(\mu+\beta\right)+\left(\mu+\alpha+\gamma\right) > \rho_1\sum_{i=1}^{n} iP\left(i\right)$, which means $p > 0$. Note that $R_0 < 1$ is equivalent to $q > 0$ and that $R_0 < 1$ also implies and, which means $p > 0$. Therefore, there exists a unique positive eigenvalue λ of J if and only if $R_0 > 1$, otherwise, if $R_0 < 1$, all real-valued eigenvalues of J are negative. By the Perron-Frobenius theorem, it implies that the maximal real part of all eigen-

values of J is positive if and only if $R_0 > 1$. Then, a theorem of Lajmanovich and York [18] yields the results of this theorem. The proof is thus completed.

Next, the globally attractivity of positive endemic equilibrium is discussed. The main result is given in the following theorem.

Lemma 1. [19] if $a > 0$, $b > 0$ and $\dfrac{\mathrm{d}x(t)}{\mathrm{d}t} \geq b - ax$, when $t \geq 0$ and $x(0) \geq 0$, we have $t \geq 0 \lim_{t\to\infty} \inf x(t) \geq \dfrac{b}{a}$, if $a > 0$, $b > 0$ and $\dfrac{\mathrm{d}x(t)}{\mathrm{d}t} \leq b - ax$, when $t \geq 0$ and $x(0) \geq 0$, we have $\lim_{t\to\infty} \sup x(t) \leq \dfrac{b}{a}$.

Theorems 3. Suppose that $\left(L_k(t), S_k(t), G_k(t)\right)$ is a solution of (2.4), with $L_k(0) > 0$, $S_k(0) > 0$, $G_k(0) > 0$ and $R_0 > 1$. If $R_0 > 1$, then $\lim_{t\to\infty}\left(L_k(t), S_k(t), G_k(t)\right) = \left(L_k^*(t), S_k^*(t), G_k^*(t)\right)$, where $\left(L_k^*(t), S_k^*(t), G_k^*(t)\right)$ is the unique smoking equilibrium of (2.4) for $k = 1, 2, \cdots, n$.

Proof: In the following, k is fixed to be any integer in $(1, 2, \cdots, n)$. There exists a sufficiently small constant $\xi (0 < \xi < 1)$ and a larger enough constant $T > 0$ such that $L_k(t) \geq \xi$ and $S_k(t) \geq \xi$ for $t > T$, therefore $(\rho_1 + \rho_2)\xi < \rho(t) < \rho_1 + \rho_2$ for $t > T$. Submit this into the first equation of (2.4) gives

$$L_k'(t) \leq k(\rho_1 + \rho_2)(1 - L_k(t)) - (\alpha + \mu + \gamma)L_k(t), \ t > T$$

By **Lemma 1**, for any given constant $0 < \xi < \dfrac{\mu + \alpha + \gamma}{k(\rho_1 + \rho_2)(\mu + \alpha + \gamma)}$, there exists a $t_1 > T$, such that $L_k(t) \leq X_k^{(1)} + \xi_1$ for $t > t_1$, where

$$L_k(t) \leq X_k^{(1)} + \xi_1 = \dfrac{k(\rho_1 + \rho_2)}{k(\rho_1 + \rho_2) + (\alpha + \mu + \gamma)} + \xi_1 < 1, \ t > t_1 \qquad (4.1)$$

From the second equation of (2.4), it follows that

$$S_k'(t) \leq \alpha(1 - S_k(t)) - (\mu + \beta)S_k(t), \ t > t_1 \qquad (4.2)$$

Hence, for any given constant $0 < \xi_2 < \min\left\{1/2, \xi_1, (\beta + \mu)(\mu + \alpha + \beta)^{-1}\right\}$, there exists a $t_2 > t_1$, such that $S_k(t) \leq Y_k^{(1)} - \xi_2$ for $t > t_2$, where

$$S_k(t) \leq Y_k^{(1)} + \xi_2 < \alpha(\alpha + \mu + \beta)^{-1} + \xi_2 < 1, \ t > t_2 \qquad (4.3)$$

Then, it follows from the third equation of (2.4),

$$G_k'(t) \leq \beta(1 - G_k(t)) + \gamma(1 - G_k(t)) - (\mu + \delta)G_k(t), \ t > t_2 \qquad (4.4)$$

Similarly, for any given constant $0 < \xi_3 < \min\left\{1/3, \xi_2, (\delta + \mu)(\mu + \delta + \beta + \gamma)^{-1} + \xi_3\right\}$, there exists a $t_3 > t_2$, such that $G_k(t) \leq Z_k^{(1)} + \xi_3$ for $t > t_3$, where

$$G_k(t) \leq Z_k^{(1)} + \xi_3 < (\beta + \gamma)(\mu + \beta + \gamma + \delta)^{-1} + \xi_3 < 1, \ t > t_3 \qquad (4.5)$$

Since $(\rho_1 + \rho_2)\xi < \rho(t)$, we substitute this into the first equation of (2.4)

$$L'_k(t) \geq k\xi(\rho_1 + \rho_2)\left(1 - L_k(t) - Y_k^{(1)} - Z_k^{(1)}\right) - (\alpha + \mu + \gamma)L_k(t), \ t > T \quad (4.6)$$

So for any given enough small constant

$$0 < \xi_4 < \min\left\{1/4, \xi_3, \frac{k\xi(\rho_1 + \rho_2)\left(1 - L_k(t) - Y_k^{(1)} - Z_k^{(1)}\right)}{k\xi(\rho_1 + \rho_2) + (\alpha + \mu + \gamma)}\right\}, \text{ there exists a } t_4 > t_3,$$

such that $L_k(t) \geq x_k^{(1)} - \xi_4$ for $t > t_4$, where

$$L_k(t) \geq x_k^{(1)} - \xi_4 = \frac{k\xi(\rho_1 + \rho_2)\left(1 - L_k(t) - Y_k^{(1)} - Z_k^{(1)}\right)}{k\xi(\rho_1 + \rho_2) + (\alpha + \mu + \gamma)} - \xi_4, \ t > t_4 \quad (4.7)$$

It follows that

$$S'_k(t) \geq \alpha x_k^{(1)} - (\mu + \eta)S_k(t), \ t > t_4 \quad (4.8)$$

So for any given enough small constant $0 < \xi_5 < \min\left\{1/5, \xi_4, \alpha x_k^{(1)}\left[(\mu + \beta)\right]^{-1}\right\}$,

there exists a $t_5 > t_4$, such that $S'_k(t) \geq y_k^{(1)} - \xi_5$ for $t > t_5$, where

$$S'_k(t) \geq y_k^{(1)} - \xi_5 = \alpha x_k^{(1)}(\mu + \beta)^{-1} - \xi_5, \ t > t_5 \quad (4.9)$$

From the third equation of (2.1) implies that

$$G'_k(t) \geq \beta y_k^{(1)} + \gamma x_k^{(1)} - (\mu + \delta)G_k(t), \ t > t_5 \quad (4.10)$$

So for any given enough small constant
$0 < \xi_6 < \min\left\{1/6, \xi_5, \left[\beta y_k^{(1)} + \gamma x_k^{(1)}\right](\mu + \delta)^{-1}\right\}$, there exists a $t_6 > t_5$, such that
$G_k(t) \geq z_k^{(1)} - \xi_6$ for $t > t_6$, where

$$G_k(t) \geq z_k^{(1)} - \xi_6 = \left(\beta y_k^{(1)} + \gamma x_k^{(1)}\right)(\mu + \delta)^{-1}, \ t > t_6 \quad (4.11)$$

Due to ξ is a small positive constant, we can derive that $0 < x_k^{(1)} \leq X_k^{(1)} < 1$, $0 < y_k^{(1)} \leq Y_k^{(1)} < 1$ and $0 < z_k^{(1)} \leq Z_k^{(1)} < 1$. Let

$$q^{(j)} = \frac{1}{\langle k \rangle}\sum_{j=1}^{n} P(i)\left(\rho_1 x_i^{(j)} + \rho_2 y_i^{(j)}\right),$$

$$Q^{(j)} = \frac{1}{\langle k \rangle}\sum_{j=1}^{n} P(i)\left(\rho_1 X_i^{(j)} + \rho_2 Y_i^{(j)}\right), \ j = 1, 2, \cdots \quad (4.12)$$

We can easily get $0 < q^{(j)} \leq \rho(t) \leq Q^{(j)} < \rho_1 + \rho_2$, $t > t_6$.

Again, from the first equation of (2.1), it has

$$L'_k(t) \leq kQ^{(1)}\left(1 - L_k(t) - y_k^{(1)} - z_k^{(1)}\right) - (\mu + \alpha + \gamma), \ t > t_6 \quad (4.13)$$

Hence, for any given constant $0 < \xi_7 < \min\{1/7, \xi_6\}$, there exists a $t_7 > t_6$, such that

$$I_k(t) \leq X_k^{(2)}$$

$$= \min\left\{X_k^{(1)} + \xi_1, kQ^{(1)}\left(1 - y_k^{(1)} - z_k^{(1)}\right)\left[kQ^{(1)} + \mu + \alpha + \gamma\right]^{-1} + \xi_7\right\}, t > t_7 \quad (4.14)$$

Then, from the second equation of (2.1), we have

$$S_k'(t) \le Y_k^{(2)} - (\mu + \beta) S_k(t), \ t > t_7 \tag{4.15}$$

So, for any given constant $0 < \xi_8 < \min\{1/8, \xi_7\}$, there exists a $t_8 > t_7$, such that

$$S_k(t) \le Y_k^{(2)} = \min\left\{ Y_k^{(2)} + \xi_2, \alpha X_k^{(2)} (\mu + \beta)^{-1} + \xi_8 \right\}, \ t > t_8 \tag{4.16}$$

Consequently, from the third equation of (2.1), we have

$$G_k'(t) \le \beta Y_k^{(2)} + \gamma X_k^{(1)} - (\mu + \delta) G_k(t), \ t > t_8 \tag{4.17}$$

Hence, for any given constant $0 < \xi_9 < \min\{1/9, \xi_8\}$, there exists a $t_8 > t_7$, such that

$$G_k(t) \le Z_k^{(2)} = \min\left\{ Z_k^{(1)} - \xi_3, \left(\beta Y_k^{(2)} + \gamma X_k^{(2)} \right)(\delta + \mu)^{-1} + \xi_9 \right\}, \ t_9 > t_8 \tag{4.18}$$

Turning back, one has

$$L_k'(t) \ge kq^{(1)}\left(1 - L_k(t) - Y_k^{(1)} - Z_k^{(1)}\right) - (\alpha + \mu + \gamma) L_k(t), \ t > t_9 \tag{4.19}$$

So, for any given enough small constant $0 < \xi_{10} < \min\{1/10, \xi_9\}$, there exists a $t_{10} > t_9$, such that $G_k(t) \ge x_k^{(2)} + \xi_{10}$ for $t > t_{10}$, where

$$L_k(t) \ge x_k^{(2)} = \max\left\{ x_k^{(1)} - \xi_4, kq^{(1)}\left(1 - Y_k^{(1)} - Z_k^{(1)}\right)\left(kq^{(1)} + (\alpha + \mu + \gamma)\right)^{-1} \right\} \tag{4.20}$$

$$S_k'(t) \ge \alpha x_k^{(2)} - (\mu + \beta) S_k(t), \ t > t_{10} \tag{4.21}$$

So for any given enough small constant $0 < \xi_{11} < \min\{1/11, \xi_{10}\}$, there exists a $t_{11} > t_{10}$, such that $S_k(t) \ge y_k^{(2)} + \xi_{11}$ for $t > t_{10}$, where

$$S_k(t) \ge y_k^{(2)} = \max\left\{ y_k^{(1)} - \xi_5, \alpha x_k^{(2)}(\gamma + \mu) + \xi_{11} \right\}, \ t > t_{11} \tag{4.22}$$

From the third equation of (2.1) implies that

$$G_k'(t) \ge \beta y_k^{(2)} + \gamma x_k^{(2)} - (\mu + \delta) G_k(t) \tag{4.23}$$

So, for any given enough small constant $0 < \xi_{12} < \min\{1/12, \xi_{11}\}$, there exists a $t_{12} > t_{11}$, such that $G_k(t) \ge z_k^{(2)} + \xi_{12}$ for $t > t_{12}$, where

$$G_k(t) \ge z_k^{(2)} = \max\left\{ z_k^{(1)} - \xi_6, \left[\left(\beta y_k^{(2)} + \gamma x_k^{(2)} \right)(\mu + \delta)^{-1} + \xi_{12} \right](\mu + \delta)^{-1} \right\} \tag{4.24}$$

Repeating the above analyses and calculation, we get six sequences $X_k^{(i)}, Y_k^{(i)}, Z_k^{(i)}, x_k^{(i)}, y_k^{(i)}, z_k^{(i)}, \ i = 1, 2, \cdots$. Due to the first three are monotone decreasing sequences and the last three are monotone increasing, there exists a sufficiently large positive integer $L \ge 2$, such that $l \ge L$:

$$X_k^{(l)} = \frac{kQ^{(l-1)}\left(1 - y_k^{(l-1)} - z_k^{(l-1)}\right)}{kQ^{(l-1)} + \mu + \alpha + \gamma} + \xi_{6l-5}, \quad Y_k^{(l)} = \frac{\alpha X_k^{(l)}}{\mu + \beta} + \xi_{6l-4},$$

$$Z_k^{(l)} = \frac{\beta Y_k^{(l)} + \gamma X_k^{(l)}}{\delta + \mu} + \xi_{6l-3}, \quad x_k^{(l)} = \frac{kq^{(l-1)}\left(1 - Y_k^{(l-1)} - Z_k^{(l-1)}\right)}{kq^{(l-1)} + \mu + \alpha + \gamma} + \xi_{6l-2}, \tag{4.25}$$

$$y_k^{(l)} = \frac{\alpha x_k^{(l)}}{\mu + \beta} + \xi_{6l-1}, \quad z_k^{(l)} = \frac{\beta y_k^{(l)} + \gamma x_k^{(l)}}{\delta + \mu} + \xi_{6l}.$$

We can easy get that

$$x_k^{(l)} \leq L_k(t) \leq X_k^{(l)}, \; y_k^{(l)} \leq S_k(t) \leq Y_k^{(l)}, \; z_k^{(l)} \leq G_k(t) \leq Z_k^{(l)}, \; t > t_{6l} \qquad (4.26)$$

Since the sequential limits of (4.26) exist, let $\lim_{l \to \infty} \Delta_k^{(l)} = \Delta_k$, where $\Delta_k^{(l)} \in \left\{ X_k^{(l)}, Y_k^{(l)}, Z_k^{(l)}, x_k^{(l)}, y_k^{(l)}, z_k^{(l)}, Q_k^{(l)}, q_k^{(l)} \right\}$ and $\Delta_k \in \left\{ X_k, Y_k, Z_k, x_k, y_k, z_k, Q_k, q_k \right\}$,

Noting that $0 < \xi_1 < 1/l$, one has $\xi_1 \to 0$ as $l \to \infty$. In the six sequences of (4.26), by taking $l \to \infty$, it follows from (4.26) that

$$X_k = \frac{kQ(1 - y_k - z_k)}{kQ + \mu + \alpha + \gamma}, \quad Y_k = \frac{\alpha X_k}{\mu + \beta}, \quad Z_k = \frac{\beta Y_k + \gamma X_k}{\delta + \mu},$$

$$x_k = \frac{kq(1 - Y_k - Z_k)}{kq + \mu + \alpha + \gamma}, \quad y_k = \frac{\alpha x_k}{\mu + \beta}, \quad z_k = \frac{\beta y_k + \gamma x_k}{\delta + \mu}. \qquad (4.27)$$

$$q^{(j)} = \frac{1}{\langle k \rangle} \sum_{j=1}^{n} P(i) \left(\rho_1 x_i^{(j)} + \rho_2 y_i^{(j)} \right), Q^{(j)} = \frac{1}{\langle k \rangle} \sum_{j=1}^{n} P(i) \left(\rho_1 X_i^{(j)} + \rho_2 Y_i^{(j)} \right), (4.28)$$

Substituting (4.27) and (4.28) into q and Q, respectively, one has

$$1 = \left(\rho_1 + \rho_1 \frac{\alpha}{\mu + \beta} \right) \sum_{i=1}^{n} \frac{iP(i)}{M_i} \left[Q(\delta + \mu)^2 (\beta + \mu)^2 (iq + \mu + \gamma + \alpha) \right.$$

$$\left. - iqQH(\delta + \mu)(\beta + \mu) \right]$$

$$1 = \left(\rho_1 + \rho_1 \frac{\alpha}{\mu + \beta} \right) \sum_{i=1}^{n} \frac{iP(i)}{G_i} \left[q(\delta + \mu)^2 (\beta + \mu)^2 (iQ + \mu + \gamma + \alpha) \right.$$

$$\left. - iqQH(\delta + \mu)(\beta + \mu) \right] .$$

where

$$M_i = \left[(\delta + \mu)^2 (\beta + \mu)^2 (iq + \mu + \gamma + \alpha)(iQ + \mu + \gamma + \alpha) - i^2 qQH^2 \right],$$

$$H = \alpha(\delta + \mu)(\mu + \gamma + \alpha) + \gamma(\beta + \mu)(\mu + \gamma + \alpha) + \alpha\beta(\mu + \gamma + \alpha).$$

By subtracting the above two equations, it arrives at

$$0 = (Q - q)(\delta + \mu)(\beta + \mu) \left(\rho_1 + \rho_1 \frac{\alpha}{\mu + \beta} \right)$$

$$\times \sum_{i=1}^{n} \frac{i^2 P(i)}{G_i} \left[(\mu + \gamma + \alpha)(\delta + \mu)(\mu + \sigma) - H \right] .$$

It is obviously that $q = Q$, so $\frac{1}{\langle k \rangle} \sum_{i=1}^{n} iP(i) \left(\rho_1 (X_i - x_i) + \rho_2 (Y_i - y_i) \right) = 0$, which sees that $X_i = x_i, Y_i = y_i$, for $i = 1, 2, \cdots, n$. From (4.26) and (4.27), it follows that

$$\lim_{t \to \infty} L_k(t) = X_k = x_k, \quad \lim_{t \to \infty} S_k(t) = Y_k = y_k, \quad \lim_{t \to \infty} G_k(t) = Z_k = z_k.$$

Finally, substituting $q = Q$ into (4.26), in view of (3.2) and (4.28), it obtains $L_k = L_k^*$, $S_k = S_k^*$, and $G_k = G_k^*$. The proof is completed.

5. Numerical Simulations

In this section, some sensitivity analyses are presented to illustrate the result of the smoking model (2.1). We consider the system (2.1) on a scale-free network with the degree distribution $P(k) = \xi k^{-r}, 2 < r \leq 3$ and $\sum_{i=1} P(i) = 1$. Consider system (2.1) with $r = 3$ and $n = 1000$.

Parameters used in the simulations list as follows: in **Figure 2**, we choose $\alpha = 0.25$, $b = 0.2$, $\mu = 0.2$, $\gamma = 0.6$, $\beta = 0.5$, $\rho_1 = 0.15$, $\rho_2 = 0.2$, $\delta = 0.15$, thus the threshold value $R_0 = 0.708 < 1$. The figure show that when $R_0 < 1$, L_k and S_k approach to zero, the smoking population will ultimately disappear, which means that smoking will disappear. In **Figure 3**, we choose $\alpha = 0.2$, $b = 0.2$, $\mu = 0.2$, $\gamma = 0.2$, $\beta = 0.5$, $\rho_1 = 0.5$, $\rho_2 = 0.2$, $\delta = 0.1$, thus the threshold value $R_0 = 4.18 > 1$. The figure show that when $R_0 > 1$, L_k and S_k maintain at a positive stationary level, which means that the smoking become endemic "disease".

Figure 4 and **Figure 5** show the dynamic behavior of light problem smoking and heavier problem smoking with different degree when $R_0 < 1$. **Figure 6** and **Figure 7** show the dynamic behavior of light problem smoking and heavier problem smoking with different degree when $R_0 > 1$. We find that the larger degree leads to larger value of the smoking level.

6. Conclusion

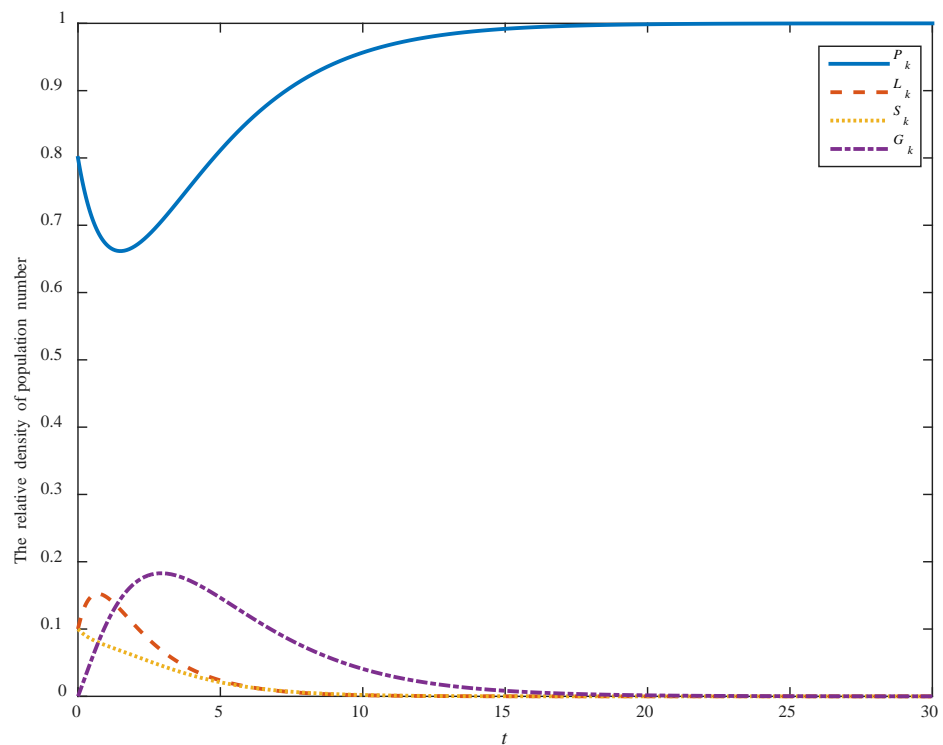In this paper, we propose a *PLSGP* giving up smoking model on scale-free network.

Figure 2. The relative density of population number when $R_0 < 1$.

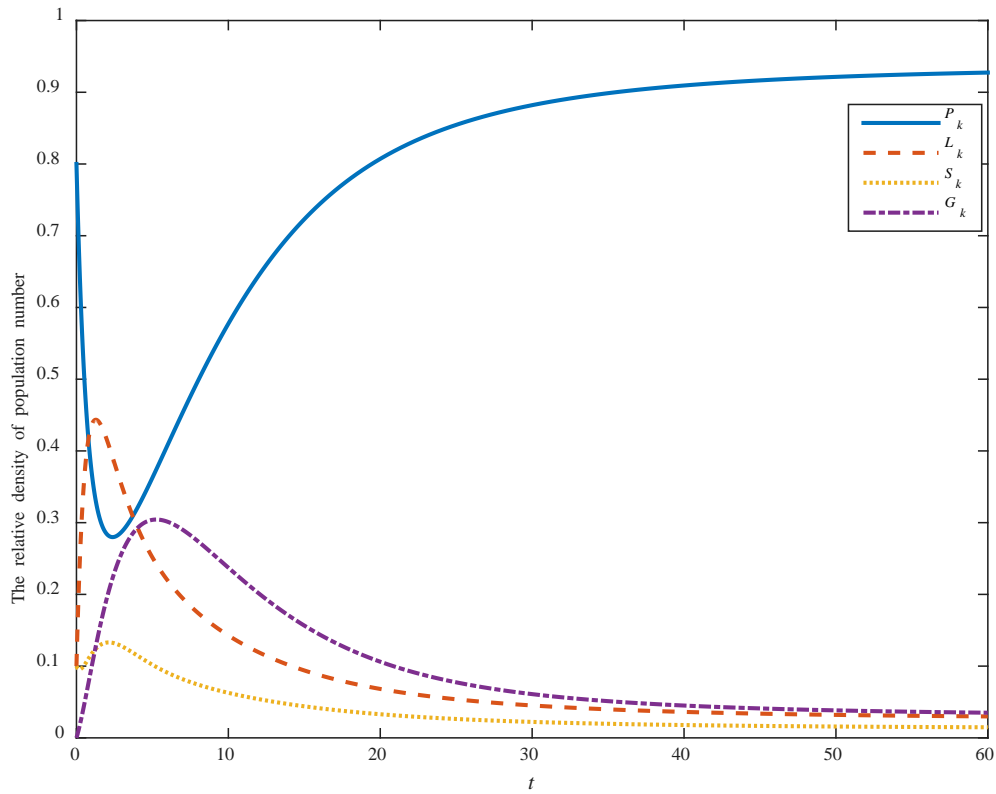**Figure 3.** The relative density of population number when $R_0 > 1$.
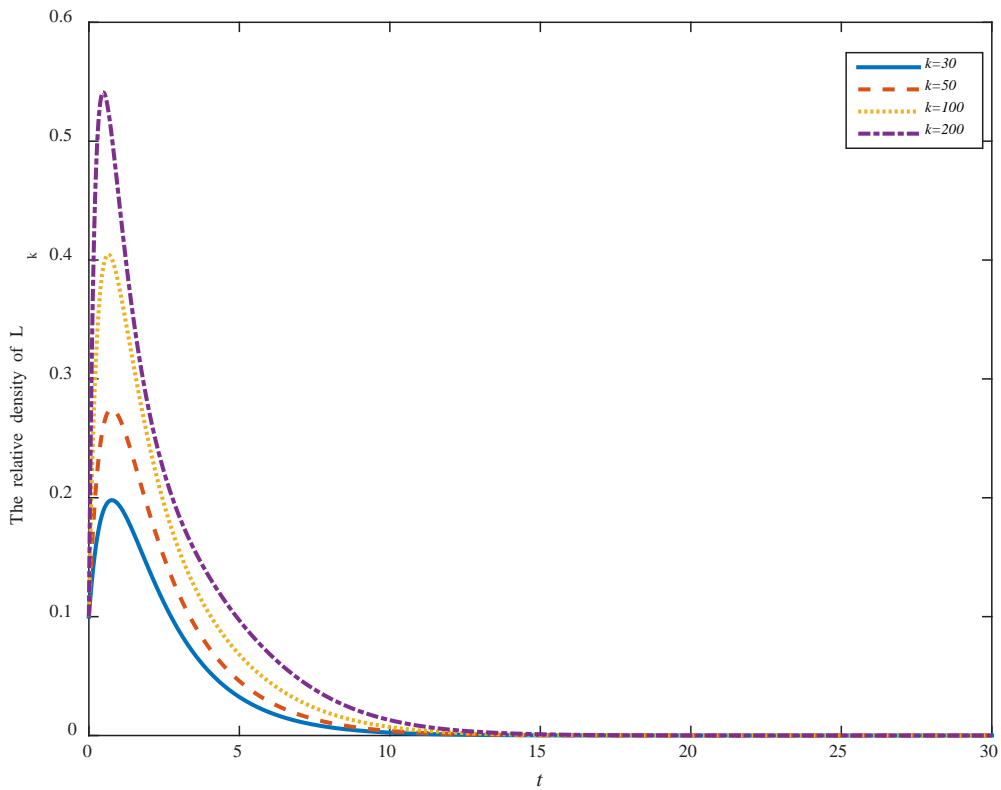

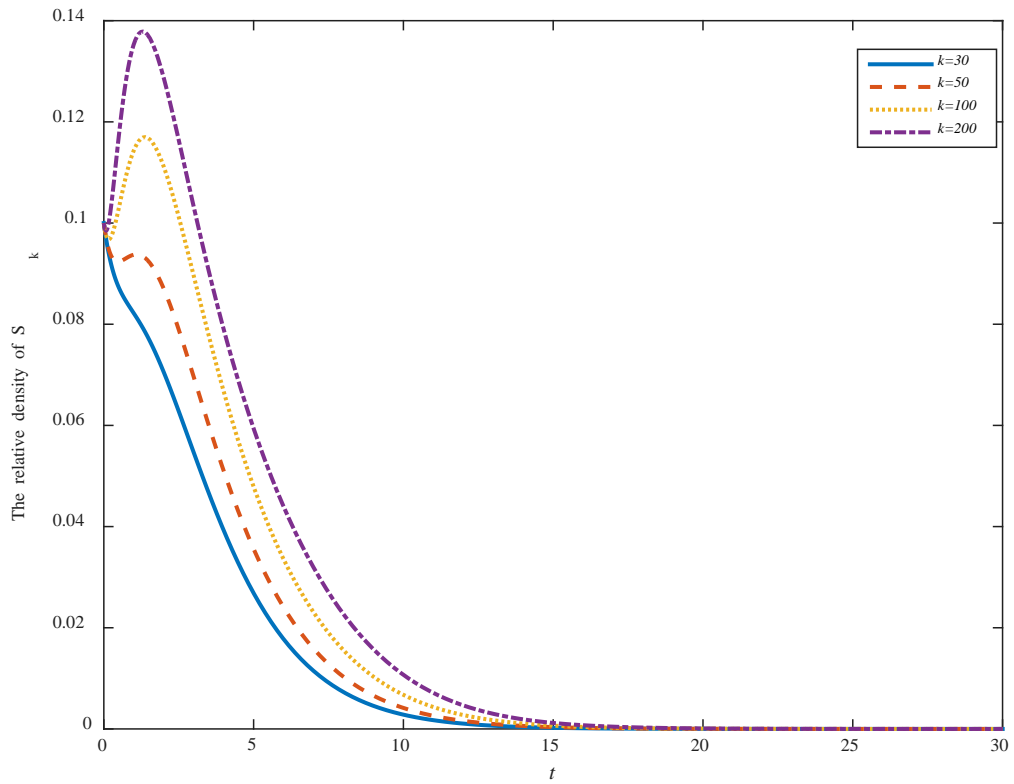**Figure 4.** Dynamic behavior of light smoker with different degree when $R_0 < 1$.

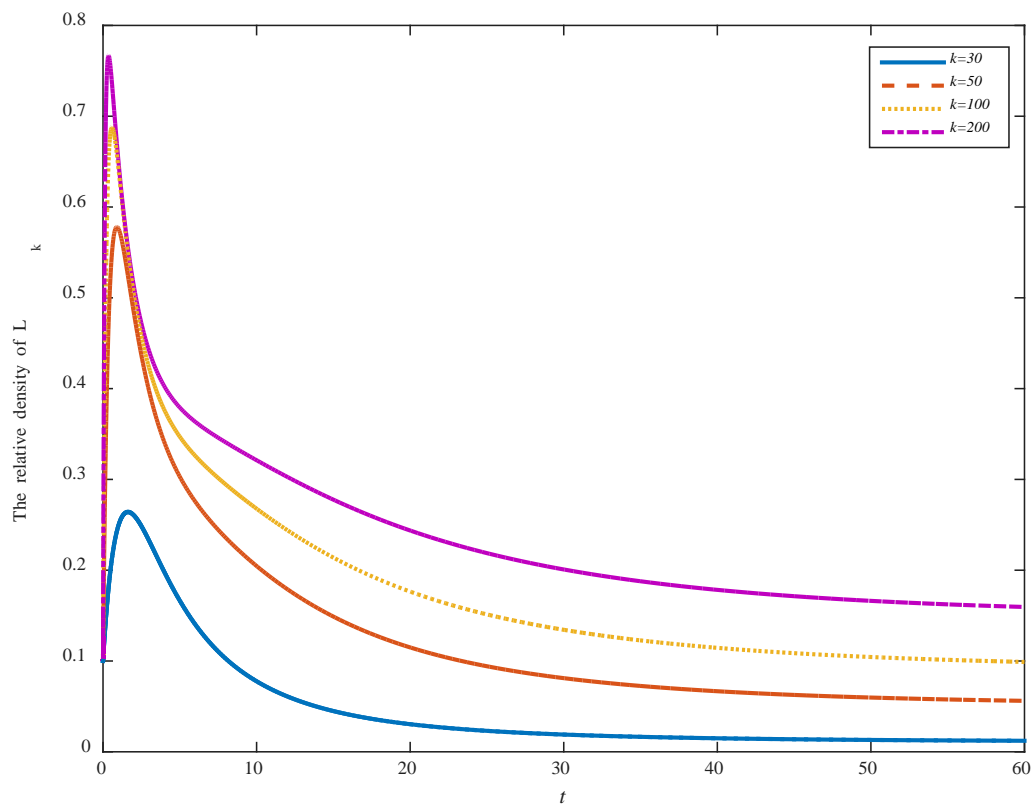**Figure 5.** Dynamic behavior of light smoker with different degree when $R_0 > 1$.

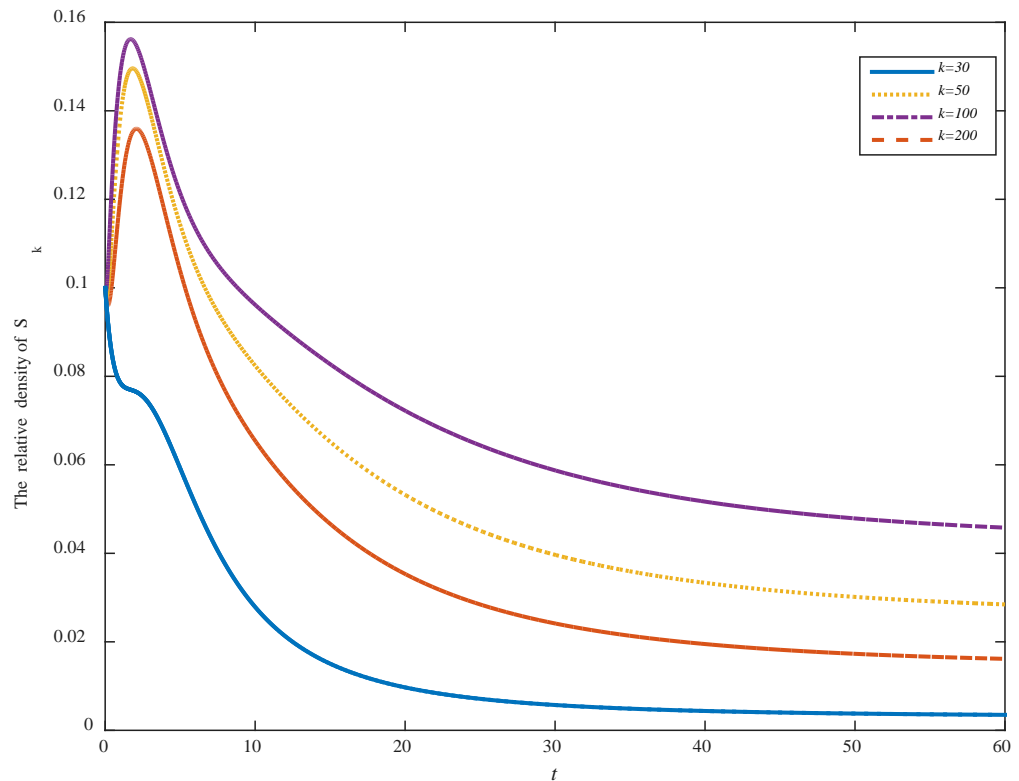**Figure 6.** Dynamic behavior of light smoker with different degree when $R_0 > 1$.

Figure 7. Dynamic behavior of light smoker with different degree when $R_0 > 1$.

We divide the smoker into two groups, light smoker and persistent smoker, considering individual's birth and death rates. Through the mathematical calculation, we obtain the basic reproduction number and equilibriums. Using the comparison theorem and the iteration principle, we analyze the stability of the smoking free equilibrium, and also give the persistence and global attractivity of the smoking. If $R_0 < 1$, the smoking-free equilibrium of the model is globally stable and smoking will disappear. If $R_0 > 1$, the smoking-present equilibrium is global attractivity and maintaining a positive constant. Furthermore, the dynamic behavior has been analyzed in our model. The study may give us valuable guiding in effectively controlling the behavior of smoking.

Acknowledgements

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