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Collocation Technique for Numerical Solution of Integral Equations with Certain Orthogonal Basis Function in Interval [0, 1]

O. L. Babasola[1], I. Irakoze[2]

[1]University of Ilorin, Ilorin, Nigeria
[2]Université du Burundi, Bujumbura, Burundi
Email: babasolaoluwatosin@yahoo.com, irene.irakoze@aims-cameroon.org

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Abstract

This paper is concerned with the construction of a class of polynomial orthogonal with respect to the weight function $w(x)=1-x^2$ over the interval $[0,1]$. The zeros of these polynomials were employed as points of collocation for the orthogonal collocation technique in the solution of integral equations. The method is illustrated with some numerical examples and the results obtained show that the method is effective.

Subject Areas

Integral Equation, Numerical Mathematics

Keywords

Collocation Method, Integral Equations, Orthogonal Polynomials

1. Introduction

Many problems arising in mathematics and in particular, applied mathematics can be formulated into two distinct but connected ways: differential equations and integral equations. Over the years, much emphasis has been placed on the solution of differential equations (ordinary differential equations and partial differential equations) more than the solution of integral equations because one may easily accept that the solution of integral equations are more tasking to obtain compared to the differential equations.

According to [1], Integral equations can be used as the mathematical model in which many physical problems are modelled. The numerical solution of such integral equations has been studied by various authors and in recent years, great works have been focused on the development of more advanced and efficient methods for integral equations as they have several applications.

Integral equations can be applied in the radioactive transfer and oscillation problems such as oscillation of string, axle and membrane [1]. Recently, the applications of integral equations have become prominent. However, mathematicians have so far devoted their attention mainly into two peculiarly types of integral equation: the linear equations of the first and second kinds.

An Integral equation is an equation in which the unknown function appears under one or more integral sign [2]. The standard integral equations of the form

$$\int_{\alpha}^{\beta} K(x,t) g(t) \mathrm{d}t = \varphi(x) \tag{1.1}$$

and

$$\lambda \int_{\alpha}^{\beta} K(x,t) g(t) \mathrm{d}t = g(x) + \varphi(x) \tag{1.2}$$

are known as the *linear Fredholm integral equations* of the first and second kinds respectively. In each case, $g(x)$ is the unknown function and it occurs to the first degree while the kernel $K(x,t)$ and $\varphi(x)$ are the known functions. If the constant β in (1.1) and (1.2) is replaced by x (the variable of integration), then the equations become Volterra integral equations. Thus, the integral equations of the form

$$\int_{a}^{x} K(x,t) g(t) \mathrm{d}t = \varphi(x) \tag{1.3}$$

and

$$\lambda \int_{a}^{x} K(x,t) g(t) \mathrm{d}t = g(x) + \varphi(x) \tag{1.4}$$

are called the *Volterra integral equations* of the first and second kinds.

If $\varphi(x) = 0$ in (1.3) and (1.4), then we say the equation is homogeneous, otherwise nonhomogeneous.

2. Literature Review

Collocation method involves evaluating of approximate solution in a suitable set of functions called basis function or trial solution. This method for obtaining the approximate solution to an integral equation has its origin in the 1930s when [3] consider an integral equation using the line collocation procedure. [4] used orthogonal collocation to solve a boundary value problems where he developed the set of orthogonal polynomials using both the boundary conditions and the roots of the polynomials as the collocation points.

Recently, many researchers have developed the numerical method to obtain the solution to an integral equation using several well known polynomials and in particular, orthogonal polynomials.

[5] obtained the numerical solution of the Volterra integral equation of second kind using the Gelerkin method and he used the Hermite polynomials as

the basis function. Similarly, [6] explore the solution of both the linear and nonlinear Volterra integral equation using the Gelerkin method but they used the Hermite and Chebyshev polynomials as the basis function. [7] also considered the first kind boundary integral equation and obtained its numerical solution by the means of attenuation factors. [8] did a work by using an extrapolation techniques and collocation method for some integral equations. [9] obtained a numerical solution of the integral equation of second kind and he compared the error with that of analytics solution. [10] uses the numerical expansion methods to solve Fredholm-Volterra linear integral equation by interpolation and quadrature rules while [11] formulated and use the collocation technique to obtain the numerical solution of the fredholm second kind integral equation.

However in this paper, the orthogonal collocation techniques will be use to obtain the numerical solution of linear integral equation and the zeros of a constructed orthogonal polynomials will be used as the points of collocation. Thereafter, the result obtained will be compare with the analytic solution to show that the method is effective and accurate.

3. Construction of Orthogonal Polynomials

Let $\varphi_n(x)$ be a polynomial of exact degree *n*, then φ_n is said to be orthogonal with respect to a weight function $w(x)$ within the interval $[\alpha,\beta]\in\mathbb{R}$ with $\alpha<\beta$ if

$$\int_\alpha^\beta \phi_m(x)\phi_n(x)w(x)\mathrm{d}x=\delta_{mn}, \tag{3.1}$$

with δ_{mn} is the Kronecker symbol defined by:

$$\delta_{mn}=\begin{cases}1 & \text{if } m=n,\\ 0 & \text{if } m\neq n.\end{cases} \tag{3.2}$$

The weight function $w(x)$ should be continuous and also positive on $[\alpha,\beta]$ such that the moments

$$\int_\alpha^\beta w(x)x^n\mathrm{d}x,\quad n\in\mathbb{N}$$

exists and finite. Then

$$\langle\phi_m,\phi_n\rangle=\int_\alpha^\beta \phi_m(x)\phi_n(x)w(x)\mathrm{d}x \tag{3.3}$$

defines the inner product of the polynomial ϕ_m and ϕ_n.

We shall adopt the weight function $w(x)=1-x^2$ in the interval $[0,1]$. Hence, we use the property below to construct our basis function.

$$\begin{gathered}\phi_n(x)=\sum_{r=0}^{n}C_r^{(n)}x^r\\ \langle\phi_m,\phi_n\rangle=0\\ \phi_n(1)=1\end{gathered} \tag{3.4}$$

For $\phi_0(x)$, we have

$$\phi_0(x)=\sum_{r=0}^{0}C_r^{(0)}x^r=C_0^{(0)}$$

$$\phi_0(1)=C_0^{(0)}=1$$

$$\phi_0(x)=1$$

For $\phi_1(x)$, we have

$$\phi_1(x)=\sum_{r=0}^{1}C_r^{(1)}x^r=C_0^{(1)}+C_1^{(1)}x.$$

When $x=1$,

$$\phi_1(1)=C_0^{(1)}+C_1^{(1)}=1 \tag{3.5}$$

$$\begin{aligned}\langle\phi_0,\phi_1\rangle&=\int_0^1(1-x^2)\phi_0(x)\phi_1(x)\mathrm{d}x=0\\&=\int_0^1(1-x^2)\left(C_0^{(1)}+C_1^{(1)}x\right)\mathrm{d}x=0\\&=\frac{2}{3}C_0^{(1)}+\frac{1}{4}C_1^{(1)}=0\end{aligned} \tag{3.6}$$

solving (3.5) and (3.6), we obtain

$$C_0^{(1)}=-\frac{3}{5},\quad C_1^{(1)}=\frac{8}{5}$$

Hence,

$$\phi_1(x)=\frac{1}{5}(8x-3) \tag{3.7}$$

Similarly,

For $\phi_2(x)$, we have

$$\phi_2(x)=\sum_{r=0}^{2}C_r^{(2)}x^r=C_0^{(2)}+C_1^{(2)}x+C_2^{(2)}x^2.$$

For $x=1$, we obtain

$$\phi_2(1)=C_0^{(2)}+C_1^{(2)}+C_2^{(2)}=1 \tag{3.8}$$

and

$$\langle\phi_0,\phi_2\rangle=\int_0^1(1-x^2)\phi_0(x)\phi_2(x)\mathrm{d}x=0$$

$$\int_0^1(1-x^2)\left(C_0^{(2)}+C_1^{(2)}x+C_2^{(2)}x^2\right)\mathrm{d}x=0$$

$$\frac{2}{3}C_0^{(2)}+\frac{1}{4}C_1^{(2)}+\frac{2}{15}C_2^{(2)}=0 \tag{3.9}$$

$$\langle\phi_1,\phi_2\rangle=\int_0^1(1-x^2)\phi_1(x)\phi_2(x)\mathrm{d}x=0$$

$$\frac{1}{5}\int_0^1(1-x^2)(8x-3)\left(C_0^{(2)}+C_1^{(2)}x+C_2^{(2)}x^2\right)\mathrm{d}x=0$$

$$\frac{19}{300}C_1^{(2)}+\frac{4}{75}C_2^{(2)}=0 \tag{3.10}$$

solving (3.8), (3.9) and (3.10), we obtain

$$C_0^{(2)}=\frac{11}{26},\quad C_1^{(2)}=-\frac{80}{26},\quad C_2^{(2)}=\frac{95}{26}.$$

Hence,

$$\phi_2(x)=\frac{1}{26}\left(95x^2-80x+11\right). \tag{3.11}$$

Following the same procedure,

$$\phi_3(x)=\frac{1}{46}\left(448x^3-595x^2+208x-15\right)$$

$$\phi_4(x)=\frac{1}{743}\left(21042x^4-38304x^3+22232x^2-4424x+197\right)$$

$$\phi_5(x)=\frac{1}{4043}\left(352176x^5-815430x^4+6669x^3-229320x^2+29840x-903\right)$$

$$\phi_6(x)=\frac{1}{22180}\left(6180603x^6-17379648x^5+18440235x^4-9144960x^3+2116935x^2-195264x+4279\right)$$

4. Numerical Examples

We consider here three problems for illustration of the proceeding discourse. For this purpose, we seek approximant of degree 3, 4 and 5 (**Tables 1-5**).

Example 1 *Consider the integral equation*

$$\int_0^1 xe^{xs}y(s)\mathrm{d}s=e^x-y(x),\quad \text{whose analytic solution is } y(x)=1.$$

Solving with $N=3$ as the degree of approximation, we have

$$\int_0^1 xe^{xs}\sum_{r=0}^{3}a_r\phi_r(s)\mathrm{d}s=e^x-\sum_{r=0}^{3}a_r\phi_r(x)$$

$$\int_0^1 xe^{xs}\left\{a_0+a_1\left(\frac{8}{5}s-\frac{3}{5}\right)+a_2\left(\frac{95}{26}s^2-\frac{80}{26}s+\frac{11}{26}\right)+a_3\left(\frac{224}{23}s^3-\frac{595}{46}s^2+\frac{104}{23}s-\frac{15}{46}\right)\right\}\mathrm{d}s$$
$$=e^x-\left\{a_0+a_1\left(\frac{8}{5}x-\frac{3}{5}\right)+a_2\left(\frac{95}{26}x^2-\frac{80}{26}x+\frac{11}{26}\right)+a_3\left(\frac{224}{23}x^3-\frac{595}{46}x^2+\frac{104}{23}x-\frac{15}{46}\right)\right\}$$

This gives

$$\frac{1}{2990x^3}\{29120x^6a_3+10925x^5a_2-38675x^5a_3+2990e^xx_3\left(a_0+a_1+a_2+a_3\right)$$
$$+4784x^4a_1-9200x^4a_2+13520x^4a_3-4784e^xx^2a_1-12650e^xx^2a_2$$
$$-23530e^xx^2a_3+21850e^xxa_2+97370e^xxa_3+4784x^2a_1-9200x^2a_2$$
$$+13520x^2a_3-174720e^xa_3-21850xa_2+77350xa_3+174720a_3\}=e^x$$

As there are four unknown coefficients in this equation, we shall collocate at the zeros of the fourth degree polynomial earlier constructed, This results into the linear system of equations:

$$1.063634023a_0 - 0.4880435909a_1 + 0.2538803066a_2 - 0.0901090390a_3 = 1.063634023$$

$$1.338950765a_0 - 0.0520191234a_1 - 0.1239293627a_2 + 0.1566142614a_3 = 1.338950765$$

$$1.821140964a_0 + 0.5886120730a_1 + 0.0023209561a_2 - 0.1022283745a_3 = 1.821140964$$

$$2.380518708a_0 + 1.221493409a_1 + 0.7117809561a_2 + 0.3397428175a_3 = 2.380518708$$

Solving the equations above, we have

$$a_0 = 0.9999999999,\ a_1 = a_2 = a_3 = 0$$

Thus, the approximate solution,

$$y_3(x) = 0.9999999999$$

Next, we seek an approximant of degree $N = 4$ and for this, we shall engage the zeros of the fifth degree orthogonal polynomial constructed. This leads to the equations

$$0.9 - 0.4574733239a_0 + 0.77337959868a_1 - 0.1951129745a_2 + 0.2172348194a_3 - 0.02283297131a_4 = 0.7695433192$$

$$0.9 - 0.2916717999a_0 + 0.5416738531a_1 + 0.3668860626a_2 - 0.07218775521a_3 + 0.1350402311a_4 = 0.5208410331$$

$$0.9 - 0.0477684829a_0 + 0.2002092093a_1 + 0.366886062a_2 + 0.11103526617a_3 - 0.053569893a_4 = 0.1549860577$$

$$0.90.2065322412a_0 - 0.1558118044a_1 + 0.098736803a_2 + 0.252593850a_3 + 0.194777949a_4 = -0.226465028$$

$$0.90.4057861632a_0 - 0.4347672951a_1 - 0.441575320a_2 - 0.284119350a_3 - 0.10098142a_4 = -0.5253459117$$

Solving the equations,

$$a_0 = 1.000000,\ a_1 = a_2 = a_3 = a_4 = 0$$

Therefore

$$y_4(x) = 1$$

Similarly, for a fifth degree approximant, we use the zeros of the sixth degree orthogonal polynomial as our point of collocation to also get $y_5(x) = 1$ as the desired approximation.

Example 2 *Consider*

$\int_0^1 (x+s)y(s)\mathrm{d}s = y(x) - \frac{3}{2}x + \frac{5}{6}$, whose analytic solution is $y(x) = -1 + x$.

For a third degree approximant of $y(x)$, we have

$$\int_0^1 (x+s)\left\{\left(a_0-\frac{3}{5}a_1+\frac{11}{26}a_2-\frac{15}{46}a_3\right)+\left(\frac{8}{5}a_1-\frac{80}{26}a_2+\frac{208}{46}a_3\right)s\right.$$
$$\left.+\left(\frac{95}{26}a_2-\frac{595}{46}a_3\right)s^2+\frac{224}{23}a_3s^3\right\}\mathrm{d}s$$
$$=\left\{\left(a_0-\frac{3}{5}a_1+\frac{11}{26}a_2-\frac{15}{46}a_3\right)+\left(\frac{8}{5}a_1-\frac{80}{26}a_2+\frac{208}{46}a_3\right)x\right.$$
$$\left.+\left(\frac{95}{26}a_2-\frac{595}{46}a_3\right)x^2+\frac{224}{23}a_3x^3-\frac{3}{2}x+\frac{5}{6}\right\}$$

This gives

$$\frac{1061}{2760}a_3-\frac{308}{69}xa_3-\frac{101}{312}a_2-\frac{124}{39}xa_2+\frac{5}{6}a_1-\frac{7}{5}xa_1-\frac{1}{2}a_0+xa_0$$
$$-\frac{95}{26}x^2a_2+\frac{595}{46}x^2a_3-\frac{224}{23}a_3x^3+\frac{3}{2}x-\frac{5}{6}=0$$

collocating this we obtain the system of equations

$$-0.4383086312a_0+0.7469654169a_1-0.1414769314a_2+0.1559852267a_3$$
$$=0.7407962800$$
$$-0.2081137040a_0+0.4246925189a_1+0.2930318285a_2-0.0586727298a_3$$
$$=0.3955038893$$
$$0.0994632079a_0-0.0059151577a_1+0.2692355947a_2+0.258737951a_3$$
$$=-0.0658614785$$
$$0.3673184086a_0-0.3809124389a_1-0.314663883a_2-0.111138066a_3$$
$$=-0.4676442797$$

We solve these to obtain

$$a_0=-0.6249999992,\ a_1=0.6250000003,$$
$$a_2=1.211249628\times10^{-10},\ a_3=-2.268186446\times10^{-10}$$

and hence

$$y_3(x)=-0.9999999992+0.9999999986x+3.376421840\times10^{-9}x^2$$
$$-2.209016365\times10^{-9}x^3.$$

For a quartic approximation of $y(x)$ we obtain by using the zeros of the fifth degree polynomial and we have

$$y_4(x)=-1.000000001+1.000000002x-2.101878150\times10^{-8}x^2$$
$$+3.718048589\times10^{-8}x^3-1.995546318\times10^{-8}x^4$$

Similarly, for an approximation of degree 5 we obtained

$$y_5(x)=-0.9999999981+0.9999999995x+2.545793296\times10^{-8}x^2$$
$$-7.244844114\times10^{-8}x^3+9.516056846\times10^{-8}x^4$$
$$-4.631806729\times10^{-8}x^5$$

Example 3 *Consider the integral equation*

$$y(x)=1+\int_0^1 x^2s^3y(s)\mathrm{d}s,\quad \text{whose analytic solution is } y(x)=1+\frac{3}{10}x^2.$$

By seeking a cubic approximation to $y(x)$ we have

$$1+\int_0^1 x^2 s^3 \sum_{r=0}^{3} a_r \phi_r(s)\,ds = \sum_{r=0}^{3} a_r \phi_r(x)$$

$$1+\int_0^1 x^2 s^3 \left\{\left(a_0 - \frac{3}{5}a_1 + \frac{11}{26}a_2 - \frac{15}{46}a_3\right) + \left(\frac{8}{5}a_1 - \frac{80}{26}a_2 + \frac{208}{46}a_3\right)s + \left(\frac{95}{26}a_2 - \frac{595}{46}a_3\right)s^2 + \frac{224}{23}a_3 s^3\right\}ds$$
$$= \left(a_0 - \frac{3}{5}a_1 + \frac{11}{26}a_2 - \frac{15}{46}a_3\right) + \left(\frac{8}{5}a_1 - \frac{80}{26}a_2 + \frac{208}{46}a_3\right)x + \left(\frac{95}{26}a_2 - \frac{595}{46}a_3\right)x^2 + \frac{224}{23}a_3 x^3$$

That is

$$\frac{35861}{2760}x^2 a_3 - \frac{1109}{312}x^2 a_2 + \frac{17}{100}x^2 a_1 + \frac{1}{4}x^2 a_0 - a_0 + \frac{3}{5}a_1 - \frac{11}{26}a_2 + \frac{15}{4}a_3 - \frac{8}{5}x a_1 + \frac{40}{13}x a_2 - \frac{104}{23}x a_3 - \frac{224}{23}a_3 x^3 = -1$$

collocating this at the four points, we have the linear system

$$-0.9990485438a_0 + 0.5019408002a_1 - 0.2467850828a_2 + 0.0942975884a_3 = -1$$
$$-0.9787005976a_0 + 0.1474655200a_1 + 0.1722009453a_2 - 0.1289571472a_3 = -1$$
$$-0.9101609656a_0 - 0.2980505892a_1 + 0.1440984716a_2 + 0.186615570a_3 = -1$$
$$-0.8119396945a_0 - 0.6598284463a_1 - 0.4282366757a_2 - 0.175869988a_3 = -1$$

We solve this to have

$$a_0 = 1.060000000,\ a_1 = 0.1578947373,$$
$$a_2 = 0.08210526282,\ a_3 = 1.260103578\times 10^{-10}$$

so that

$$y_3(x) = 0.9999999996 + 2.4\times 10^{-9}x + 0.2999999972x^2 + 1.227231311\times 10^{-9}x^3$$

is our desired approximant of $y(x)$

Similarly, for the quartic and quintic approximant of $y(x)$, we obtained respectively

$$y_4(x) = 1.0000 + 9.0\times 10^{-10}x + 0.3000000052x^2 - 9.297606497\times 10^{-9}x^3 + 5.11785260\times 10^{-8}x^4$$

$$y_5(x) = 1.000000000 - 6.8\times 10^{-9}x + 0.3000000373x^2 - 8.393008256\times 10^{-8}x^3 + 8.191152506\times 10^{-8}x^4 - 2.880832986\times 10^{-8}x^5$$

5. Conclusion

A method for the numerical solution of integral equations has been presented. The method employs the idea of collocation and it uses a class of orthogonal polynomials with respect to the weight function $w(x)=1-x^2$ over the interval [0, 1]. The zeros or roots of the orthogonal polynomials were chosen as collocation

Table 1. Numerical results for 1.

x	Exact Solution	Approximate Solution $N = 3$	Approximate Solution $N = 4$	Approximate Solution $N = 5$
0.01	1.000000	0.999999999	1.000000	1.000000
0.02	1.000000	0.999999999	1.000000	1.000000
0.03	1.000000	0.999999999	1.000000	1.000000
0.04	1.000000	0.999999999	1.000000	1.000000
0.05	1.000000	0.999999999	1.000000	1.000000
0.06	1.000000	0.999999999	1.000000	1.000000
0.07	1.000000	0.999999999	1.000000	1.000000
0.08	1.000000	0.999999999	1.000000	1.000000
0.09	1.000000	0.999999999	1.000000	1.000000
0.10	1.000000	0.999999999	1.000000	1.000000

Table 2. Error results for 1.

x	Error, $N = 3$	Error, $N = 4$	Error, $N = 5$
0.01	1.00e−09	0.00e+00	0.00e+00
0.02	1.00e−09	0.00e+00	0.00e+00
0.03	1.00e−09	0.00e+00	0.00e+00
0.04	1.00e−09	0.00e+00	0.00e+00
0.05	1.00e−09	0.00e+00	0.00e+00
0.06	1.00e−09	0.00e+00	0.00e+00
0.07	1.00e−09	0.00e+00	0.00e+00
0.08	1.00e−09	0.00e+00	0.00e+00
0.09	1.00e−09	0.00e+00	0.00e+00
0.10	1.00e−09	0.00e+00	0.00e+00

Table 3. Numerical results for 2.

x	Exact Solution	Approximate Solution $N = 3$	Approximate Solution $N = 4$	Approximate Solution $N = 5$
0.01	−0.99000000	−0.9899999992	−0.9900000010	−0.99000000
0.02	−0.98000000	−0.9799999992	−0.9800000010	−0.98000000
0.03	−0.97000000	−0.9699999992	−0.9700000010	−0.97000000
0.04	−0.96000000	−0.9599999993	−0.9600000010	−0.96000000
0.05	−0.95000000	−0.9499999993	−0.9500000010	−0.95000000
0.06	−0.94000000	−0.9399999993	−0.9400000010	−0.94000000
0.07	−0.93000000	−0.9299999993	−0.9300000010	−0.93000000
0.08	−0.92000000	−0.9199999993	−0.9200000010	−0.92000000
0.09	−0.91000000	−0.9099999993	−0.9100000010	−0.91000000
0.10	−0.90000000	−0.8999999993	−0.9000000010	−0.90000000

Table 4. Error results for 2. 7.00e−10 1.00e−10 0.00e+00

x	Error, $N = 3$	Error, $N = 4$	Error, $N = 5$
0.01	8.00e−10	1.00e−10	0.00e+00
0.02	8.00e−10	1.00e−10	0.00e+00
0.03	8.00e−10	1.00e−10	0.00e+00
0.04	7.00e−10	1.00e−10	0.00e+00
0.05	7.00e−10	1.00e−10	0.00e+00
0.06	7.00e−10	1.00e−10	0.00e+00
0.07	7.00e−10	1.00e−10	0.00e+00
0.08	7.00e−10	1.00e−10	0.00e+00
0.09	7.00e−10	1.00e−10	0.00e+00
0.10	7.00e−10	1.00e−10	0.00e+00

Table 5. Numerical results for 3.

x	Exact	Approximate	Approximate	Approximate
	Solution	Solution,	Solution,	Solution,
		$N = 3$	$N = 4$	$N = 5$
0.01	1.0000300	1.0000300	1.0000300	1.0000300
0.02	1.0001200	1.0001200	1.0001200	1.0001200
0.03	1.0002700	1.0002700	1.0002700	1.0002700
0.04	1.0004800	1.0004800	1.0004800	1.0004800
0.05	1.0007500	1.0007500	1.0007500	1.0007500
0.06	1.0010800	1.0010800	1.0010800	1.0010800
0.07	1.0014700	1.0014700	1.0014700	1.0014700
0.08	1.0019200	1.0019200	1.0019200	1.0019200
0.09	1.0024300	1.0024300	1.0024300	1.0024300
0.10	1.0030000	1.0030000	1.0030000	1.0030000

points for an orthogonal collocation technique. Three numerical examples were considered to illustrate the proposed method. However, the numerical evidences show that method is effective and gives better approximation solution compared to the one in the literatures.

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