



Lorentz Transformation Properties of Currents for the Particle-Antiparticle Pair Wave Functions

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Abstract

The Lorentz transformation properties of charge current four vector for Dirac spinor particles are examined once more especially for the zitterbewegung terms which are integral parts of this theory.

Subject Areas

Quantum Mechanics

Keywords

Dirac Equation, Zitterbewegung, Lorentz Transformation

1. Introduction

Four vectors like electric or charge particle currents transform under Lorentz boost obeying certain transformation laws in classical physics [1]. It is expected that the quantum mechanical charge currents for elementary particles (and/or antiparticles) as derived from the probability currents [2], by multiplying the particles' charge with it, follow these transformation laws if the correspondence principle [3] relating quantum and classical physics is to remain valid. As is stated in reference [2] the Dirac equation describing elementary particles like the electron give rise to terms in the probability current four vector which have rapid oscillation or "zitterbewegung" in cases where states are formed by combining particle and antiparticle wave functions. In this present letter we show that these zitterbewegung terms do not follow the Lorentz transformation law. It is also demonstrated that the wave function for a scalar particle (along with its antiparticle) which follows the Klein Gordon equation [4] gives rise to no such zitterbewegung terms at least in the zeroth component of charge currents which vi-

ulate its Lorentz transformation properties.

2. Lorentz Transformation Properties of Classical and Quantum Mechanical Currents

Any classical four vector J^μ transforms from a reference frame S to another S' which is moving with respect to S with a constant velocity v along the x direction according to (we are writing the zeroth that is the $\mu=0$ component only)

$$J^{0'} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \left(J^0 - \frac{v}{c} J^1 \right) = J^0 \cosh \omega - J^1 \sinh \omega$$

as given for example by Jackson [1] (see p. 526 of this reference). If in this frame the value of J^1 (the x component of current) is zero then the ratio of charge density $c\rho' = J^{0'}$ in S' to its value $c\rho$ in S is given by

$$\frac{\rho'}{\rho} = \cosh \omega \quad (1)$$

On the other hand the Dirac particle-antiparticle pair, like the electron and positron can be represented in frame S in which it is assumed to be at rest by the wave function as given for example by Bjorken and Drell [2] (see p. 10 of this book) to be

$$\psi(x, t) = A e^{\frac{-imc^2}{\hbar}t} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + B e^{\frac{imc^2}{\hbar}t} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad (2)$$

Here the complex coefficients A and B represent the proportion in which the particle and antiparticle components are present in the combined state. There should not be any objection to forming such combined states with the help of Dirac spinors where both particle and antiparticle wave functions are involved as such states are included in forming wave packets as given for example by Equation (3.30) of reference [2]. Also we need not normalize the wave function to study the Lorentz transformation properties of the charge current four vector (that is the electronic charge e multiplied by the probability current four vector). This quantity can be expressed as

$$J^\mu(x, t) = ce\psi^\dagger(x, t)\gamma^0\gamma^\mu\psi(x, t) \quad (3)$$

following the prescriptions of pages 23 and 9 of reference [2]. Here $\psi^\dagger(x, t)$ is the complex conjugate of the transpose of $\psi(x, t)$. The zeroth component of this vector $J^0(x, t)$ is the charge density $c\rho(x, t)$ and can be evaluated in both S and S' frames to see if they follow Equation (1) above provided $\psi(x, t)$ in the frame S is given by Equation (2) implying that particle and antiparticle are both at rest in this frame. Thus in the S frame

$$c\rho(x, t) = J^0(x, t) = ce\psi^\dagger(x, t)\psi(x, t) \quad (4)$$

into which substitution of the value of $\psi(x, t)$ from Equation (2) yields

$$c\rho(x, t) = ce(|A|^2 + |B|^2) \quad (5)$$

Before applying Equation (4) to the frame S' we must transform the spinor $\psi(x, t)$ to $\psi'(x', t')$ using the matrix operator which appears in Equation (3.5) of reference [2] that is

$$\psi'(x', t') = \begin{bmatrix} \cosh \omega/2 & 0 & 0 & -\sinh \omega/2 \\ 0 & \cosh \omega/2 & -\sinh \omega/2 & 0 \\ 0 & -\sinh \omega/2 & \cosh \omega/2 & 0 \\ -\sinh \omega/2 & 0 & 0 & \cosh \omega/2 \end{bmatrix} \begin{bmatrix} Ae^{-\frac{imc^2}{\hbar}t} \\ 0 \\ 0 \\ Be^{\frac{imc^2}{\hbar}t} \end{bmatrix} \quad (6)$$

Thus from Equations (3) and (6) we get

$$\begin{aligned} c\rho'(x', t') &= ec\psi^{*'}(x', t')\psi'(x', t') \\ &= ce(|A|^2 + |B|^2) \cosh \omega - ce \left(A^* B e^{\frac{2imc^2}{\hbar}t} + AB^* e^{-\frac{2imc^2}{\hbar}t} \right) \sinh \omega \end{aligned} \quad (7)$$

where A^* is the complex conjugate of A . Thus the second term in Equation (7) is the zitterbewegung term and if the equation is divided by Equation (5) we get

$$\frac{\rho'}{\rho} = \cosh \omega - \frac{A^* B e^{\frac{2imc^2}{\hbar}t} + AB^* e^{-\frac{2imc^2}{\hbar}t}}{|A|^2 + |B|^2} \sinh \omega \quad (8)$$

Comparison of Equations (1) and (8) shows that classical Lorentz covariance is violated by the zitterbewegung terms which are essential features of the Dirac description of a particle even though these may exist in a small proportion (see p 39 of reference [2]). Furthermore charge density is an observable in quantum mechanics and so this will lead to some consequences as far as experimental results are concerned.

3. Scalar Particle Charge Density and Its Lorentz Transformation

The quantum mechanical wave function $\phi(x, t)$ of a scalar particle follows the Klein-Gordon equation and can be written following the guidelines of reference [4] as

$$\frac{1}{c^2} \frac{\partial^2 \phi(x, t)}{\partial t^2} - \nabla^2 \phi(x, t) + \frac{c^2}{\hbar^2} m^2 \phi(x, t) = 0 \quad (9)$$

where we have retained the velocity c of light explicitly. The charge density $c\rho(x, t)$ can be expressed following Equation (3.17) of this above reference as

$$c\rho(x, t) = cei \left[\phi(x, t)^* \frac{\partial \phi(x, t)}{\partial t} - \phi(x, t) \frac{\partial \phi(x, t)^*}{\partial t} \right] \quad (10)$$

We evaluate this in both the S and S' frames to obtain from

$$\phi(x, t) = Ae^{\frac{-imc^2}{\hbar}t} + Be^{\frac{imc^2}{\hbar}t} \quad \text{and}$$

$$\phi'(x', t') = Ae^{\frac{-imc^2}{\hbar}\left(t' \cosh \omega + \frac{x'}{c} \sinh \omega\right)} + Be^{\frac{imc^2}{\hbar}\left(t' \cosh \omega + \frac{x'}{c} \sinh \omega\right)} \quad \text{the expressions}$$

$$c\rho(x, t) = -\frac{2mc^3 e}{\hbar} \left(-|A|^2 + |B|^2\right) \quad (11)$$

$$c\rho'(x', t') = -\frac{2mc^3 e}{\hbar} \left(-|A|^2 + |B|^2\right) \cosh \omega \quad (12)$$

The expressions for $\phi'(x', t')$ above are obtained from $\phi(x, t)$ by using the Lorentz transformation equation for t which is $t = t' \cosh \omega + \frac{x'}{c} \sinh \omega$.

Here the complex numbers A and B have the same meaning as in Section 2. Now we are in a position to check from Equations (11) and (12) how the scalar field charge density transform. Indeed one obtains the same relation between ρ and ρ' as in Equation (1) if we divide Equation (12) by Equation (11).

4. Conclusion and Response to the Constructive Criticism of the Referee

We would like to thank the referee for his constructive comments especially on the need to make a contrastive analysis of the results obtained in sections 2 and 3. The immediate point that is to be noted is the fact that Equations (5) and (7) in contrast to Equations (11) and (12) have the terms involving $|A|^2$ and $|B|^2$ with the same signature. Antiparticles are supposed to be oppositely charged as compared to particles and this fact is made explicit in Equations (11) and (12). In fact on page 76 of reference [4] it is made clear that the concept of charged current for the particle-antiparticle pair as given in the case of the zeroth component by Equation (10) above was introduced for the first time by Pauli and Weisskopf in 1934 which consists of terms of both signatures. The Dirac equation fails to show such consistency if we are to accept his definition of the particle (and antiparticle) current and this as we know is a direct result of the desire to keep probability density “positive”. However we would like to add that the Dirac equation has explained the fine structure of hydrogen atom along with a host of experiments in particle physics so it would be premature to put a question mark on this equation.

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