Open Access Library Journal
2017, Volume 4, e3589
ISSN Online: 2333-9721
ISSN Print: 2333-9705

A Kind of Neither Keynesian Nor Neoclassical Model (6): The Ending State of Economic Growth

Ming'an Zhan[1], Zhan Zhan[2]

[1]Yunnan University, Kunming, China
[2]Westa College, Southwest University, Chongqing, China
Email: 1792481782@qq.com

How to cite this paper: Zhan, M.A. and Zhan, Z. (2017) A Kind of Neither Keynesian Nor Neoclassical Model (6): The Ending State of Economic Growth. *Open Access Library Journal*, **4**: e3589.
https://doi.org/10.4236/oalib.1103589

Received: April 10, 2017
Accepted: May 14, 2017
Published: May 18, 2017

Abstract

In traditional macroeconomics argues that the decision and fluctuation of output level is short-term theory, and the growth of output is a long-term theory. The former is determined by the demand; the latter is determined by the production. No one has questioned why the former is determined by production and the latter is determined by demand. This paper argues that the factors that affect the output are the same in the short and long term, but there is no need to analyze the problem of fluctuation in the long term. Based on the analysis of the growth path in our previous paper, this paper first examines whether our model is applicable to the Pontryagin maximum principle, and then analyzes the difference of the Cobb-Douglas function in the exogenous growth model and the endogenous growth model. Reveal the special role of parameter A in Cobb-Douglas function: as long as A is considered as output-related variables, there is no substantial difference between the so-called exogenous model and endogenous mode. Finally, according to the trend of the growth path and the nature of A, the paper derives the final state of Cobb-Douglas function.

Subject Areas

Economics

Keywords

Cobb-Douglas Function, Dynamic Optimization, Exogenous Growth, Endogenous Growth

1. Preface

Lucas has been attracted by the questions on economic growth: "Is there some action a government of India could take that would lead the Indian economy to grow like Indonesia's or Egypt's? … Once one starts to think about them, it is hard to think about anything else". [1] While it may be necessary to swap out the role, Lucas's question still has important theoretical and practical significance before figuring out whether economic growth is endogenous or exogenous.

Unlike the economics of the Adam Smith era, modern economics, which is close to various labels, is not content with the literary logic of explaining economic phenomena. However, more use of the mathematical logic does not guarantee that the analysis results are better than the literary logic. The advantage of mathematical logic is that it is easier to find bugs in the process of reasoning, and that problems such as preconditions, variable settings, and implicit conditions are even worse than literary logic. Dynamic Stochastic General Equilibrium (DSGE) has been used for economic problems extensively since the 1960s, and the Keynesian and Neoclassical schools have used it to demonstrate their theory. Of course, we can also use it in our model. What will happen?

2. Dynamic Optimization Analysis on Condition $\mathrm{d}K = rK$ or $\mathrm{d}Y = rY$

In the Solow model, output growth depends on capital growth, capital growth is a function of net investment, and net investment is part of the output, which can use the mathematical tool of DSGE to analyze the time path and the ending state of the output changes. In the dynamic analysis of the utility function of consumption, we do not know the specific form of the utility function, but as long as the utility function is convergent and satisfies some constraints, we can get some important results according to the standard solution of the dynamic maximum principle.

The study of Cass shows that assume the utility function of per capita consumption $U(c)$ at a certain time satisfies the condition $\mathrm{d}U(c)/\mathrm{d}c > 0$, $\mathrm{d}^2U(c)/\mathrm{d}c^2 < 0$, $\lim\limits_{c\to 0}\left(\mathrm{d}U(c)/\mathrm{d}c\right) = \infty$, $\lim\limits_{c\to\infty}\left(\mathrm{d}U(c)/\mathrm{d}c\right) = 0$, and the Solow model $\mathrm{d}k/\mathrm{d}t = I_g - (\delta + n)k$ is used as the transfer equation (among them $I_g = y - c$) in the dynamic optimization, then the shadow price λ of k, the time path of the per capita consumption c and the capital k can be solved according to the Pontryagin maximum principle. The results have no difference from the original Solow model [2].

However, our study shows $\Delta K \gg I$. If the constraint condition $\mathrm{d}K = I$ in Solow model is replaced by $\mathrm{d}K = rK$ (where $r = \partial Y/\partial K = \alpha Y/K$) [3], then we need to re-verify the following three conditions of the Hamilton function in the dynamic maximum principle:

$$\lim_{t\to\infty} H = 0 \tag{1}$$

$$\frac{\partial H}{\partial c} = 0 \tag{2}$$

$$\frac{\partial H}{\partial k} = -\frac{\mathrm{d}\lambda}{\mathrm{d}t} \tag{3}$$

Let $K = Nk$ (where N is the labor population), substituting it into the identity $\mathrm{d}K = N\mathrm{d}k + k\mathrm{d}N$, and the following transfer equation is obtained from $\mathrm{d}K = rK$ [3]:

$$\mathrm{d}k = \frac{\mathrm{d}K - k\mathrm{d}N}{N} = \frac{rK}{N} - \frac{\mathrm{d}N}{N}k = rk - nk = (r-n)k \tag{4}$$

If the objective function is assumed to be the maximum utility of per capita consumption as the dynamic model of Cass, its dynamic optimization problem is as follows:

$$\begin{cases} \mathrm{Max}\int_0^\infty U(c)\mathrm{e}^{-(r-n)t}\mathrm{d}t \\ \text{s.t.}\ \frac{\mathrm{d}k}{\mathrm{d}t} = (r-n)k,\ k(0) = k_0 \end{cases} \tag{5}$$

The corresponding Hamilton function is:

$$H = U(c)\mathrm{e}^{-(r-n)t} + \lambda k(r-n) \tag{6}$$

Among them, $c = C/N$ is the per capita consumption, the discount rate r in $\mathrm{e}^{-(r-n)t}$ is assumed to be the market interest rate, λ is the costate variable, $k = K/N$ is the per capita capital, $n = \mathrm{d}N/N$ is the growth rate of labor force. The Hamilton function is differentiated for c and k respectively:

$$\frac{\partial H}{\partial c} = \mathrm{e}^{-(r-n)t}\frac{\partial U(c)}{\partial c} \tag{7}$$

$$\frac{\partial H}{\partial k} = \lambda(r-n) \tag{8}$$

According to the necessary condition shown by Equation (2) $\partial H/\partial c = 0$,

$$\mathrm{e}^{-(r-n)t}\frac{\partial U(c)}{\partial c} = 0 \tag{9}$$

In the above equation, the discount factor $\mathrm{e}^{-(r-n)t}$ is not 0 at time t, so we can only make $\partial U(c)/\partial c = 0$. This means that the utility $U(c)$ is a constant greater than 0. Therefore, the first part of the Hamilton function $U(c)\mathrm{e}^{-(r-n)t}$ is not 0 when $t \to \infty$. From the necessary condition shown by Equation (3) $\partial H/\partial k = -\mathrm{d}\lambda/\mathrm{d}t$,

$$\frac{\mathrm{d}\lambda}{\mathrm{d}t} = -\lambda(r-n) \tag{10}$$

where r and n are exogenous variables and assume that the rate of population growth n is less than the market interest rate r, that is, $(r-n) > 0$, the general solution of Equation (10):

$$\lambda = \frac{\lambda_0}{\mathrm{e}^{(r-n)t}} \tag{11}$$

From the transfer equation $\mathrm{d}k/\mathrm{d}t = k(r-n)$, the general solution of k is:

$$k = k_0\mathrm{e}^{(r-n)t} \tag{12}$$

Then, the Equation (11) (12) is brought into the second part $\lambda k(r-n)$ in the Hamilton function,

$$\lim_{t\to\infty}\left[\lambda k(r-n)\right] = \lim_{t\to\infty}\left[\frac{\lambda_0}{\mathrm{e}^{(r-n)t}}\cdot k_0\mathrm{e}^{(r-n)t}\cdot(r-n)\right] = \lim_{t\to\infty}\lambda_0 k_0(r-n) \neq 0.$$

It is shown that unless $\lim_{t\to\infty} U(c)\mathrm{e}^{-(r-n)t} = -\lambda_0 k_0 (r-n)$ in the Hamilton function $H = U(c)\mathrm{e}^{-(r-n)t} + \lambda k(r-n)$, otherwise $\lim_{t\to\infty} H = \lim_{t\to\infty}\left[U(c)\mathrm{e}^{-(r-n)t} + \lambda k(r-n)\right] \neq 0$. It is not possible to satisfy the necessary conditions shown in Equation (1). Equation (9) shows that the consumption utility $U(c)$ is a constant and $\lambda_0 k_0 (r-n) > 0$, so $\lim_{t\to\infty} U(c)\mathrm{e}^{-(r-n)t} = -\lambda_0 k_0 (r-n)$ cannot be established.

The reason for this is that $\mathrm{d}k/k$ is decreasing in the transfer equation $\mathrm{d}k/\mathrm{d}t = y(k) - c - (\delta + n)k$ of the Solow model, but $\mathrm{d}k/k$ is not diminish when the increase of k in our transfer equation $\mathrm{d}k/\mathrm{d}t = (r-n)k$.

We can also construct dynamic models in another way. Since $\mathrm{d}Y$ and Y are already present in our basic equation $\mathrm{d}Y/Y = r$ [3], there is no need to introduce the labor population N in order to convert Y and C to per capita output y and consumption c as the dynamic model of Cass. We directly use the total consumption C in the utility function, the dynamic optimization problem can be simplified as:

$$\begin{cases} \mathrm{Max}\int_0^\infty U(C)\mathrm{e}^{-rt}\mathrm{d}t \\ \text{s.t.}\ \dfrac{\mathrm{d}Y}{\mathrm{d}t} = rY,\ Y(0) = Y_0 \end{cases} \tag{13}$$

The corresponding Hamilton function is:

$$H = U(C)\mathrm{e}^{-rt} + \lambda rY \tag{14}$$

$$\frac{\partial H}{\partial C} = \mathrm{e}^{-rt}\frac{\partial U(C)}{\partial C} + \lambda r\frac{\partial Y}{\partial C} \tag{15}$$

$$\frac{\partial H}{\partial Y} = \lambda r \tag{16}$$

According to the necessary conditions $\partial H/\partial C = 0$ and the Equation (15),

$$\lambda = -\frac{1}{r\mathrm{e}^{rt}(\partial Y/\partial C)}\cdot\frac{\partial U(C)}{\partial C} \tag{17}$$

Since we do not know the specific form of the utility function $U(C)$, the limit of the costate variable λ can not be determined by Equation (17). From the necessary conditions shown in Equations (16) and (3),

$$\frac{\mathrm{d}\lambda}{\mathrm{d}t} = -\lambda r \tag{18}$$

Since r in Equation (18) is an exogenous variable greater than 0, we can get the general solution of λ:

$$\lambda = \frac{\lambda_0}{\mathrm{e}^{rt}} \tag{19}$$

$$\lim_{t\to\infty}\lambda = \lim_{t\to\infty}\frac{\lambda_0}{\mathrm{e}^{rt}} = 0 \tag{20}$$

Equation (20) can not guarantee $\lim_{t\to\infty} H(t) = 0$, because the function $H = U(C)\mathrm{e}^{-rt} + \lambda rY$ also contains the variable Y, we need to examine the changes in Y. According to the transfer equation $\mathrm{d}Y/\mathrm{d}t = rY$, the general solution of Y is:

$$Y = Y_0 e^{rt} \tag{21}$$

From the Equation (19) and (21),

$$\lambda Y = \frac{\lambda_0}{e^{rt}} \cdot Y_0 e^{rt} = \lambda_0 Y_0$$

$$\lim_{t\to\infty} \lambda Y = \lambda_0 Y_0 \neq 0$$

Therefore, unless $\lim_{t\to\infty} U(C) e^{-rt} = -r\lambda_0 y_0$, otherwise $\lim_{t\to\infty} H = \lim_{t\to\infty}\left[U(C)e^{-rt} + \lambda rY\right] = \lim_{t\to\infty}\left[U(C)e^{-rt} + r\lambda_0 Y_0\right] \neq 0$. In the economic sense, the utility $U(C)$ should not be less than 0 (if utility less than 0 the consumption is meaningless), it is, $\lim_{t\to\infty} U(C)e^{-rt} \geq 0$, so there is no $\lim_{t\to\infty} U(C)e^{-rt} = -r\lambda_0 y_0$.

In the Solow model, since $\lim_{t\to\infty} e^{(r-n)t} = 0$, $U(c)$ is bounded by the assumption of $\lim_{c\to\infty}(dU(c)/dc) = 0$. Therefore, the limit of the first term in the Hamilton function is always 0; the second term consists of the costate variable λ and the transfer equation $dk/dt = y - c - (\delta + n)k$. The effect of c and δ on the dk in the transfer equation is negative and the margin of y is decreasing, so the limit of the second term of the Hamilton function is also 0.

When the transfer equation is changed to our $dK = rK$ or $dY = rY$, the consumption and the depreciation do not restrict the growth of K and Y because dY has already includes increased consumption and depreciation. If the fluctuation of output in short-term are ignored, the output growth rate dY/Y is independent of the size of K and Y at any time (including $t \to \infty$), only by the marginal state variable r. As long as $r > 0$, dK or dY will not be 0, so the limit of Hamilton function can not be 0 and the transversality condition $\lim_{t\to\infty} H = 0$ can not be satisfied. The model of per capita variables, which considers the change in the labor force as in the Equation (5), is no exception.

From the Cobb-Douglas function $Y = AK^{\alpha}L^{\beta}$, $\partial Y/\partial K = \alpha Y/K$, let $\partial Y/\partial K = r$, then $K = \alpha Y/r$. Assuming that α and r are independent of K and Y changes (we will see that the assumption is reasonable from the equation $Y = 2rK$), then $dK = (\alpha/r)dY$, and then by the statistical identity $dY = dC + dD + I$ [1],

[1]There are three types of variables in the macroeconomic analysis: incremental, stock, and mixed. In a closed economic system that ignores foreign trade, the statistical identity of the output is expressed as $Y = C + I_g = C + I + D$, where C is consumption, I_g is the gross investment, I is the net investment, and D is depreciation. Exactly, there is a theoretical bug in this identity. Assuming that the output of this period (t) is Y_t and the output of the previous period ($t - 1$) is Y_{t-1}, the exact identity should be expressed as follows: $Y_t = Y_{t-1} + \Delta Y = (C_{t-1} + D_{t-1}) + (\Delta C + \Delta D + I_t) = (C_{t-1} + \Delta C) + (D_{t-1} + \Delta D) + I_t = C_t + D_t + I_t$. Therefore, in the identity, ΔY, ΔC, ΔD and I_t are incremental variables, Y_{t-1}, C_{t-1} and D_{t-1} are stock variables, Y_t, C_t and D_t should be called the amount of mixing because Y_t both incremental and stock variable. Since I is the increment rather than the stock, theoretically it can only appear in the incremental expression: $\Delta Y = \Delta C + \Delta D + I$.

$$\mathrm{d}K = \frac{\alpha}{r}\mathrm{d}Y = \frac{\alpha}{r}\left(\mathrm{d}C + \mathrm{d}D + I\right)$$

In the Cobb-Douglas function, C and I are not the cause of the change in output, but the result. Since the investment and depreciation increment is only part of the output increment, $I + \mathrm{d}D$ is always less than $\mathrm{d}Y$, and because of $Y < K$, $\mathrm{d}Y < \mathrm{d}K$, the investment is much smaller than the capital increase. Therefore, it is impossible to obtain the correct conclusion for all dynamic optimal analysis based on the hypothesis of $\mathrm{d}K = I$.

The statistics show that about 80% of the newly increased output ΔY is used for the newly increased consumption ΔC. In the above equation because $\alpha/r > 1$, so $\mathrm{d}K > I$, I is only a small part of $\mathrm{d}K$.

$\mathrm{d}K > I$ is not unreasonable in reality. Because net investment I in addition to generating equal assets with I, but also bring the production technology upgrades, improve production efficiency and other non-physical assets changes, it is these changes make greedy investors willing to take possible risks.

3. Exogenous and Endogenous Growth

Assuming L in $Y = AK^{\alpha}L^{\beta}$ as a labor population can give us a benefit that simplifies $Y = AK^{\alpha}L^{\beta}$ to a formal univariate function $y = Ak^{\alpha}$, where $y = Y/N$, $k = K/N$, N is the labor population. Full differential of $y = Ak^{\alpha}$:

$$\frac{\mathrm{d}y}{y} = \frac{\mathrm{d}A}{A} + \alpha\frac{\mathrm{d}k}{k} + \alpha\ln k\frac{\mathrm{d}\alpha}{\alpha} \tag{22}$$

Assuming that α and A are constant,

$$\frac{\mathrm{d}y}{y} = \alpha\frac{\mathrm{d}k}{k} \tag{23}$$

If $\mathrm{d}K = I$ is further assumed, then $\mathrm{d}K = I = I_g - \delta K$, where I_g is the gross investment and δ is the depreciation rate. By the identity $\mathrm{d}K = \mathrm{d}\left(kN\right) = k\mathrm{d}N + N\mathrm{d}k$, $I_g - \delta K = k\mathrm{d}N + N\mathrm{d}k$, the growth rate of population $n = \mathrm{d}N/N$, then

$$\frac{\mathrm{d}k}{k} = \frac{I_g}{K} - \left(\delta + n\right) = \frac{sY}{K} - \left(\delta + n\right) = \frac{sA}{k^{1-\alpha}} - \left(\delta + n\right) \tag{24}$$

Among them, $Y = C + I_g$. Assuming that the consumption rate $c = C/Y$, then $I_g = Y - cY = \left(1 - c\right)Y = sY$. Obviously, the savings rate s is determined by the consumption rate c. Substituting Equation (24) into Equation (23):

$$\frac{\mathrm{d}y}{y} = \alpha\left[\frac{I_g}{K} - \left(\delta + n\right)\right] = \alpha\left[\frac{sA}{k^{1-\alpha}} - \left(\delta + n\right)\right] \tag{25}$$

The Equation (23) and (24) show that since the marginal output of k decreases, the contribution of k to $\mathrm{d}y$ is getting smaller and smaller, and the depreciation δk grows linearly with k increases. Therefore, regardless of the current state of growth of the economic system, even if the population growth rate n is 0, the economic system can not avoid the end state, that is, $\mathrm{d}y/y = 0$. For a finite time, when k grows to $sA/k^{1-\alpha} = \delta + n$, or $k = \left[sA/\left(\delta + n\right)\right]^{1/(1-\alpha)}$, the net capital growth will be lost to the depreciation of capital stock. Thus, even if people do not consume at all, output all for investment ($s=1$), it can only extend the time that the output growth rate is reduced to 0.

To make $\mathrm{d}y/y$ in the Solow model not diminish, a simple improvement is to think of the A in the Cobb-Douglas function as a variable that grows with k. The Equation (23) shows that there is $\mathrm{d}y/y>0$ as long as $sA/k^{1-\alpha}>\delta+n$, or $A>(\delta+n)k^{1-\alpha}/s$. people generally believed that A is the technical condition in the production process, is exogenous factor and does not adjust automatically with k changes. Therefore, the Solow model is considered a kind of "exogenous growth theory".

In the subsequent "endogenous growth theory", someone direct assumptions that the production function is the marginal output does not the diminish functions [4] [5] [6], another one derived such equations by re-assuming the meaning of the production factor L.

In fact, the A as a constant or that it is an exogenous variable, but people's subjective assumptions. By $Y=AK^{\alpha}L^{\beta}$,

$$\frac{\partial Y}{\partial K}=\frac{\alpha Y}{K},\frac{\partial Y}{\partial L}=\frac{\beta Y}{L} \tag{26}$$

Let $\partial Y/\partial K=r,\partial Y/\partial L=w$, when $r>0$, $w>0$,

$$A=\frac{Y}{K^{\alpha}L^{\beta}}=\frac{rK/\alpha}{K^{\alpha}L^{\beta}}=\frac{r}{\alpha}\left(\frac{K}{L}\right)^{\beta}=\frac{r}{\alpha}\left(\frac{\alpha w}{\beta r}\right)^{\beta}=\frac{r^{\alpha}w^{\beta}}{\alpha^{\alpha}\beta^{\beta}} \tag{27}$$

Assume $L=N$, then

$$A=\frac{r^{\alpha}w^{\beta}}{\alpha^{\alpha}\beta^{\beta}}=\frac{r^{\alpha}\left(\frac{\beta Y}{N}\right)^{\beta}}{\alpha^{\alpha}\beta^{\beta}}=\left(\frac{r}{\alpha}\right)^{\alpha}y^{\beta} \tag{28}$$

Since $y=Ak^{\alpha}$, so we can have:

$$A=\frac{r}{\alpha}k^{\beta} \tag{29}$$

This shows that A is closely related to the marginal state of the production r and the distribution state (α/β) of the production factors and should be a comprehensive variable that is affected by various factors in the Cobb-Douglas function.

The Equation (28) is obtained at the hypothesis $L=N$, and $L=N$ is not necessary to analyze the per capita output level y. Without assuming $L=N$, the per capita output should be expressed as:

$$y=\frac{Y}{N}=\frac{AK^{\alpha}L^{\beta}}{N}=A\left(\frac{K}{N}\right)^{\alpha}\left(\frac{L}{N}\right)^{\beta}=Ak^{\alpha}\left(\frac{L}{N}\right)^{\beta} \tag{30}$$

Among them,

$$A=\frac{r^{\alpha}w^{\beta}}{\alpha^{\alpha}\beta^{\beta}}=\frac{r}{\alpha}\left(\frac{N}{L}\right)^{\beta}k^{\beta} \tag{31}$$

When $L\neq N$, the size of A is related to the size of N and L, except for r, α and k. Therefore, suppose $L=N$,we can transform the Cobb-Douglas function into a simple function $y = Ak^{\alpha}$ for easy analysis, but also confuse the difference between per capita output and hypothesis $L = N$.

If L is human resource and let $L = EN$, E is the coefficient of worker quality $(E > 0)$, then the production function is $y = AE^{\beta}k^{\alpha}$. If E is a constant, this assumption is not substantially different from the Solow model and the diminishing of output growth rate can not be changed.

Like the "Learning by doing model" [7], if the quality of human resources is considered to grow with the capital, that is, $E = K$, then $L = EN = KN$, the corresponding production function is $y = AN^{\beta}k$. This is free from the impact of diminishing marginal output of capital with output growth, and the growth of population N not only does not impede per capita output growth as the Solow model, but also promote the growth of per capita output.

If $E = K/N$, then $L = EN = K$, so $Y = AK^{\alpha}L^{\beta} = AK^{\alpha}K^{\beta} = AK$, or $y = Ak$. This is called the "AK model" and is the most concise endogenous growth model [5] [6].

Note that there is no A as an endogenous variable in the above-mentioned production function, which derived from the various assumptions about human resource L above. If A is regarded as an endogenous variable and is substituted into the "Learning by doing model" and the "AK model", it is found that there is no essential difference between the two models.

From Equation (31), in the "Learning by doing" model and the "AK" model, the expression of A should be:

"Learning by doing model":

$$A = \frac{r}{\alpha}\left(\frac{N}{L}\right)^{\beta} k^{\beta} = \frac{r}{\alpha}\left(\frac{N}{KN}\right)^{\beta} k^{\beta} = \frac{r}{\alpha}\left(\frac{1}{N}\right)^{\beta}$$

"AK model":

$$A = \frac{r}{\alpha}\left(\frac{N}{L}\right)^{\beta} k^{\beta} = \frac{r}{\alpha}\left(\frac{N}{K}\right)^{\beta} k^{\beta} = \frac{r}{\alpha}$$

They are substituted into the "Learning by doing model" $y = AN^{\beta}k$ and the "AK model" $y = Ak$, respectively, the,

"Learning by doing model":

$$y = AN^{\beta}k = \frac{r}{\alpha}\left(\frac{1}{N}\right)^{\beta} \cdot N^{\beta}k = \frac{r}{\alpha}k$$

"AK model":

$$y = Ak = \frac{r}{\alpha}k$$

It like a game in mathematics. As long as A is regarded as an endogenous variable, all models are nothing but an expression of the equation $y = (r/\alpha)k$. The problem is that when $r > 0$, $Y = AK^{\alpha}L^{\beta}$ and $Y = (r/\alpha)K$ can be used to express the relationship between Y and K, but A contains variables associated with r in $Y = AK^{\alpha}L^{\beta}$.

Lucas discovered the gap between rich and poor countries in capital marginal output in his analysis of Indian data in 1990 [8]. This is called "Lucas's Paradox". In fact, if a clear endogenous A, we will understand the so-called "Lucas's Paradox". The equation $k = \alpha y/r$ (from $r = \alpha y/k$) is brought into the Cobb-Douglas function $y = Ak^{\alpha}$, then $y = A(\alpha y/r)^{\alpha}$, this is,

$$r = \alpha A^{\frac{1}{\alpha}} y^{\frac{\alpha-1}{\alpha}} \tag{32}$$

where, r is $\partial Y/\partial K$. The trap of Equation (32) is that if A is seen as a constant, we can predict the marginal output $\partial y/\partial k(=r)$ by the per capita output of a country.

For example, the per capita output of the United States is 15 times that of India, assuming that $\alpha = 0.4$ both countries in Equation (32), then India's marginal output $\partial Y/\partial K(=r)$ will be 58 times that of the United States. In such a big gap, American capital should go to India to get amazing benefits, so Lucas said that neoclassical theory simply can not reasonably explain the real world.

However, if the Equation (29) $A = (\alpha/r)k^{\beta}$ is brought into Equation (32), we find that Equation (32) is another expression of $r = \alpha y/k$. The equation $r = \alpha y/k$ indicates that the difference in interest rate between the two countries is determined by the difference in output-capital ratio y/k between the two countries, not determined by the difference in the level of per capita output y. Although the per capita output of the United States is 15 times that of India, if the US per capita capital k is also 15 times that of India, the interest rate (capital marginal output) of the two countries will be very similar (assuming the difference of α between the two countries is not significant). Therefore, although the per capita income of countries may have a big gap, but the interest rate gap is not large (especially the real interest rate).

Lucas said there were several ways to make the economy of India rapidly grow, the first one is to improve the original production efficiency, the second one is to make full use of domestic resources through international trade, the third one is to earn other country's transfer payments through diplomatic access that have significant effect on its own economy.

The essence of "trade to be rich" is to turn the idle resources of a country into a real income that can improve people's living standards. If use some revenues from trading to import technologies and to improve the domestic production efficiency, the process of getting rich will be faster and more sustainable. The export of cheap oil in the Middle East, and export of cheap labor in China with the help of "two outsides (products and raw materials), two large (export and import)" are the example of "trade to be rich" in economic globalization.

In theory, as long as the clear $A = r^{\alpha} w^{\beta} / \alpha^{\alpha} \beta^{\beta}$, no matter how the assumption that L can be, it will only cause the formal changes of the production function. For example, assuming that the monetary value of human resources is the income it obtains in the market: $L = \beta Y$, then $Y = AK^{\alpha} L^{\beta} = AK^{\alpha} (\beta Y)^{\beta}$, that is, $Y = (A\beta^{\beta})^{1/\alpha} K$.

In the philosophical sense, if the contribution of material resource K to output is also derived from human wisdom, all income is the contribution of human resources, that is, assuming $L = Y$, then $Y = AK^{\alpha}L^{\beta} = AK^{\alpha}Y^{\beta} = A^{1/\alpha}K$. Obviously, this hypothesis derived production function has also the characteristics of "Learning by doing model" and the "AK model" that the marginal output does not diminish. Table 1 summarizes the changes in the Cobb-Douglas function under various assumptions of L.

In addition to these assumptions shown in Table 1, we can also think of more reasons to assume more. In short, by assuming a different L and subjective interpretation of L, any desired endogenous growth or exogenous growth model can be obtained. However, as long as the corresponding change of $r = \partial Y/\partial K$ and A is introduced, all models are not different, and are the deformation of the marginal state equation $r = \alpha Y/K$ or $Y = rK/\alpha$ of the Cobb-Douglas function. Equation $Y = rK/\alpha$ is not only the starting point for the analysis of the cyclical changes in output [9], but also implies the output growth path and the

Table 1. The influence in the various hypotheses of L on the Cobb-Douglas function and the parameter A.

Hypothesis	The production function expressed by total output Y	The production function expressed by the per capita output y ($y = Y/N$, N is the labor population)	The variable A corresponding to the hypothesis of L	Note
$L = L$	$Y = AK^{\alpha}L^{\beta}$	$\frac{Y}{N} = A\left(\frac{K}{N}\right)^{\alpha}\left(\frac{L}{N}\right)^{\beta}$ $y = Ak^{\alpha}\left(\frac{L}{N}\right)^{\beta}$	$A = \frac{r^{\alpha}w^{\beta}}{\alpha^{\alpha}\beta^{\beta}}$ $A = \left(\frac{r}{\alpha}\right)^{\alpha}\left(\frac{Y}{L}\right)^{\beta}$	Assuming $r = \partial Y/\partial K$, $w = \partial Y/\partial L$, then $Y/K = r/\alpha$, $Y/L = w/\beta$, so $A = \left(r^{\alpha}w^{\beta}\right)/\left(\alpha^{\alpha}\beta^{\beta}\right)$
$L = K^{\alpha/\beta}$ where $K = \alpha Y/r$	$Y = AK^{\alpha}L^{\beta}$	$\frac{Y}{N} = A\cdot\frac{K^{2\alpha}}{N}$ $y = \frac{A}{N^{1-2\alpha}}k^{2\alpha}$	$A = \left(\frac{r}{\alpha}\right)^{\alpha}\left(\frac{Y}{K^{\alpha/\beta}}\right)^{\beta}$ $A = \frac{r}{\alpha}\cdot K^{\beta-\alpha}$	K is calculated by the definition $K = \alpha Y/r$ (where α, Y and r are from the statistical data), rather than calculated by the fixed assets.
$L = N$	$Y = AK^{\alpha}N^{\beta}$	$\frac{Y}{N} = A\left(\frac{K}{N}\right)^{\alpha}\left(\frac{N}{N}\right)^{\beta}$ $y = Ak^{\alpha}$	$A = \left(\frac{r}{\alpha}\right)^{\alpha}\left(\frac{Y}{N}\right)^{\beta} = \left(\frac{r}{\alpha}\right)^{\alpha}y^{\beta}$ $A = \frac{r}{\alpha}k^{\beta}$	A is different in the Cobb-Douglas function with the assumption of L. In $A = \left(r^{\alpha}w^{\beta}\right)/\left(\alpha^{\alpha}\beta^{\beta}\right)$, assuming $L = N$, then The result of A is not only related to the calculation of w, but also to the dimensions of N and other variables. If the Equation (28) $A = \left(r/\alpha\right)^{\alpha}y^{\beta}$ is calculated (where $y = Y/N$), then Y and N using the different dimensions (such as Y and N dimensions were billion and thousands of people, or million and people), size A will be different. If the Equation (29) $A = \left(r/\alpha\right)k^{\beta}$ is calculated (where $k = K/N$), when K is calculated from $K = \alpha Y/r$, The result is not different from Equation (28); If the statistics "fixed assets and consumer goods" as K, then the Equation (29) A will be different from the results of Equation (28). Only the A calculated by assuming $L = K^{\alpha/\beta}$ will not show different results, and is a dimensionless number.
$L = KN$	$Y = AK^{\alpha}\left(KN\right)^{\beta}$ $Y = AN^{\beta}K$	$\frac{Y}{N} = A\left(\frac{K}{N}\right)^{\alpha}\left(\frac{KN}{N}\right)^{\beta}$ $y = Ak^{\alpha}K^{\beta}$	$A = \left(\frac{r}{\alpha}\right)^{\alpha}\left(\frac{Y}{KN}\right)^{\beta} = \left(\frac{r}{\alpha}\right)^{\alpha}\left(\frac{r}{\alpha N}\right)^{\beta}$ $A = \frac{r}{\alpha N^{\beta}}$	
$L = K$	$Y = AK^{\alpha}K^{\beta}$ $Y = AK$	$\frac{Y}{N} = A\cdot\frac{K}{N}$ $y = Ak$	$A = \left(\frac{r}{\alpha}\right)^{\alpha}\left(\frac{Y}{K}\right)^{\beta} = \left(\frac{r}{\alpha}\right)^{\alpha}\left(\frac{r}{\alpha}\right)^{\beta}$ $A = \frac{r}{\alpha}$	
$L = \beta Y$	$Y = AK^{\alpha}\left(\beta Y\right)^{\beta}$ $Y = A^{1/\alpha}\beta^{\beta/\alpha}K$	$\frac{Y}{N} = A^{1/\alpha}\beta^{\beta/\alpha}\cdot\frac{K}{N}$ $y = \left(A\beta^{\beta}\right)^{1/\alpha}k$	$A = \left(\frac{r}{\alpha}\right)^{\alpha}\left(\frac{Y}{\beta Y}\right)^{\beta} = \left(\frac{r}{\alpha}\right)^{\alpha}\left(\frac{1}{\beta}\right)^{\beta}$ $A = \frac{r^{\alpha}}{\alpha^{\alpha}\beta^{\beta}}$	
$L = Y$	$Y = AK^{\alpha}Y^{\beta}$ $Y = A^{1/\alpha}K$	$\frac{Y}{N} = A^{1/\alpha}\cdot\frac{K}{N}$ $y = A^{1/\alpha}k$	$A = \left(\frac{r}{\alpha}\right)^{\alpha}\left(\frac{Y}{Y}\right)^{\beta}$ $A = \frac{r^{\alpha}}{\alpha^{\alpha}}$	

final state of the information.

Figure 1 shows the difference in A assuming $L = K^{\alpha/\beta}$ and $L = N$. Under the assumption of $L = K^{\alpha/\beta}$), the change of A is obviously decreasing, and in the other way, A shows a slight upward trend. Therefore, A is the variable associated with the hypothesis L, not a constant.

Gollop and others on the statistical analysis think that the contribution of capital is the first, the labor force is second, and the contribution of the productivity is less than one quarter in the economic growth of the United States [10].

After the Cobb-Douglas function $Y = AK^{\alpha}L^{\beta}$ is fully differentiated, the $\mathrm{d}A/A$ in Equation (33) is called Total Factor Productivity (TFP).

$$\frac{\mathrm{d}Y}{Y} = \frac{\mathrm{d}A}{A} + \alpha\frac{\mathrm{d}K}{K} + \beta\frac{\mathrm{d}L}{L} \tag{33}$$

Since the sum of $\alpha(\mathrm{d}K/K)$ and $\beta(\mathrm{d}N/N)$ is less than $\mathrm{d}Y/Y$ in the statistical data, It is believed that the residual $\mathrm{d}A/A$ is the contribution of technological progress. If L is determined by $L = K^{\alpha/\beta}$, the average of $\mathrm{d}A/A$ will be negative (during 1970-2016, the average value of $\mathrm{d}A/A$ is −0.0268). Because A decreases with output (see Figure 1). This difference is due to the fact that Gollop's estimates do not take into account the marginal relationship between the variables in the Cobb-Douglas function. If the marginal equation $Y = rK/\alpha$ is fully different,

$$\frac{\mathrm{d}Y}{Y} = \frac{\mathrm{d}r}{r} + \frac{\mathrm{d}K}{K} - \frac{\mathrm{d}\alpha}{\alpha} \tag{34}$$

Substituting Equation (34) into Equation (33), according to $\alpha + \beta = 1$ and $\mathrm{d}K/K = \alpha(\mathrm{d}K/K) + \beta(\mathrm{d}K/K)$,

$$\frac{\mathrm{d}A}{A} = \beta\left(\frac{\mathrm{d}K}{K} - \frac{\mathrm{d}L}{L}\right) + \frac{\mathrm{d}r}{r} - \frac{\mathrm{d}\alpha}{\alpha} \tag{35}$$

In Equation (35), the first term shows that $\mathrm{d}A/A$ is also affected by $\mathrm{d}K/K$ and $\mathrm{d}L/L$, the second term $\mathrm{d}r/r$ is the fluctuating variable that causes r periodic change, and the third term $\mathrm{d}\alpha/\alpha$ is increased and fluctuates periodically

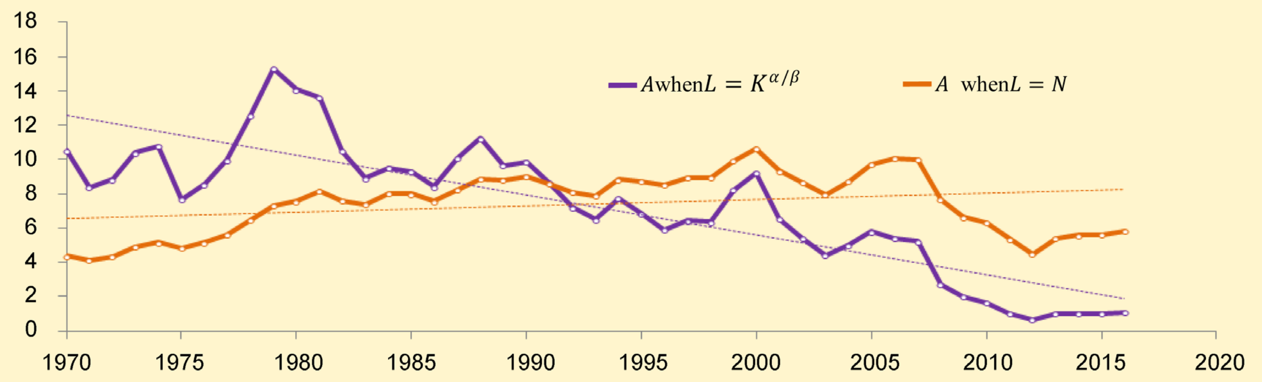

Figure 1. Compare A under different assumptions. Source: 1) A is calculated from Equation (10) $A = r^{\alpha}w^{\beta}/\alpha^{\alpha}\beta^{\beta}$, where $w = \beta Y/L$, r, Y, α and β are statistical data. They are from http://www.federalreserve.gov and http://www.bea.gov. 2) When L is calculated from $L = K^{\alpha/\beta}$, K is calculated from $K = \alpha Y/r$. 3) Data of civilian labor force N are from http://www.bls.gov/cps/cpsaat01.htm, when $L = N$. 4) The dotted line is automatically generated by Excel in the figure.

in the statistical data. It would be easier to understand the nature of A by re-writing the Equation (27)as follows:

$$A = \left(\frac{r}{\alpha}\right)^{\alpha}\left(\frac{w}{\beta}\right)^{\beta}$$

where r and w are the marginal states of K and L, and the function of α and β is the effect-proportion of r and w in the production. In this way, A is an endogenous variable that contributes to the marginal contribution of factors K and L with the distribution parameters α and β. It plays an important role as bridge in the Cobb-Douglas function $Y = AK^{\alpha}L^{\beta}$ and the marginal state equation $r = \alpha Y/K$. Ignoring the endogeneity of A ignores the relation between $Y = AK^{\alpha}L^{\beta}$ and $r = \alpha Y/K$.

4. The Final State of Economic Growth

According to "A Kind of Neither Keynesian Nor Neoclassical Model (5): The Path of Economic Growth" [11] we know that regardless the production state of the system is $\partial Y/\partial K < \partial Y/\partial L$ (on the left side of the Cobb-Douglas function bisector $K = L$), or $Y/\partial K > \partial Y/\partial L$ (on the right side of the Cobb-Douglas function bisector $K = L$), the state will close to the bisector $K = L$ driven by market competition. And the state of limit is $\partial Y/\partial K = \partial Y/\partial L$, namely $r = w$, at that point $\alpha = \beta = 1/2$. Therefore the final state of variable A is:

$$\lim_{\alpha/\beta \to 1} A = \lim_{\alpha/\beta \to 1} \frac{r^{\alpha} w^{\beta}}{\alpha^{\alpha}\beta^{\beta}} = 2r \tag{36}$$

In Cobb-Douglas function $Y = AK^{\alpha}L^{\beta}$, because $L^{\beta} = K^{\alpha}$ [9], when $\alpha/\beta \to 1$, $L \to K$, therefore,

$$\lim_{\alpha/\beta \to 1} Y = \lim_{\alpha/\beta \to 1} AK^{\alpha}L^{\beta} = \lim_{\alpha/\beta \to 1} A \cdot \lim_{\alpha/\beta \to 1} K^{\alpha}L^{\beta} = 2rK \tag{37}$$

This is the result of the "AK model" or the "Learning by doing model": the contribution of K to the growth of output is not marginally diminishing. The "AK model" and "Learning by doing model" are derived from the subjective interpretation of L, and our endogenous model is derived from the final state of A and Y.

Equation (37) shows that the final state of output Y is still a function but not a constant. In the final state function $Y = 2rK$, the relation between Y and K evolves from an exponential relationship of $Y = AK^{\alpha}L^{\beta}$ to a linear relationship, and the output growth does not terminate due to the growth of K.

Differentiate $A = 2r$, then $\mathrm{d}A/A = \mathrm{d}r/r$. In the final state, $\mathrm{d}A/A$ only has the second item in Equation (35). This shows that A will fluctuate due to the fluctuation of r (r fluctuation reasons see reference [9]).

It is simpler to derive the final state of the output Y from the marginal state equation $Y = rK/\alpha$ of the Cobb-Douglas function. Since $\alpha = \beta = 1/2$ when $\partial Y/\partial K = \partial Y/\partial L$,

$$\lim_{\alpha/\beta \to 1} Y = \lim_{\alpha/\beta \to 1} \frac{rK}{\alpha} = 2rK \tag{38}$$

Comparing Equations (37) and (38), $Y = AK^{\alpha}L^{\beta}$ and $Y = rK/\alpha$ are the equivalent expression of Cobb-Douglas function. From the equation $Y = 2rK$ we can see that when r is an exogenous variable, $\mathrm{d}Y/\mathrm{d}K = 2r$. This suggests that "the marginal output of production factors is decreasing" is a relative definition. When the marginal output of all factors in production is equal to each other, the output increment $\mathrm{d}Y$ is not necessarily diminishing compared to the factor increment $\mathrm{d}K$, since the increment of another factor $\mathrm{d}L(=\mathrm{d}K)$ also contributes the same to $\mathrm{d}Y$.

Microeconomics divides production into three states. In the first, the marginal output of K is greater than the average output ($MPK > APK$). In the second, $MPK < APK$, the third is the inefficient state, because $MPK < 0$. In the macroeconomic field, since the marginal output $MPK = \partial Y/\partial K = \alpha Y/K$, the average output of K $APK = Y/K$, $\alpha < 1$, so $MPK < APK$. This suggests that, as long as $\partial Y/\partial K > 0$, the production in the macroscopic sense is always in the second state of efficiency.

We have defined $Y/(K+L)$ as "unit capital output" [11], and now its economic significance is more obvious. Since $K = \alpha Y/r$, $L = \beta Y/w$, when $\alpha + \beta = 1$, $\alpha/\beta = 1$, $r = w$, then,

$$\lim_{\alpha/\beta \to 1} \frac{Y}{K+L} = \lim_{\alpha/\beta \to 1} \frac{Y}{\frac{\alpha Y}{r} + \frac{\beta Y}{w}} = r \tag{39}$$

"Unit Resource Output" $Y/(K+L)$ is actually "Total Factor Average Product (TFAP)". At $\partial Y/\partial K \neq \partial Y/\partial L$, TFAP is always greater than the marginal output $\partial Y/\partial K$ or $\partial Y/\partial L$. As the output Y grows and $\alpha/\beta \to 1$, the TFAP will become more optimized [11] and will be close to the marginal output $\partial Y/\partial K$ or $\partial Y/\partial L$.

According to statistical data of the United States, as shown in **Figure 2**, there are always $A/2 > Y/(K+L) > r$. And based on Equation (36) and (39), we can get the following inequality, where the equation is satisfied when $\alpha/\beta = 1$.

$$\frac{A}{2} \geq \frac{Y}{K+L} \geq r \tag{40}$$

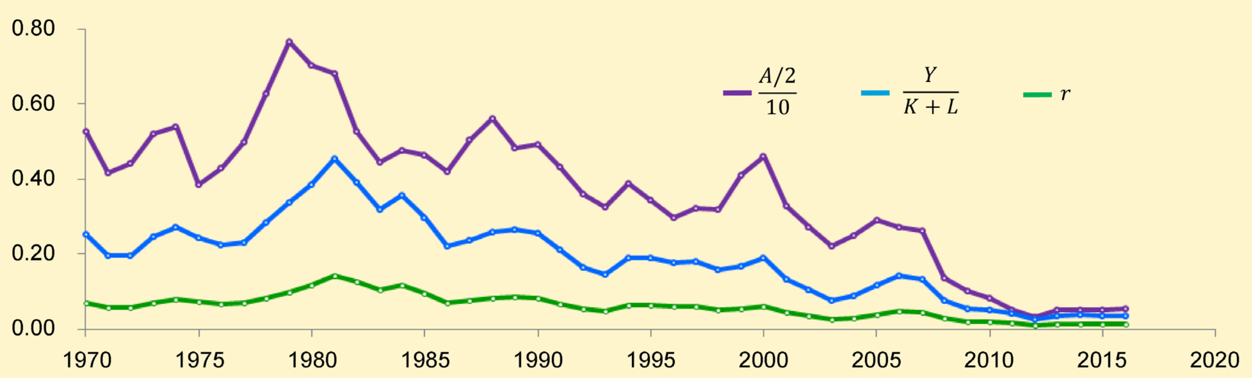

Figure 2. The trend of "unit resource output" $Y/(K+L)$. Source: 1) In the calculation of $Y/(K+L)$ and A, $L = K^{\alpha/\beta}$, $K = \alpha Y/r$, among them r, the sources of Y, α and β are the same as in **Figure 1**. 2) Reducing $A/2$ to 1/10 is to show the fluctuation of all variables in the same ordinate.

As in the case of traditional theory, it is impossible to have the above inequality if L is the labor force N. The most straightforward reason is that there is no comparability between A, $Y/(K+L)$ and r, because their dimensions are inconsistent. For example, when L is the employment population, the dimension of thousands of people, What is the dimension of A and $Y/(K+L)$?

The final state of the production function does not mean that the output growth will eventually stop. Differentiated the final state equation $Y = 2rK$, then:

$$\frac{\mathrm{d}Y}{Y} = \frac{\mathrm{d}K}{K} + \frac{\mathrm{d}r}{r} \tag{41}$$

This is not contradictible to the fully differential equation of $Y = rK/\alpha$, since $Y = rK/\alpha$ is not the final state, so $\mathrm{d}\alpha/\alpha \neq 0$. In the final state $\alpha = 1/2$, so $\mathrm{d}\alpha/\alpha = 0$. From the basic equation $\mathrm{d}K/K = r$ [3], then

$$\frac{\mathrm{d}Y}{Y} = r + \frac{\mathrm{d}r}{r} \tag{42}$$

This is the cycle equation of the growth of output that we have previously derived [9], but at that time we assumed that in the short term $\mathrm{d}\alpha/\alpha = 0$. Therefore, although the final state production function $Y = 2rK$ is different from the production function $Y = AK^{\alpha}L^{\beta}$ or $Y = rK/\alpha$, the factors that determine the core output growth rate in short-, medium- and long-term are the same, which is the core interest rate $\overline{r}$. And as long as the core interest rate $\overline{r}$ is greater than 0, Even if the production function evolves to $Y = 2rK$, the business cycle will not disappear.

If we neglect the periodic fluctuations in output growth, then $\mathrm{d}Y/Y = r$. Rewrite it as a differential equation varies along time $(\mathrm{d}Y/\mathrm{d}t)/Y = r$, the general solution is:

$$Y = Y_0 \mathrm{e}^{rt} \tag{43}$$

Although this time path function of Y does not have abundant production state information as the Cobb-Douglas function, it is not like $Y = AK^{\alpha}L^{\beta}$, when $r = \partial Y/\partial K = \partial Y/\partial L = 0$, the result is not understandable:
$Y = (\partial Y/\partial K)K + (\partial Y/\partial L)L = 0$. For the time path function $Y = Y_0 \mathrm{e}^{rt}$, when $r = \partial Y/\partial K = \partial Y/\partial L = 0$, then $Y = Y_0$, the system is in a simple reproduction state.

The Harrod-Domar model assumes that the production function is $Y = aK$ [12] [13]. This can be seen as a simplified form of the marginal state equation $Y = (r/\alpha)K$ expressed. Although the subsidiary hypothesis $\Delta K = I$ is not correct, when introduce coefficient b in $\Delta K = I$, then $\Delta K = bI$. And with the production function $Y = aK$, then $\Delta Y = a\Delta K = abI$. According to the statistical data, I is a part of the output $Y(= Y_{t-1} + \Delta Y)$, assuming $I = sY$, where s is another coefficient that is different from b (note: we do not call s for the savings rate), so $\Delta Y = abI = absY$. Rewrite it as the differential equation considering time $\mathrm{d}Y/\mathrm{d}t = abI = absY$, then its solution is $Y = Y_0 \mathrm{e}^{(abs)t}$. As long as the coefficient (abs) is regards as the interest rate r, there is no difference with Equation

(43). Therefore, regardless of whether the analysis of Harrod-Domar model is correct, the time path of its output is roughly the same as our conclusion.

The Solow model uses $Y = AK^{\alpha}L^{\beta}$ as the production function and regards A as a constant. Although the auxiliary assumptions $\Delta K = I$ and $I = sY$ are the same as those of the Harrod-Domar model, the output will finally stop increase in a limited time since the growth of output is constrained by dual factors of the "diminishing capital marginal contribution per capita" and "stock capital depreciation" [14].

Comparing Equations (41) and (34), it can be seen that the difference between the growth rate of the final state and the short-term is that there is less ($-\mathrm{d}\alpha/\alpha$) in Equation (41). The average number of $\mathrm{d}\alpha/\alpha$ in the United States from 1970 to 2016 is 0.0070, so before the Cobb-Douglas function evolves to the final state equation $Y = 2rK$, the real output growth rate $\mathrm{d}Y_r/Y_r$ for the removal of the inflation rate should be slightly reduced.

α of the United States increased from 0.2770 in 1970 to 0.3691 in 2016, and experienced an annual growth of about 0.00196 for 47 years. Assuming a linear increase at this rate, after another 71 years, α in the United States will reach 1/2. As the theoretical growth rate of α is decreasing, so in 2087 we may not see the actual value of α equal to 1/2.

Since α is not a variable that determines whether the marginal output is greater than zero or not, the growth of output is determined by the interest rate r in the production function $Y = rK/\alpha$ or $Y = 2rK$. The speed of output growth is not the production function itself, but on the production efficiency.

Adam Smith said in *An Inquiry into the Nature and Causes of the Wealth of Nations* that there was an "invisible hand" that led self-interest people promoted public interest [15], but Smith had never revealed the metaphor in the book. Some people say "invisible hand" is the relationship between supply and demand, some said that it is the market price, and some said that it is market competition, and so on. We believe that "invisible hand" means "market competition". Market competition raises the production efficiency and makes it necessary for producers to return the surplus generated by the increase of efficiency to the society.

During 1970-2016, the average of real interest rate in the United States r_r is 0.0229. Assuming that the number can last 200 years, the actual output Y_r will also increase at the average annual rate of 0.0229. Then after another 200 years in year 2216, the actual GDP will be 92.61 times the current GDP. According to the annual population growth rate 0.010 of the United States in 1970-2016, by 2216 the total population will be 7.32 times the current. At that time, the real per capita GDP in the United States would be 12.65 times that of 2016, or per capita GDP of 651,684 USD (real GDP in 2016 is 16,660 billion USD, 323,391 thousands population, 51,517 USD/person).

The question is with the income level of up to real 650,000 USD/person, will people be involved in the exhausting competition for higher income? Perhaps when it has not yet reached this figure, many people have already voted for the party who advocates for the universal welfare policy. Therefore, the terminator of economic growth is not the Cobb-Douglas production function, but at what level of income, those who believe in "market competition" will become social minorities.

5. Conclusions

The issues we are concerned with today are not much different from what Adam Smith was concerned about in his great work of two hundred years ago, but at that time most of the research methods were empirical and conjecture, and we now have to use strict analytical methods and data validation. Our model is derived from the Cobb-Douglas function, and only $r > 0$ will have $\mathrm{d}Y/Y > 0$, $r > 0$ is determined by the marginal state of real production and inflation, so we conclude that the equation of growth is endogenous, the cause of growth is exogenous.

No matter the growth of output is generated by the economic system revolution, the improvement of production management or the progress of science and technology, ultimately the result is the increase of production efficiency. These are in the model that the marginal output is greater than 0. So, regardless of

Table 2. Summary of macroeconomic model based on Cobb-Douglas function.

Name of model	Hypothesis or cause	Inference or conclusion	Note
Precondition	$Y = AK^{\alpha}L^{\beta}$	$r = \frac{\partial Y}{\partial K} = \frac{\alpha Y}{K}$ $w = \frac{\partial Y}{\partial L} = \frac{\beta Y}{L}$	Cobb-Douglas function
Fundamental equation, Investment decision [3]	$\frac{\Delta\alpha}{\alpha} = 0$ $\frac{\Delta r}{r} = 0$ $\Delta Y = \Delta C + I + \Delta D$	$\frac{\Delta Y}{Y} = r, \frac{\Delta K}{K} = r$ $\frac{I}{Y} = r(1-r)$ $\Delta K > I$	If separating core variables and fluctuating variables, then $\dot{Y} = \frac{\mathrm{d}Y}{Y} = \overline{\left(\frac{\mathrm{d}Y}{Y}\right)} + \widetilde{\left(\frac{\mathrm{d}Y}{Y}\right)} = \bar{\dot{Y}} + \tilde{\dot{Y}}$ then $\bar{\dot{Y}} = \bar{r}, \bar{\dot{K}} = \bar{r}$. Exactly, all equations that ignore fluctuation of variable should be expressed as such.
Business cycle equation [9]	$\frac{\mathrm{d}\alpha}{\alpha} = 0$ $L^{\beta} = K^{\alpha}$ $\frac{\mathrm{d}E}{E} = \frac{\mathrm{d}w_r}{w_r}$	$\frac{\mathrm{d}Y}{Y} = r + \frac{\mathrm{d}r}{r}$ $\frac{\mathrm{d}R_E}{R_E} = \left(1-\frac{\alpha}{\beta}\right)r_r + \frac{\mathrm{d}r_r}{r_r} - \frac{\mathrm{d}N}{N}$	The simplified symbol can be expressed as: $\dot{Y} = \frac{\mathrm{d}Y}{Y} = r + \frac{\mathrm{d}r}{r} = r + \dot{r}$ $\dot{R}_E = \left(1-\frac{\alpha}{\beta}\right)r_r + \dot{r}_r - \dot{N}$
Inflation equation [16] [17]	$M = aY$ $Y = PY_r$, $\dot{Y} = \dot{P} + \dot{Y}_r$ $\dot{V} = \dot{Y} - \dot{M}$ $\dot{V} = \theta\ddot{P}$	$\dot{P} - \theta\ddot{P} = \dot{M} - \dot{Y}_r$ $\dot{M} = \bar{\dot{M}} + \tilde{\dot{M}}$ $r_r = \bar{r}_r + \tilde{r}_r$ $\dot{P} = \bar{\dot{P}} + \tilde{\dot{P}}$ $\dot{Y}_r = r_r + \dot{r}_r$ (when $\dot{P} = 0$ and $\dot{P} = r_r$)	$\dot{P} = \frac{\mathrm{d}P}{P}$ $\ddot{P} = \frac{\mathrm{d}\dot{P}}{\dot{P}}$
The path and the ending state of economic growth [11]	$t \to \infty$ $\frac{r}{w} = \frac{\partial Y}{\partial K} \Big/ \frac{\partial Y}{\partial L} \to 1$ $\frac{\alpha}{\beta} \to 1$	$A = \frac{r^{\alpha}w^{\beta}}{\alpha^{\alpha}\beta^{\beta}}$ $\frac{K}{L} \to 1, A \to 2r, \frac{Y}{K+L} \to r$ $Y = 2rK$, $Y = Y_0\mathrm{e}^{rt}$	$\frac{A}{2} \ge \frac{Y}{K+L} \ge r$

whether the ending state of $Y = AK^{\alpha}L^{\beta}$ is $Y = 2rK$, as long as the marginal state of production is greater than 0, Adam Smith said "The Wealth of Nations" will continue to grow.

The 2008 financial crisis made the fabulous former Federal Reserve Chairman Alan Greenspan into a nude swimmer and humiliated the once brilliant traditional macroeconomic theory, but the textbooks still propagate those misleading theories. We abandon the traditional analytical framework, trying to use axiomatic methods to derive the various models of macroeconomic problems, thus forming a new system of macroeconomics with an interest rate as the core variable. Table 2 summarizes these models and helps us understand the link between economic growth and other macroeconomic problems.

Compared with the traditional theory, these models are not only very rigorous in theory, but also have the important practical significance: Based on these models, we will identify the characteristics and causes of the financial crisis, and make a contribution to the prediction or avoiding the next financial crisis.

References

[1] Lucas Jr., R.E. (1988) On the Mechanics of Economic Development. *Journal of Monetary Economics*, **22**, 3-42.

[2] Cass, D. (1965) Optimum Growth in an Aggregate Model of Capital Accumulation. *The Review of Economic Studies*, **32**, 233-240. https://doi.org/10.2307/2295827

[3] Zhan, M.A. and Zhan, Z. (2016) A Kind of Neither Keynesian nor Neoclassical Model (1): The Fundamental Equation. *Open Access Library Journal*, **3**, e3207. https://doi.org/10.4236/oalib.1103207

[4] Romer, P.M. (1986) Increasing Returns and Long-Run Growth. *Journal of Political Economy*, **94**, 1002-1037.

[5] Barro, R.J. (1990) Government Spending in a Simple Model of Endogenous Growth. *Journal of Political Economy*, **98**, 103-125. https://doi.org/10.1086/261726

[6] Rebelo, S.T. (1991) Long-Run Policy Analysis and Long-Run Growth. *Journal of Political Economy*, **99**, 500-521. https://doi.org/10.1086/261764

[7] Arrow, K.J. (1962) The Economic Implications of Learning by Doing. *The Review of Economic Studies*, **29**, 155-173. https://doi.org/10.2307/2295952

[8] Lucas Jr., R.E. (1990) Why Doesn't Capital Flow From Rich to Poor Countries, *American Economic Review* (*Papers and Proceedings*), **80**, 92-96.

[9] Zhan, M.A. and Zhan Z. (2016) A Kind of Neither Keynesian nor Neoclassical Model (2): The Business Cycle. *Open Access Library Journal*, **3**, e3215. https://doi.org/10.4236/oalib.1103215

[10] Gollop, F.M., Fraumeni, B.M. and Jorgenson, D.W. (1987) Productivity and U.S. Economic Growth. Harvard University Press, Cambridge, MA. (Reprinted, 1999)

[11] Zhan, M.A. and Zhan, Z. (2017) A Kind of Neither Keynesian nor Neoclassical Model (5): The Path of Economic Growth. *Open Access Library Journal*, **4**, e3525. https://doi.org/10.4236/oalib.1103525

[12] Harrod, R.F. (1939) An Essay in Dynamic Theory. *The Economic Journal*, **49**, 14-33. https://doi.org/10.2307/2225181

[13] Domar, E.D. (1946) Capital Expansion, Rate of Growth, and Employment. *Econometrica*, **14**, 137-147. https://doi.org/10.2307/1905364

[14] Solow, R.M. (1956) A Contribution to the Theory of Economic Growth. *The Quarterly Journal of Economics*, **70**, 65-94. https://doi.org/10.2307/1884513

[15] Smith, A. (1776) An Inquiry into the Nature and Causes of the Wealth of Nations. 5th Edition, Methuen Co. Ltd., London.

[16] Zhan, M.A. and Zhan, Z. (2017) A Kind of Neither Keynesian nor Neoclassical Model (3): The Inflation Equation. *Open Access Library Journal*, **4**, e3333. https://doi.org/10.4236/oalib.1103333

[17] Zhan, M.A. and Zhan, Z. (2017) A Kind of Neither Keynesian nor Neoclassical Model (4): The Nature of Philips Curve. *Open Access Library Journal*, **4**, e3388. https://doi.org/10.4236/oalib.1103388