



Backstepping Technique Based on Error

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Abstract

Backstepping technique usually adopts back step design to construct the Lyapunov function gradually, and then to design the corresponding virtue controller. The backstepping technique based on error also adopts back step design process, but the design of virtue controllers depends on the corresponding errors which are designed to satisfy some expected behaviors. Six different error equations are deduced by changing the results of the virtue controls arbitrarily while guaranteeing the system behaviors such as stability, and an example shows the effectiveness of these six versions. Simulated results illustrate that these six versions of backstepping technique based on error are effective.

Subject Areas

Dynamical System

Keywords

Backstepping Technique, Lyapunov Function, Error, Strictly Feedback Nonlinear Control System

1. Introduction

Backstepping design methodology is among the most important nonlinear control design techniques with numerous applications. It was first presented by P. V. Kokotovic and his coauthors, see [1] [2] [3] and references therein, and was formulated by Krstic *et al.* in [4], and was advanced in many papers and not limited in the following. Murat Arcaç *et al.* [5] presented robustification of backstepping design methodology by two versions: cancellation backstepping and L_GV -backstepping. In reference [6], about ten-year development of this recursive design was introduced. Dragan B. Dacic *et al.* [7] studied the use of backstepping design methodology in power integrator triangular systems. Dimitrios Karagiannis *et al.* [8] considered an alternative to adaptive backstep-

ping. Keng Peng Tee *et al.* [9] employed a barrier Lyapunov function for the control of output-constrained nonlinear systems.

In all the references mentioned above, the purpose of backstepping design methodology is the construction of various types of control Lyapunov functions: stable, adaptive, robust, etc. D. Swaroop *et al.* [10] introduced Dynamic Surface Control (DSC), which was similar to backstepping and multiple surface control methods, but with an important addition, one low pass filter was included in the design which ended the complexity arising due to the “explosion of terms” that had made backstepping methods difficult to implement in practice. In recent years, DSC has received a great deal attention, see [11] [12] and references therein.

A new redesign backstepping technique, backstepping technique based on error is presented in this paper, which also adopts backstepping design process, but doesn't construct the system, control Lyapunov function; the design of the virtue controller depends on the corresponding errors which are designed to satisfy some expected behaviors. The method is also more flexible than DSC. Based on some results of [13], we deduce six different error equations by changing the result of the virtue control law arbitrarily while guaranteeing the system behaviors such as stability.

In this paper, we call backstepping technique based on control Lyapunov functions as conventional backstepping, and call backstepping technique based on error as error backstepping. Section 2 presents the design and six results of error backstepping, in Section 3 an example shows the effectiveness of these six versions, and Section 4 concludes this paper.

2. Backstepping Technique Based on Error

Consider a usual strictly feedback nonlinear system as follows:

$$\begin{aligned} \dot{x}_i &= f_i(\bar{x}_i) + g_i(\bar{x}_i)x_{i+1} \\ \dot{x}_n &= f_n(\bar{x}_n) + g_n(\bar{x}_n)u \quad i \in \{1, \dots, n-1\} \\ y &= x_1 \end{aligned} \quad (1)$$

where $\bar{x}_i = [x_1, x_2, \dots, x_i]^T$, $f_i(\bar{x}_i) \rightarrow 0$, when $x_i \rightarrow 0$ ($x_i \in \mathfrak{R}$), $g_i(\bar{x}_i) \neq 0$, $i \in \{1, \dots, n\}$. The design objective is to make $y \rightarrow y_i$.

The backstepping design process is as follows.

Step 1: Consider the control goal $y = x_1 \rightarrow y_i$, take x_2 as the first virtual control and pretend that x_{2d} satisfies the following equation.

$$dx_1/dt = f_1(x_1) + g_1(x_1)x_{2d} \quad (2)$$

Define the first tracking error

$$e_1 = x_1 - x_{1d} = x_1 - y_i \quad (3)$$

Error e_1 is wanted to converge to zero exponentially, therefore select the desired behaviour to be $\dot{e}_1 = -\lambda_1 e_1$ x_{2d} is chosen to satisfy the required dynamic characteristic.

$$\dot{e}_1 = \dot{x}_1 - \dot{x}_{1d} = f_1(x_1) + g_1(x_1)x_{2d} - \dot{x}_{1d} = -\lambda_1 e_1 \quad (4)$$

Then the following equality corresponds to desired behaviour

$$x_{2d} = \frac{1}{g_1(x_1)}(-\lambda_1 e_1 - f_1(x_1) + \dot{x}_{1d}) \quad (5)$$

Step 2: but we cannot just choose x_2 to be x_{2d} , so we “step back” one integrator to the \dot{x}_2 equation. Choose x_3 as the second virtual control to solve the $x_2 \rightarrow x_{2d}$ tracking problem.

Introduce

$$\dot{x}_2 = f_2(\bar{x}_2) + g_2(\bar{x}_2)x_{3d} \quad (6)$$

Define the second tracking error:

$$e_2 = x_2 - x_{2d} \quad (7)$$

The error e_2 is also wanted to converge to zero exponentially, and select the desired behaviour to be:

$$\dot{e}_2 = -\lambda_2 e_2 \quad (8)$$

x_{3d} is chosen to satisfy the required dynamic characteristic.

$$\dot{e}_2 = \dot{x}_2 - \dot{x}_{2d} = f_2(\bar{x}_2) + g_2(\bar{x}_2)x_{3d} - \dot{x}_{2d} = -\lambda_2 e_2 \quad (9)$$

Then the following equality corresponds to desired behavior.

$$x_{3d} = \frac{1}{g_2(\bar{x}_2)}(-\lambda_2 e_2 - f_2(\bar{x}_2) + \dot{x}_{2d}) \quad (10)$$

Step i: Choose x_{i+1} as the i virtual control to solve the $x_i \rightarrow x_{id}$ tracking problem. Define the i tracking error

$$e_i = x_i - x_{id} \quad (11)$$

Select its time derivative to satisfy the following behaviour

$$\dot{e}_i = -\lambda_i e_i \quad (12)$$

Introduce

$$\dot{x}_i = f_i(\bar{x}_i) + g_i(\bar{x}_i)x_{(i+1)d} \quad (13)$$

Choose $x_{(i+1)d}$ to satisfy the required behaviour

$$\dot{e}_i = \dot{x}_i - \dot{x}_{id} = f_i(\bar{x}_i) + g_i(\bar{x}_i)x_{(i+1)d} - \dot{x}_{id} = -\lambda_i e_i \quad (14)$$

Then has

$$x_{(i+1)d} = \frac{(-\lambda_i e_i - f_i(\bar{x}_i) + \dot{x}_{id})}{g_i(\bar{x}_i)} \quad (15)$$

Step n: Choose u to solve the $x_n \rightarrow x_{nd}$ tracking problem. Define the n tracking error:

$$e_n = x_n - x_{nd} \quad (16)$$

Its derivative satisfies. $\dot{e}_n = -\lambda_n e_n$

In fact:

$$\dot{e}_n = \dot{x}_n - \dot{x}_{nd} = f_n(\bar{x}_n) + g_n(\bar{x}_n)u - \dot{x}_{nd} = -\lambda_n e_n \quad (17)$$

The last equality corresponds to what we are forcing.

$$u = \frac{(-\lambda_n e_n - f_n(\bar{x}_n) + \dot{x}_{nd})}{g_n(\bar{x}_n)} \quad (18)$$

The deduced error equation is:

$$\begin{aligned} \dot{e}_1 &= -\lambda_1 e_1 + g_1(x_1) e_2 \\ \dot{e}_2 &= -\lambda_2 e_2 + g_2(\bar{x}_2) e_3 \\ &\vdots \\ \dot{e}_{n-1} &= -\lambda_{n-1} e_{n-1} + g_{n-1}(\bar{x}_{n-1}) e_n \\ \dot{e}_n &= -\lambda_n e_n \end{aligned} \quad (19)$$

Defining $E = (e_1, e_2, \dots, e_n)^T$, then

$$\dot{E} = \begin{bmatrix} -\lambda_1 & g_1(x_1) & \cdots & 0 & 0 \\ 0 & -\lambda_2 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & -\lambda_{n-1} & g_{n-1}(\bar{x}_{n-1}) \\ 0 & 0 & \cdots & 0 & -\lambda_n \end{bmatrix} E \quad (20)$$

Proposition 2.1: When choose parameters

$$\lambda_i > 0 \quad (i = 1, 2, \dots, n) \quad \text{and} \quad g_i(\bar{x}_i) = c_i, i = 1, 2, \dots, n$$

where c_i is constant. It is obvious that errors e_1, \dots, e_n converge to origin exponentially, that is, $y \rightarrow y_i, x_i \rightarrow x_{id}, i = 1, 2, \dots, n$.

Proposition 2.2: When $g_i(\bar{x}_i) (i = 1, 2, \dots, n)$ is the function of \bar{x}_i , and when select appropriate parameters $\lambda_i > 0 (i = 1, 2, \dots, n)$, It can make errors e_1, \dots, e_n globally stable at origin, then $y \rightarrow y_i, x_i \rightarrow x_{id}, i = 1, 2, \dots, n$.

Proof: error Equation (19) is in short as $\dot{E} = AE$, where

$$A = \begin{bmatrix} -\lambda_1 & g_1(x_1) & \cdots & 0 & 0 \\ 0 & -\lambda_2 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & -\lambda_{n-1} & g_{n-1}(\bar{x}_{n-1}) \\ 0 & 0 & \cdots & 0 & -\lambda_n \end{bmatrix} \quad (21)$$

Introduce

$$V = \frac{1}{2} E^T E \quad (22)$$

Its time derivative is

$$\begin{aligned} \dot{V} &= -\sum_{i=1}^n \lambda_i e_i^2 + \sum_{i=2}^n g_{i-1}(\bar{x}_{i-1}) e_{i-1} e_i \\ &\leq -\sum_{i=1}^n \lambda_i e_i^2 + \frac{1}{2} \sum_{i=2}^n (g_{i-1}(\bar{x}_{i-1}) e_{i-1})^2 + \frac{1}{2} \sum_{i=2}^n e_i^2 \\ &= -\left(\lambda_1 - \frac{1}{2} g_1^2(x_1) \right) e_1^2 - \left(\lambda_n - \frac{1}{2} \right) e_n^2 - \frac{1}{2} \sum_{i=2}^n (2\lambda_i - (g_{i-1}(\bar{x}_{i-1}))^2 - 1) e_{i-1}^2 \end{aligned} \quad (23)$$

If parameters $\lambda_i (i=1,2,\dots,n)$ λ_i satisfy $\lambda_1 \geq \frac{1}{2}g_1^2(x_1)$, $\lambda_i \geq \frac{(g_{i-1}(\bar{x}_{i-1}))^2 + 1}{2}$, $i \in [2,\dots,n-1]$, $\lambda_n \geq \frac{1}{2}$, $x_i \in Q, Q \in \mathfrak{R}$, then $\dot{V} \leq 0$, so all the errors converge to origin globally.

End of proof.

Proposition 2.3: Assume the expression of virtual control (14) is changed into:

$$x_{(i+1)d} = \frac{(-\lambda_i e_i - f_i(\bar{x}_i) - g_{i-1}(\bar{x}_{i-1})e_{i-1} + \dot{x}_{id})}{g_i(\bar{x}_i)} \quad (24)$$

where $x_{n+1} = u$, when choose the parameters $\lambda_1, \lambda_2, \dots, \lambda_n > 0$ and $g_i(\bar{x}_i) (i=1,2,\dots,n)$ is the function of the corresponding states, errors e_1, \dots, e_n converge to origin globally, and $y \rightarrow y_i, x_i \rightarrow x_{id}$, $i=1,2,\dots,n$.

Proof: when virtual control (24) takes place (14) in recursive procedure, the new error equation is changed into

$$\begin{aligned} \dot{e}_1 &= -\lambda_1 e_1 + g_1(x_1)e_2 \\ \dot{e}_2 &= -g_1(x_1)e_1 - \lambda_2 e_2 + g_2(\bar{x}_2)e_3 \\ \dot{e}_3 &= -g_2(\bar{x}_2)e_2 - \lambda_3 e_3 + g_3(\bar{x}_3)e_4 \\ &\vdots \\ \dot{e}_{n-1} &= -g_{n-2}(\bar{x}_{n-2})e_{n-2} - \lambda_{n-1}e_{n-1} + g_{n-1}(\bar{x}_{n-1})e_n \\ \dot{e}_n &= -g_{n-1}(\bar{x}_{n-1})e_{n-1} - \lambda_n e_n \end{aligned}$$

or

$$\dot{E} = \begin{bmatrix} -\lambda_1 & g_1(x_1) & \cdots & 0 & 0 \\ 0 & -\lambda_2 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & -\lambda_{n-1} & g_{n-1}(\bar{x}_{n-1}) \\ 0 & 0 & \cdots & -g_{n-1}(\bar{x}_{n-1}) & -\lambda_n \end{bmatrix} E \quad (25)$$

It is the same as (2.27). Then all the errors converge to origin globally.

End of proof.

Proposition 3.4: Similarly assume the expression of virtual control (8) is changed into:

$$x_{(i+1)d} = \frac{(-\lambda_i e_i - f_i(\bar{x}_i) - c_{i-1}g_{i-1}(\bar{x}_{i-1})e_{i-1} + \dot{x}_{id})}{g_i(\bar{x}_i)} \quad (26)$$

where $i=1,2,\dots,n$, $x_{(n+1)d} = u$, and $g_i(\bar{x}_i) (i=1,2,\dots,n)$ is the function of the corresponding states, then the new error equation is:

$$\begin{aligned} \dot{e}_1 &= -\lambda_1 e_1 + g_1(x_1)e_2 \\ \dot{e}_2 &= -c_1 g_1(x_1)e_1 - \lambda_2 e_2 + g_2(\bar{x}_2)e_3 \\ \dot{e}_3 &= -c_2 g_2(\bar{x}_2)e_2 - \lambda_3 e_3 + g_3(\bar{x}_3)e_4 \\ &\vdots \\ \dot{e}_{n-1} &= -c_{n-2} g_{n-2}(\bar{x}_{n-2})e_{n-2} - \lambda_{n-1}e_{n-1} + g_{n-1}(\bar{x}_{n-1})e_n \\ \dot{e}_n &= -c_{n-1} g_{n-1}(\bar{x}_{n-1})e_{n-1} - \lambda_n e_n \end{aligned}$$

or

$$\dot{E} = \begin{bmatrix} -\lambda_1 & g_1(x_1) & \cdots & 0 & 0 \\ -c_1 g_1(x_1) & -\lambda_2 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & -\lambda_{n-1} & g_{n-1}(\bar{x}_{n-1}) \\ 0 & 0 & \cdots & -c_{n-1} g_{n-1}(\bar{x}_{n-1}) & -\lambda_n \end{bmatrix} E \quad (27)$$

When choose the parameters $\lambda_1, \lambda_2, \dots, \lambda_n > 0$ and $c_1, c_2, \dots, c_{n-1} > 0$, errors e_1, \dots, e_n are globally asymptotically stable at origin, and $y \rightarrow y_i, x_i \rightarrow x_{id}, i = 1, 2, \dots, n$.

Proof:

Defining

$$Y = \Lambda^{-1} E \quad (28)$$

$$\tilde{g}_{i-1}(\bar{x}_{i-1}) = \sqrt{c_{i-1}} g_{i-1}(\bar{x}_{i-1})$$

where

$$\Lambda = \text{diag} \{d_1, \dots, d_n\} \quad i = 2, \dots, n \quad (29)$$

$$d_1 = 1, d_2 = \sqrt{c_1}, d_i = \sqrt{c_{i-1}} d_{i-1}$$

Then

$$\dot{Y} = \begin{bmatrix} -\lambda_1 & \tilde{g}_1(x_1) & \cdots & 0 & 0 \\ -\tilde{g}_1(x_1) & -\lambda_2 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & -\lambda_{n-1} & \tilde{g}_{n-1}(\bar{x}_{n-1}) \\ 0 & 0 & \cdots & -\tilde{g}_{n-1}(\bar{x}_{n-1}) & -\lambda_n \end{bmatrix} Y \quad (30)$$

Introduce a positive definite quadratic function

$$V = \frac{1}{2} Y^T Y \quad (31)$$

It can be obtained

$$\dot{V} = -\sum_{i=1}^n \lambda_i y_i^2 \quad (32)$$

Then Y converge to origin globally, so errors are globally asymptotically stable at origin and $y \rightarrow y_i, x_i \rightarrow x_{id}, i = 1, 2, \dots, n$.

End of proof.

Proposition 2.5: The virtual control can also be choose as

$$x_{(i+1)d} = \frac{(-\lambda_i e_i - f_i(\bar{x}_i) - c_{i-1}(t) g_{i-1}(\bar{x}_{i-1}) e_{i-1} + \dot{x}_{id})}{g_i(\bar{x}_i)} \quad (33)$$

where $i = 1, 2, \dots, n$, $x_{(n+1)d} = u$, and $g_i(\bar{x}_i) (i = 1, 2, \dots, n)$ is the function of the corresponding states, then the new error equation is:

$$\begin{aligned}
\dot{e}_1 &= -\lambda_1 e_1 + g_1(x_1) e_2 \\
\dot{e}_2 &= -c_1(t) g_1(x_1) e_1 - \lambda_2 e_2 + g_2(\bar{x}_2) e_3 \\
\dot{e}_3 &= -c_2(t) g_2(\bar{x}_2) e_2 - \lambda_3 e_3 + g_3(\bar{x}_3) e_4 \\
&\vdots \\
\dot{e}_{n-1} &= -c_{n-2}(t) g_{n-2}(\bar{x}_{n-2}) e_{n-2} - \lambda_{n-1} e_{n-1} + g_{n-1}(\bar{x}_{n-1}) e_n \\
\dot{e}_n &= -c_{n-1}(t) g_{n-1}(\bar{x}_{n-1}) e_{n-1} - \lambda_n e_n
\end{aligned}$$

or

$$\dot{E} = AE = \begin{bmatrix} -\lambda_1 & g_1(x_1) & \cdots & 0 & 0 \\ -c_1(t) g_1(x_1) & -\lambda_2 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & -\lambda_{n-1} & g_{n-1}(\bar{x}_{n-1}) \\ 0 & 0 & \cdots & -c_{n-1}(t) g_{n-1}(\bar{x}_{n-1}) & -\lambda_n \end{bmatrix} E \quad (34)$$

When choose parameters $\lambda_1, \lambda_2, \dots, \lambda_n > 0$, $c_{\max} > c_i(t) > c_{\min}$ ($i=1, 2, \dots, n-1$) and the following inequality is satisfied

$$\frac{d}{dt} \left(\prod_{i=1}^j c_i(t) \right) + \lambda_j \prod_{i=1}^j c_i(t) > 0 \quad (35)$$

Errors e_1, \dots, e_n are globally asymptotically stable at origin, and $y \rightarrow y_i, x_i \rightarrow x_{id}, i=1, 2, \dots, n$.

Proof:

Define Λ as follows:

$$\begin{aligned}
\Lambda &= \text{diag} \{d_1, \dots, d_n\} \\
d_1 &= 1, d_2 = \sqrt{c_1(t)}, d_i = \sqrt{c_{i-1}(t)} d_{i-1} \quad i=2, \dots, n
\end{aligned} \quad (36)$$

Then it can be obtained

$$A = \Lambda A_1 \Lambda^{-1} \quad (37)$$

where

$$A_1 = \begin{bmatrix} -\lambda_1 & \tilde{g}_1 & \cdots & 0 & 0 \\ -\tilde{g}_1 & -\lambda_2 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & -\lambda_{n-1} & \tilde{g}_{n-1} \\ 0 & 0 & \cdots & -\tilde{g}_{n-1} & -\lambda_n \end{bmatrix} \quad (38)$$

$$\tilde{g}_{i-1} = \sqrt{c_{i-1}(t)} g_{i-1}(\bar{x}_{i-1}), i=2, \dots, n \quad (39)$$

Introduce a positive definite function $V = E^T P E$, where P is chosen as $P = \Lambda^{-2}$, then $V \geq \alpha E^T E$, where $\alpha = \min \{1, c_{\min}^{-2(n-1)}\}$, then the time derivative of V is

$$\begin{aligned}
\dot{V} &= E^T (PA + A^T P + \dot{P}) E \\
&= E^T (\Lambda^{-2} \Lambda A_1 \Lambda^{-1} + \Lambda^{-1} A_1^T \Lambda \Lambda^{-2}) E + E^T \dot{P} E \\
&= E^T \Lambda^{-1} (A_1 + A_1^T) \Lambda^{-1} E + E^T \dot{P} E
\end{aligned} \quad (40)$$

Because

$$A_1 + A_1^T = \Lambda_1 = \text{diag}\{-2\lambda_1, \dots, -2\lambda_n\} \quad (41)$$

So

$$\dot{V} = E^T \Lambda^{-1} \Lambda_1 \Lambda^{-1} E + E^T \dot{P} E = E^T (\Lambda_1 P + \dot{P}) E \quad (42)$$

It is obvious that $\Lambda_1 P + \dot{P}$ is a diagonal matrix, and the i diagonal unit is $-2\lambda_i d_i^{-2} + \frac{d(d_i^{-2})}{dt}$, from (35) and (36), we deduced that

$$-2\lambda_i d_i^{-2} + \frac{d(d_i^{-2})}{dt} < 0, i = 1, \dots, n \quad (43)$$

Substituting (42) into (43), it can be deduced that \dot{V} is negative definite, so errors e_1, \dots, e_n converge to origin globally.

End of proof.

Proposition 2.6: The virtual control can also be choose as

$$x_{(i+1)d} = \frac{(-\lambda_i e_i - f_i(\bar{x}_i) - c_{i-1}(\bar{x}_n) g_{i-1}(\bar{x}_{i-1}) + \dot{x}_{id})}{g_i(\bar{x}_i)} \quad (44)$$

where $i = 1, 2, \dots, n$, $x_{(n+1)d} = u$, and $g_i(\bar{x}_i) (i = 1, 2, \dots, n)$ is the function of the corresponding states, then the new error equation is:

$$\begin{aligned} \dot{e}_1 &= -\lambda_1 e_1 + g_1(x_1) e_2 \\ \dot{e}_2 &= -c_1(\bar{x}_n) g_1(x_1) e_1 - \lambda_2 e_2 + g_2(\bar{x}_2) e_3 \\ \dot{e}_3 &= -c_2(\bar{x}_n) g_2(\bar{x}_2) e_2 - \lambda_3 e_3 + g_3(\bar{x}_3) e_4 \\ &\vdots \\ \dot{e}_{n-1} &= -c_{n-2}(\bar{x}_n) g_{n-2}(\bar{x}_{n-2}) e_{n-2} - \lambda_{n-1} e_{n-1} + g_{n-1}(\bar{x}_{n-1}) e_n \\ \dot{e}_n &= -c_{n-1}(\bar{x}_n) g_{n-1}(\bar{x}_{n-1}) e_{n-1} - \lambda_n e_n \end{aligned}$$

or

$$\dot{E} = \begin{bmatrix} -\lambda_1 & g_1(x_1) & \cdots & 0 & 0 \\ -c_1(\bar{x}_n) g_1(x_1) & -\lambda_2 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & -\lambda_{n-1} & g_{n-1}(\bar{x}_{n-1}) \\ 0 & 0 & \cdots & -c_{n-1}(\bar{x}_n) g_{n-1}(\bar{x}_{n-1}) & -\lambda_n \end{bmatrix} E \quad (45)$$

When choose parameters $\lambda_1, \lambda_2, \dots, \lambda_n > 0$ $c_{\max} > c_i(X) > c_{\min} (i = 1, 2, \dots, n-1)$ and the following inequality is satisfied

$$\frac{d}{dt} \left(\prod_{i=1}^j c_i(X) \right) + \lambda_j \prod_{i=1}^j c_i(X) > 0 \quad (46)$$

Errors e_1, \dots, e_n are globally asymptotically stable at origin, and $y \rightarrow y_i, x_i \rightarrow x_{id}, i = 1, 2, \dots, n$.

Proof: it is similar to the proof of Proposition 2.5.

3. Numerical Simulation

Consider the following two-order system:

$$\begin{cases} dx_1/dt = x_1^3 + g(x_1)x_2 \\ dx_2/dt = u \\ y = x_1 \end{cases} \quad (47)$$

The control objective is to design a state feedback control to asymptotically stabilize the origin.

We adopt backstepping technique based on error to design control law. The calculated results are presented in **Table 1** by choosing parameters $\lambda_1 = 2, \lambda_2 = 3$. If the initial conditions are $x_1(0) = 0.3, x_2(0) = 1$, the simulated results are shown in **Figure 1**.

As it is shown in **Figure 1** the transients of state variables x_1, x_2 and error variables e_1, e_2 are stable, they get to origin in finite time. It is also shown that when the system structure or the control law is simple the transients of state variables x_1, x_2 and e_1, e_2 converge to origin perfectly.

Table 1. Calculated results.

	Virtual control	Control law	error equation
$g(x_1) = 1$	$x_{2d} = -2x_1 - x_1^3$	$u = u_1 = -3(x_2 + 2x_1 + x_1^3) + (2 + 3x_1^2)(x_1^3 + x_2)$	$\dot{e}_1 = -2e_1 + e_2$ $\dot{e}_2 = -3e_2$
$g(x_1) = \exp(x_1)$	$x_{2d} = \frac{-2x_1 - x_1^3}{\exp(x_1)}$	$u = u_2 = -3(x_2 + 2x_1 + x_1^3) + \frac{(2 + 3x_1^2) - (2x_1 + x_1^3)}{\exp(x_1)}(x_1^3 + x_2)$	$\dot{e}_1 = -2e_1 + \exp(x_1)e_2$ $\dot{e}_2 = -3e_2$
$g(x_1) = \exp(x_1)$	$x_{2d} = \frac{-2x_1 - x_1^3}{\exp(x_1)}$	$u = u_3 = -\lambda_2 e_2 - g(x_1)e_1 + \dot{x}_{2d}$ $= -3(x_2 + 2x_1 + x_1^3) - \exp(x_1)e_1$ $+ \frac{(2 + 3x_1^2) - (2x_1 + x_1^3)}{\exp(x_1)}(x_1^3 + x_2)$	$\dot{e}_1 = -2e_1 + \exp(x_1)e_2$ $\dot{e}_2 = -\exp(x_1)e_1 - 3e_2$
$g(x_1) = \exp(x_1)$ $c = 3$	$x_{2d} = \frac{-2x_1 - x_1^3}{\exp(x_1)}$	$u = u_4 = -\lambda_2 e_2 - cg(x_1)e_1 + \dot{x}_{2d}$ $= -3(x_2 + 2x_1 + x_1^3) - 3\exp(x_1)e_1$ $- \frac{(-(2 + 3x_1^2)) - (-2x_1 - x_1^3)}{\exp(x_1)}(x_1^3 + x_2)$	$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \end{bmatrix} = \begin{bmatrix} -2 & \exp(x_1) \\ -3\exp(x_1) & -3 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$
$g(x_1) = \exp(x_1)$ $c(t) = \exp(-t)$	$x_{2d} = \frac{-2x_1 - x_1^3}{\exp(x_1)}$	$u = u_5 = -\lambda_2 e_2 - c(t)g(x_1)e_1 + \dot{x}_{2d}$ $= -3(x_2 + 2x_1 + x_1^3) - \exp(-t)\exp(x_1)e_1$ $- \frac{(-(2 + 3x_1^2)) - (-2x_1 - x_1^3)}{\exp(x_1)}(x_1^3 + x_2)$	$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \end{bmatrix} = \begin{bmatrix} -2 & \exp(x_1) \\ -\exp(-t)\exp(x_1) & -3 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$
$g(x_1) = \exp(x_1)$ $c(t) = \exp(-3x_1/2)$	$x_{2d} = \frac{-2x_1 - x_1^3}{\exp(x_1)}$	$u = u_6 = -\lambda_2 e_2 - c(x_1)g(x_1)e_1 + \dot{x}_{2d}$ $= -3(x_2 + 2x_1 + x_1^3) - \exp(-x_1/2)e_1$ $- \frac{(-(2 + 3x_1^2)) - (-2x_1 - x_1^3)}{\exp(x_1)}(x_1^3 + x_2)$	$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \end{bmatrix} = \begin{bmatrix} -2 & \exp(x_1) \\ -\exp(-x_1/2) & -3 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$

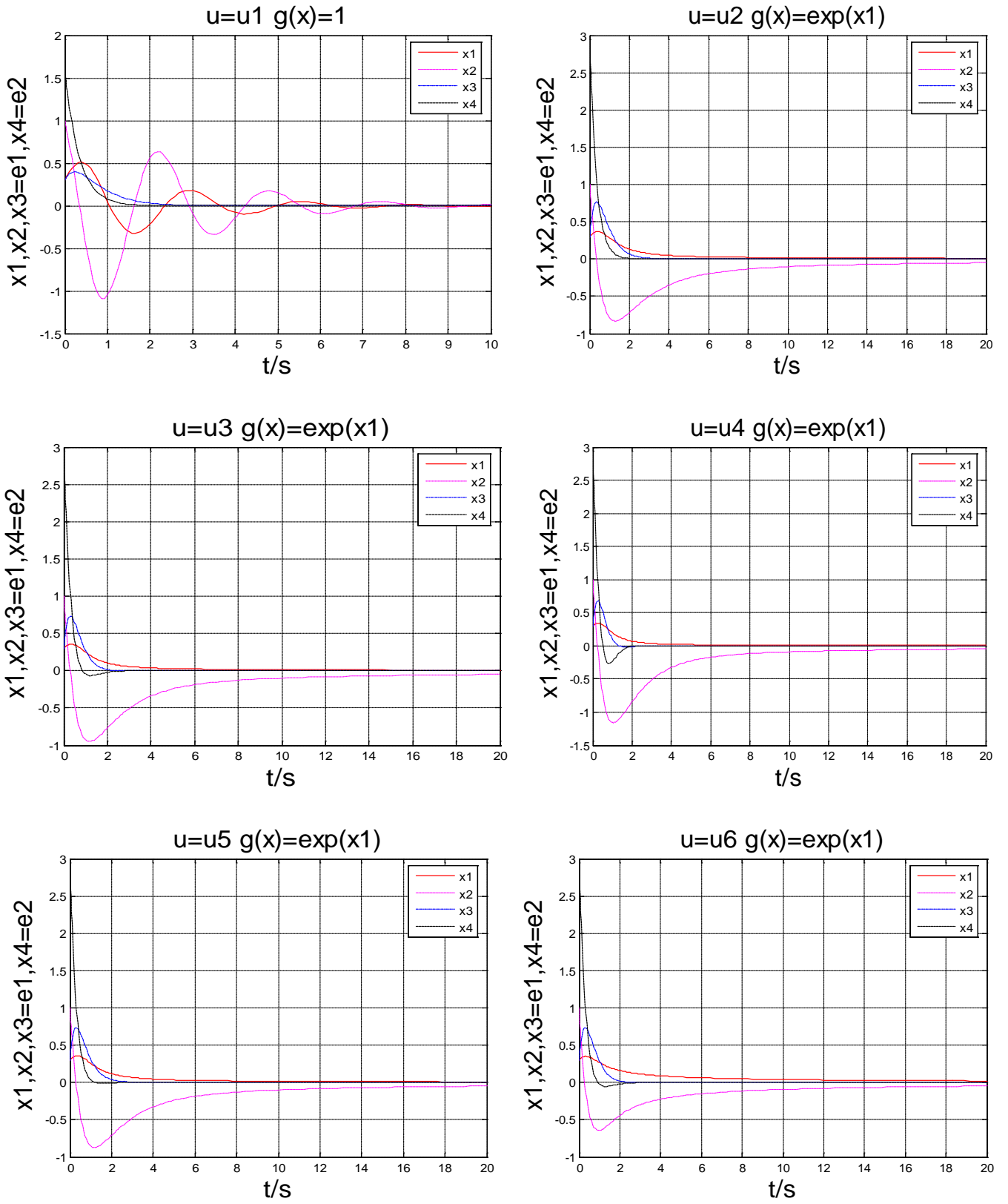


Figure 1. Simulated results.

4. Conclusion

The backstepping technique based on error is the expansion of the backstepping technique, it adopts backstepping design process, but the design of the virtue controller depends on the corresponding errors which are designed to satisfy some expected behaviors. This method makes the design systematical and structural, and it can change the result of the virtue control law arbitrarily in six forms while guaranteeing the system stability. The method can be used for both stabilization control problems and tracking control problems. Subjects of future research include the discussions of systems that contain uncertain terms, unknown parameters or unmeasured signals.

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