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Rumor Spreading of a *SICS* Model on Complex Social Networks with Counter Mechanism

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Abstract

The rumor spreading has been widely studied by scholars. However, there exist some people who will persuade infected individuals to resist and counterattack the rumor propagation in our social life. In this paper, a new *SICS* (susceptible-infected-counter-susceptible) rumor spreading model with counter mechanism on complex social networks is presented. Using the mean-field theory the spreading dynamics of the rumor is studied in detail. We obtain the basic reproductive number ρ and equilibriums. The basic reproductive number is correlated to the network topology and the influence of the counter mechanism. When $\rho < 1$, the rumor-free equilibrium is globally asymptotically stable, and when $\rho > 1$, the positive equilibrium is permanent. Some interesting patterns of rumor spreading involved with counter force have been revealed. Finally, numerical simulations have been given to demonstrate the effectiveness of the theoretical analysis.

Keywords

Rumor Spreading Model, Complex Social Networks, Counter Mechanism, Stability, Permanence

Subject Areas: **Network Modeling and Simulation**

1. Introduction

Nowadays, more and more SNS (Social Networking Services) networks are emerging in our social life, such as Facebook, WeChat, LinkedIn and so on, which are seemingly like cobwebs to connect people from different places. With the rapid increase of the number of SNS users, rumor will be quickly into people's horizons. Each coin has its two sides, as the rumors spread on the impact of our social lives. Sometimes, the rumor spreading

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may play a positive role, for instance, we can let more people to concern about something and take pertinent precaution measures by utilizing the rapid and efficient characteristic of rumor spreading [1] [2]. However, most rumors induce public panic, social disarray and severe economic loss, etc. [3] [4]. Therefore, it is very important to investigate the mechanism of rumor spreading and how to effectively control the rumor.

Rumor can be viewed as an "infection of the mind", and its spreading shows an interesting similarity to the epidemic spreading [5]-[9]. Daley and Kendal [5] first proposed the classic DK model of rumor spreading. Since then, most of the studies are based on DK model [10]-[17]. In order to overcome the weaknesses of DK model, more and more researchers consider the topological characteristics of underlying networks that they have started to study the problems of rumor spreading on complex networks [15]-[20]. Nekovee and Moreno *et al.* [16] derived a conclusion that scale-free social networks were prone to the spreading of rumors. In Ref. [17], the authors found that the degree distribution influenced directly the final rumor size. Recently, researchers [18]-[20] started to take full into account of the role of human behaviors and different mechanisms in the rumor spreading. Zhao *et al.* [18] presented a novel model by introducing the forget mechanism. Wang *et al.* [19] presented a novel *SIR* model by introducing the trust mechanism between the ignorant nodes and the spreader nodes. Han *et al.* [20] presented a novel model based on the heat energy theory to analyze the mechanisms of rumor propagation on social networks.

However, most of the previous models didn't consider that people may not agree with the rumor and counterattack it strongly. Based on some realistic perspectives, different people may have different views to the rumor on social networks. Some people may be in conflict with their beliefs when they hear rumor. They will persuade infected individuals to resist and counterattack the rumor propagation. In order to study this phenomenon, we present a *SICS* (susceptible-infected-counter-susceptible) rumor spreading model with counter mechanism on complex social networks to explain it. Obviously, the counter mechanism can change the contacts among people, *i.e.* network topology structure. Within the counter mechanism of the *SICS* model, when an infected individual contacts a counter individual, it may become a counter individual with a certain probability.

The rest of this paper is organized as follows. In Section 2, we present a *SICS* rumor spreading model and derive the corresponding mean-field equations to describe the dynamics of the model. In Section 3, the basic reproductive number obtained at first. Then we analyze the globally asymptotic stability of rumor-free equilibrium and the permanence of the rumor in detail. Simulation results of the proposed model are shown in Section 4. Finally, we conclude the paper in Section 5.

2. Model Formulation

As mentioned earlier, we present a *SICS* rumor spreading model. The population is divided into three classes: susceptible individuals who have ambiguous attitude about the rumor; infected individuals who believe and spread it actively; counter individuals who reject the rumor, refute the rumor and persuade neighbors don't believe in it. Taking into account the heterogeneity induced by the presence of vertices with different connectivities, let $S_k(t), I_k(t), C_k(t)$ be the densities of susceptible, infected and counter individuals of connectivity *k* at time *t*, respectively.

The *SICS* model has the flow diagram given in **Figure 1**. In the course of rumor spreading, a susceptible individual is infected with probability β_1 if it is connected to an infected individual. When a counter individual

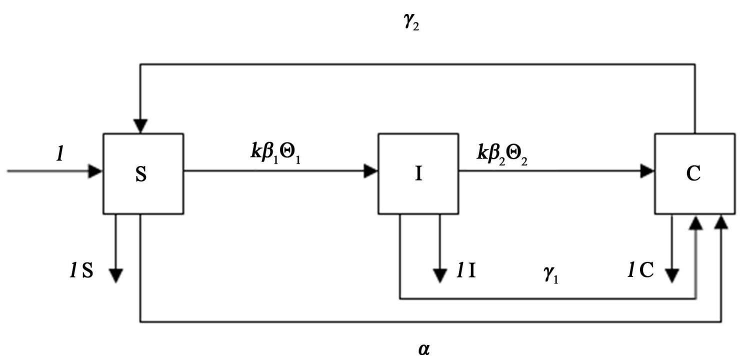

Figure 1. The flow diagram of the *SICS* model.

contacts an infected individual, the counter individual can persuade infected individual to resist and counterattack the rumor, so the infected individual becomes a counter node with probability β_2. A susceptible individual transform into a counter individual with probability α. Due to some own reason, an infected individual turns into a counter individual with probability γ_1. However, some counter individuals, due to loss of counterattack ability, join the susceptible individuals again, *i.e.*, moving back to susceptible state, with probability γ_2. We assume that the immigration rate and emigration rate are both constant l in the spreading process of rumor. All recruitment is into the susceptible class.

Thus, the dynamic mean-field reaction rate equations can be written as

$$\begin{cases} \dfrac{\mathrm{d}S_k(t)}{\mathrm{d}t} = l + \gamma_2 C_k(t) - lS_k(t) - k\beta_1\Theta_1(t)S_k(t) - \alpha S_k(t) \\ \dfrac{\mathrm{d}I_k(t)}{\mathrm{d}t} = k\beta_1\Theta_1(t)S_k(t) - k\beta_2\Theta_2(t)I_k(t) - \gamma_1 I_k(t) - lI_k(t) \\ \dfrac{\mathrm{d}C_k(t)}{\mathrm{d}t} = \alpha S_k(t) + k\beta_2\Theta_2(t)I_k(t) + \gamma_1 I_k(t) - \gamma_2 C_k(t) - lC_k(t) \end{cases} \tag{1}$$

The probability $\Theta_1(t)$ describes a link pointing to an infected individual,

$$\Theta_1(t) = \frac{\sum_k kP(k)I_k(t)}{\sum_k sP(s)} = \frac{1}{\langle k\rangle}\sum_k kP(k)I_k(t) \tag{2}$$

the probability $\Theta_2(t)$ describes a link pointing to a counter individual which satisfies the relation

$$\Theta_2(t) = \frac{\sum_k kP(k)C_k(t)}{\sum_s sP(s)} = \frac{1}{\langle k\rangle}\sum_k kP(k)C_k(t) \tag{3}$$

where $\langle k\rangle$ is the average degree within the network. And $I(t) = \sum_k P(k)I_k(t)$ is the density of infected individuals in the whole network, $C(t) = \sum_k P(k)C_k(t)$ is the density of counter individuals in the whole network, $P(k)$ is the connectivity distribution.

3. Stability Analysis

In this section, we present an analytic solution to the deterministic equations describing the dynamic of the (*SICS*) rumor spreading process.

Theorem 1. Let. $\rho = \dfrac{\beta_1(\gamma_2 + l) - \beta_2\alpha}{(\gamma_1 + l)(\gamma_2 + l + \alpha)} \cdot \dfrac{\langle k^2\rangle}{\langle k\rangle}$. There always exists a rumor-free equilibrium $E_0\left(\dfrac{\gamma_2 + l}{\gamma_2 + l + \alpha}, 0, \dfrac{\alpha}{\gamma_2 + l + \alpha}\right)$ and when $\rho > 1$, then system (1) has a positive equilibrium solution $E_+\left(S_k^\infty, I_k^\infty, C_k^\infty\right)$.

Proof. To get the equilibrium solution $E_+\left(S_k^\infty, I_k^\infty, C_k^\infty\right)$, we need to make the right side of system (1) equal to zero. Then the equilibrium $E_+\left(S_k^\infty, I_k^\infty, C_k^\infty\right)$ should satisfy

$$\begin{cases} l + \gamma_2 C_k^\infty - lS_k^\infty - k\beta_1\Theta_1^\infty S_k^\infty - \alpha S_k^\infty = 0 \\ k\beta_1\Theta_1^\infty S_k^\infty - k\beta_2\Theta_2^\infty I_k^\infty - \gamma_1 I_k^\infty - lI_k^\infty = 0 \\ \alpha S_k^\infty + k\beta_2\Theta_2^\infty I_k^\infty + \gamma_1 I_k^\infty - \gamma_2 C_k^\infty - lC_k^\infty = 0 \end{cases} \tag{4}$$

where $\Theta_1^\infty = \dfrac{1}{\langle k\rangle}\sum_k kP(k)I_k^\infty$, $\Theta_2^\infty = \dfrac{1}{\langle k\rangle}\sum_k kP(k)C_k^\infty$, one has

$$\begin{cases} S_k^\infty = \dfrac{\left(k\beta_2\Theta_2^\infty+\gamma_1+l\right)}{k\beta_1\Theta_1^\infty} I_k^\infty \\ C_k^\infty = \dfrac{k\beta_1\Theta_1^\infty\left(k\beta_2\Theta_2^\infty+\gamma_1\right)+\alpha\left(k\beta_2\Theta_2^\infty+\gamma_1+l\right)}{k\beta_1\Theta_1^\infty\left(\gamma_2+l\right)} I_k^\infty \end{cases} \tag{5}$$

According to the following normalization condition for all k:

$$S_k^\infty + I_k^\infty + C_k^\infty = 1.$$

We can obtain:

$$I_k^\infty = \frac{k\beta_1\Theta_1^\infty\left(\gamma_2+l\right)}{k\beta_1\Theta_1^\infty\left(k\beta_2\Theta_2^\infty+\gamma_1+\gamma_2+l\right)+\left(k\beta_2\Theta_2^\infty+\gamma_1+l\right)\left(\alpha+\gamma_2+l\right)} \tag{6}$$

$$C_k^\infty = \frac{k\beta_1\Theta_1^\infty\left(k\beta_2\Theta_2^\infty+\gamma_1\right)+\alpha\left(k\beta_2\Theta_2^\infty+\gamma_1+l\right)}{k\beta_1\Theta_1^\infty\left(k\beta_2\Theta_2^\infty+\gamma_1+\gamma_2+l\right)+\left(k\beta_2\Theta_2^\infty+\gamma_1+l\right)\left(\alpha+\gamma_2+l\right)} \tag{7}$$

Inserting Equation (6) into Equation (2), we obtain the following equation

$$\Theta_1^\infty = \frac{\sum_k k^2P(k)}{\langle k\rangle}\cdot\frac{\beta_1\Theta_1^\infty\left(\gamma_2+l\right)}{k\beta_1\Theta_1^\infty\left(k\beta_2\Theta_2^\infty+\gamma_1+\gamma_2+l\right)+\left(k\beta_2\Theta_2^\infty+\gamma_1+l\right)\left(\alpha+\gamma_2+l\right)} \tag{8}$$

Inserting Equation (7) into Equation (3), we obtain the following equation

$$\Theta_2^\infty = \frac{\sum_k k^2P(k)}{\langle k\rangle}\cdot\frac{\beta_1\Theta_1^\infty\left(k\beta_2\Theta_2^\infty+\gamma_1\right)+\left(k\beta_2\Theta_2^\infty+\gamma_1+l\right)\alpha/k}{k\beta_1\Theta_1^\infty\left(k\beta_2\Theta_2^\infty+\gamma_1+\gamma_2+l\right)+\left(k\beta_2\Theta_2^\infty+\gamma_1+l\right)\left(\alpha+\gamma_2+l\right)} \tag{9}$$

Equation (9) divided by Equation (8), we obtain the following equation

$$\Theta_2^\infty = \frac{\gamma_1k\beta_1\Theta_1^\infty+\alpha\left(\gamma_1+l\right)}{k\beta_1\left(\gamma_2+l\right)-k\beta_2\left(k\beta_1\Theta_1^\infty+\alpha\right)} \tag{10}$$

Inserting Equation (10) into Equation (8), we can obtain

$$\Theta_1^\infty = \frac{\sum_k k^2P(k)}{\langle k\rangle}\cdot\frac{\beta_1\Theta_1^\infty\left(\gamma_2+l\right)}{\left(\dfrac{\beta_2\left(\gamma_1k\beta_1\Theta_1^\infty+\alpha\left(\gamma_1+l\right)\right)}{\beta_1\left(\gamma_2+l\right)-\beta_2\left(k\beta_1\Theta_1^\infty+\alpha\right)}+\gamma_1+l\right)\left(k\beta_1\Theta_1^\infty+\alpha+\gamma_2+l\right)+\gamma_2k\beta_1\Theta_1^\infty} \triangleq f\left(\Theta_1^\infty\right). \tag{11}$$

Obviously, $\Theta_1^\infty = 0$ is a solution of Equation (11), *i.e.*, $f\left(\Theta_1^\infty\right)=0$. To ensure Equation (11) have a non-trivial solution, *i.e.* $0<\Theta_1^\infty\le 1$, the following conditions must be satisfied

$$\left.\frac{\mathrm{d}f\left(\Theta_1^\infty\right)}{\mathrm{d}\Theta_1^\infty}\right|_{\Theta_1^\infty=0} > 1 \text{ and } f\left(\Theta_1^\infty\right)\le 1.$$

We can obtain the basic reproductive number

$$\rho = \frac{\beta_1\left(\gamma_2+l\right)-\beta_2\alpha}{\left(\gamma_1+l\right)\left(\gamma_2+l+\alpha\right)}\cdot\frac{\langle k^2\rangle}{\langle k\rangle} > 1 \tag{12}$$

So, a nontrivial solution exists if and only if $\rho > 1$.

Substitute the nontrivial solution of (11) into (6), we can get I_k^∞. By (5) and (6), we can easily obtain

$$0 < S_k^\infty < 1, 0 < I_k^\infty < 1, 0 < C_k^\infty < 1.$$

Therefore, the positive equilibrium $E_+\left(S_k^\infty, I_k^\infty, C_k^\infty\right)$ is well-defined. Hence, when $\rho > 1$, one and only one positive equilibrium $E_+\left(S_k^\infty, I_k^\infty, C_k^\infty\right)$ of system (1) exists. This completes the proof.

Remark. The basic reproductive number is obtained by Equation (12), which depends on the fluctuations of the degree distribution and the influence of counter mechanism. The β_2 can affect the basic reproductive number.

Theorem 2. If $\rho < 1$, the rumor-free equilibrium E_0 of the system (1) is globally asymptotically stable.

Proof. We rewrite the system (1) as

$$\begin{cases} \dfrac{\mathrm{d}I_k(t)}{\mathrm{d}t} = \dfrac{k\beta_1}{\langle k\rangle}\left(1 - I_k(t) - C_k(t)\right)\sum_k kP(k)I_k(t) - \dfrac{k\beta_2}{\langle k\rangle}I_k(t)\sum_k kP(k)C_k(t) - (\gamma_1 + l)I_k(t) \\ \dfrac{\mathrm{d}C_k(t)}{\mathrm{d}t} = \alpha\left(1 - I_k(t) - C_k(t)\right) + \dfrac{k\beta_2}{\langle k\rangle}I_k(t)\sum_k kP(k)C_k(t) + \gamma_1 I_k(t) - (\gamma_2 + l)C_k(t) \end{cases} \tag{13}$$

The Jacobian matrix of system (13) at $\left\{\left(0, \dfrac{\alpha}{\gamma_2 + l + \alpha}\right)\right\}$ is a $2k_{\max} \times 2k_{\max}$ as follows

$$J = \begin{bmatrix} A_1 & B_{12} & B_{13} & \cdots & B_{1k_{\max}} \\ B_{21} & A_2 & B_{23} & \cdots & B_{2k_{\max}} \\ \vdots & \vdots & \ddots & & \vdots \\ B_{k_{\max}1} & B_{k_{\max}2} & B_{k_{\max}3} & \cdots & A_{k_{\max}} \end{bmatrix},$$

where

$$A_j = \begin{bmatrix} \dfrac{j^2P(j)}{\langle k\rangle(\gamma_2 + l + \alpha)}\left[\beta_1(\gamma_2 + l) - \beta_2\alpha\right] - \gamma_1 - l & 0 \\ \dfrac{j^2P(j)}{\langle k\rangle(\gamma_2 + l + \alpha)}\beta_2\alpha - \alpha + \gamma_1 & -\gamma_2 - l - \alpha \end{bmatrix},$$

$$B_{ij} = \begin{bmatrix} \dfrac{ijP(j)}{\langle k\rangle(\gamma_2 + l + \alpha)}\left[\beta_1(\gamma_2 + l) - \beta_2\alpha\right] & 0 \\ \dfrac{ijP(j)}{\langle k\rangle(\gamma_2 + l + \alpha)}\beta_2\alpha & 0 \end{bmatrix}.$$

By mathematical induction method, the characteristic equation can be calculated as follows

$$\left(\lambda + \gamma_2 + l + \alpha\right)^{k_{\max}-1}\left(\lambda + \gamma_1 + l - \frac{\left(\beta_1(\gamma_2 + l) - \beta_2\alpha\right)}{(\gamma_2 + l + \alpha)}\frac{\langle k^2\rangle}{\langle k\rangle}\right)^{k_{\max}-1}$$
$$\times\left(\lambda + \gamma_2 + l + \alpha\right)\left(\lambda + \gamma_1 + l - \frac{\left(\beta_1(\gamma_2 + l) - \beta_2\alpha\right)}{(\gamma_2 + l + \alpha)}\frac{\langle k^2\rangle}{\langle k\rangle}\right) = 0$$

where

$$\langle k^2\rangle = P(1) + 2^2P(2) + \cdots + k_{\max}^2P(k_{\max}).$$

The stability of E_0 is only dependent on

$$\left(\lambda + \gamma_2 + l + \alpha\right)\left(\lambda + \gamma_1 + l - \frac{\left(\beta_1(\gamma_2 + l) - \beta_2\alpha\right)}{(\gamma_2 + l + \alpha)}\frac{\langle k^2\rangle}{\langle k\rangle}\right) = 0.$$

Note that

$$\rho=\frac{\beta_1(\gamma_2+l)-\beta_2\alpha}{(\gamma_1+l)(\gamma_2+l+\alpha)}\cdot\frac{\langle k^2\rangle}{\langle k\rangle}.$$

So, we have obtained

$$(\lambda+\gamma_2+l+\alpha)\big(\lambda+(\gamma_1+l)(1-\rho)\big)=0.$$

When $\rho<1$, all real-valued eigenvalues are negative. Hence, E_0 is locally asymptotically stable if $\rho<1$.

Now we will prove that $E_0\left(\frac{\gamma_2+l}{\gamma_2+l+\alpha},0,\frac{\alpha}{\gamma_2+l+\alpha}\right)$ is globally attractive. From the second equation of system (1) we can get

$$\begin{aligned}\frac{\mathrm{d}\Theta_1(t)}{\mathrm{d}t}&=\frac{1}{\langle k\rangle}\sum_{k=1}^{n}kP(k)\big[k\beta_1\Theta_1(t)S_k(t)-k\beta_2\Theta_2(t)I_k(t)-\gamma_1I_k(t)-lI_k(t)\big]\\&<\left[\frac{1}{\langle k\rangle}\sum_{k=1}^{n}kP(k)k\beta_1S_k(t)-k\beta_2\Theta_2-(\gamma_1+l)\right]\Theta_1(t)\\&=\left[\frac{1}{\langle k\rangle}\sum_{k=1}^{n}kP(k)k\beta_1S_k(t)-\frac{1}{\langle k\rangle}\sum_{k=1}^{n}kP(k)C_k(t)k\beta_2-(\gamma_1+l)\right]\Theta_1(t)\\&<\left[\frac{\beta_1(\gamma_2+l)}{\gamma_2+l+\alpha}\frac{\langle k^2\rangle}{\langle k\rangle}-\frac{\beta_2\alpha}{\gamma_2+l+\alpha}\frac{\langle k^2\rangle}{\langle k\rangle}-(\gamma_1+l)\right]\Theta_1(t)\\&=\left[\frac{\beta_1(\gamma_2+l)-\beta_2\alpha}{\gamma_2+l+\alpha}\frac{\langle k^2\rangle}{\langle k\rangle}-(\gamma_1+l)\right]\Theta_1(t)\\&=(\gamma_1+l)(\rho-1)\Theta_1(t)\end{aligned}$$

Now we consider the comparison equation with the condition $\Theta_1(0)=\varphi(0)$ as follows

$$\frac{\mathrm{d}\varphi(t)}{\mathrm{d}t}=(\gamma_1+l)(\rho-1)\varphi(t),$$

integrating from 0 to *t* yields

$$\varphi(t)=\varphi(0)\mathrm{e}^{(\gamma_1+l)(\rho-1)t}.$$

Since $\rho<1$, we obtain $\varphi(t)\to 0$ as $t\to 0$.

According to the comparison theorem of functional differential equation, we can get $0\le\Theta_1(t)\le\varphi(t)$, for all $t>0$.

Thus, $\Theta_1(t)\to 0$ as $t\to\infty$, which implies $I_k\to 0$ as $t\to\infty$, for $k=1,2,\cdots,n$. It follows that the rumor-free equilibrium E_0 is globally attractive. This completes the proof.

Theorem 3. If $\rho>1$, the rumor is permanent on complex social networks, *i.e.*, there exists a $\xi>0$, such that

$$\liminf_{t\to\infty} I(t)=\liminf_{t\to\infty}\sum_k P(k)I_k(t)>\xi.$$

Proof. We will use the result of Thieme in Theorem 4.6 [21] to prove it. Define

$$X=\left\{\left(S_1,I_1,C_1,\cdots,S_{k_{\max}},I_{k_{\max}},C_{k_{\max}}\right):S_k,I_k,C_k\ge 0\text{ and }S_k+I_k+C_k=1,k=1,\cdots,k_{\max}\right\},$$

$$X_0=\left\{\left(S_1,I_1,C_1,\cdots,S_{k_{\max}},I_{k_{\max}},C_{k_{\max}}\right)\in X:\sum_k P(k)I_k>0\right\},$$

$$\partial X_0=X\setminus X_0.$$

In the following, we will show that (1) is uniformly persistent with respect to $(X_0,\partial X_0)$.

Obviously, X is positively invariant with respect to system (1). If $S_k(0) \ge 0$, $\sum_k P(k) I(k) > 0$ and $\sum_k P(k) C(k) > 0$ for $k = 1, \cdots, k_{\max}$, then $S_k(0) \ge 0$, $\sum_k P(k) I(k) > 0$ and $\sum_k P(k) C(k) > 0$ for all $t > 0$. Since $\left(\sum_k P(k) I_k(t)\right)' \ge -(\gamma_1 + l) \sum_k P(k) I_k(t)$, $\left(\sum_k P(k) C_k(t)\right)' \ge -(\gamma_2 + l) \sum_k P(k) C_k(t)$, $\sum_k P(k) I(0) > 0$ and $\sum_k P(k) C(0) > 0$, we have $\sum_k P(k) I_k(t) \ge \sum_k P(k) I_k(0) \mathrm{e}^{-(\gamma_1 + l)t} > 0$, $\sum_k P(k) C_k(t) \ge \sum_k P(k) C_k(0) \mathrm{e}^{-(\gamma_2 + l)t} > 0$. Thus, X_0 is also positively invariant. Furthermore, there exists a compact set B in which all solutions of (1) initiated in X will enter and remain forever after. The compactness condition (C4.2) in Thieme [21] is easily verified for this set B. Denote

$$M_\partial = \Big\{ \left(S_1(0), I_1(0), C_1(0), \cdots, S_{k_{\max}}(0), I_{k_{\max}}(0), C_{k_{\max}}(0)\right): \\ \left(S_1(t), I_1(t), C_1(t), \cdots, S_{k_{\max}}(t), I_{k_{\max}}(t), C_{k_{\max}}(t)\right) \in \partial X_0, t \ge 0 \Big\}.$$

Denote

$$\Omega = \bigcup \Big\{ \omega\left(S_1(0), I_1(0), C_1(0), \cdots, S_{k_{\max}}(0), I_{k_{\max}}(0), C_{k_{\max}}(0)\right): \\ \left(S_1(0), I_1(0), C_1(0), \cdots, S_{k_{\max}}(0), I_{k_{\max}}(0), C_{k_{\max}}(0) \in X\right) \Big\}$$

where $\omega\left(S_1(0), I_1(0), C_1(0), \cdots, S_{k_{\max}}(0), I_{k_{\max}}(0), C_{k_{\max}}(0)\right)$ is the omega limit set of the solutions of system (1) starting in $\left(S_1(0), I_1(0), C_1(0), \cdots, S_{k_{\max}}(0), I_{k_{\max}}(0), C_{k_{\max}}(0)\right)$. Restricting system (1) on M_∂ gives

$$\begin{cases} \dfrac{\mathrm{d}S_k(t)}{\mathrm{d}t} = l + \gamma_2 C_k(t) - l S_k(t) - \alpha S_k(t) \\ \dfrac{\mathrm{d}I_k(t)}{\mathrm{d}t} = -\gamma_1 I_k(t) - l I_k(t) \\ \dfrac{\mathrm{d}C_k(t)}{\mathrm{d}t} = \alpha S_k(t) - \gamma_2 C_k(t) - l C_k(t) \end{cases} \tag{14}$$

It is easy to verify that system (13) has a unique equilibrium E_0 in X. Thus E_0 is the unique equilibrium of system (1) in M_∂. It is easy to check that E_0 is locally asymptotically stable. This implies that E_0 is globally asymptotically stable for (13) is a linear system. Therefore $\Omega = \{E_0\}$. And E_0 is a covering of Ω, which is isolated and is acyclic (since there exists no solution in M_∂ which links E_0 to itself). Finally, the proof will be done if we show E_0 is a weak repeller for X_0, *i.e.*

$$\limsup_{t \to \infty} \operatorname{dist}\left(\left(S_1(t), I_1(t), C_1(t), \cdots, S_{k_{\max}}(t), I_{k_{\max}}(t), C_{k_{\max}}(t)\right), E_0\right) > 0,$$

where $\left(S_1(t), I_1(t), C_1(t), \cdots, S_{k_{\max}}(t), I_{k_{\max}}(t), C_{k_{\max}}(t)\right)$ is an arbitrarily solution with initial value in X_0. By Leenheer and Smith (2003, Proof of Lemma 3.5, [22]), we need only to prove $W^s(E_0) \cap X_0 = \varnothing$ where $W^s(E_0)$ is the stable manifold of E_0. Suppose it is not true, then there exists a solution $\left(S_1(t), I_1(t), C_1(t), \cdots, S_{k_{\max}}(t), I_{k_{\max}}(t), C_{k_{\max}}(t)\right)$ in X_0, such that

$$S_k(t) \to \frac{\gamma_2 + l}{\gamma_2 + l + \alpha}, I_k(t) \to 0, C_k(t) \to \frac{\alpha}{\gamma_2 + l + \alpha} \quad \text{as} \quad t \to \infty. \tag{15}$$

Since $\rho = \dfrac{\beta_1(\gamma_2 + l) - \beta_2 \alpha}{(\gamma_1 + l)(\gamma_2 + l + \alpha)} \cdot \dfrac{\langle k^2 \rangle}{\langle k \rangle}$, we can choose $\eta > 0$ such that

$$\frac{\left[\beta_1(\gamma_2+l)-\beta_2\alpha\right]\left(\frac{\gamma_2+l}{\gamma_2+l+\alpha}-\eta\right)}{(\gamma_1+l)(\gamma_2+\alpha+l)}\cdot\frac{\langle k^2\rangle}{\langle k\rangle}>1.$$

For $\eta>0$, by (15) there exists a $T>0$ such that

$$\frac{\gamma_2+l}{\gamma_2+l+\alpha}-\eta<S_k(t)<\frac{\gamma_2+l}{\gamma_2+l+\alpha}+\eta, 0\le I_k(t)<\eta, \frac{\alpha}{\gamma_2+l+\alpha}-\eta<C_k(t)<\frac{\alpha}{\gamma_2+l+\alpha}+\eta.$$

For all $t\ge T$ and $k=1,2,\cdots,k_{\max}$. Let

$$V(t)=\sum_k(\gamma_2+l)kP(k)I_k(t).$$

The derivative of V along the solution is given by

$$\begin{aligned}
\dot{V}(t)&=\sum_k(\gamma_2+l)kP(k)\dot{I}_k(t)\\
&=\sum_k(\gamma_2+l)kP(k)\left[k\beta_1S_k(t)\frac{\sum_k kP(k)I_k(t)}{\langle k\rangle}-k\beta_2I_k(t)\frac{\sum_k kP(k)C_k(t)}{\langle k\rangle}-(\gamma_1+l)I_k(t)\right]\\
&\ge\sum_k P(k)\frac{k^2\beta_1(\gamma_2+l)}{\langle k\rangle}\left(\frac{\gamma_2+l}{\gamma_2+l+\alpha}-\eta\right)\sum_k kP(k)I_k(t)\\
&\quad-\sum_k P(k)\frac{k^2\beta_2(\gamma_2+l)}{\langle k\rangle}\left(\frac{\alpha}{\gamma_2+l+\alpha}+\eta\right)\sum_k kP(k)I_k(t)-\sum_k(\gamma_2+l)kP(k)(\gamma_1+l)I_k(t)\\
&=\sum_k P(k)k\frac{\beta_1(\gamma_2+l)\langle k^2\rangle}{\langle k\rangle}\left(\frac{\gamma_2+l}{\gamma_2+l+\alpha}-\eta\right)I_k(t)\\
&\quad-\sum_k P(k)k\left[\frac{\beta_2\alpha\langle k^2\rangle}{\langle k\rangle}\left(\frac{\gamma_2+l}{\gamma_2+l+\alpha}+(\gamma_2+l)\eta\right)-(\gamma_2+l)(\gamma_1+l)\right]I_k(t)\\
&>\sum_k P(k)k\left[\frac{\beta_1(\gamma_2+l)\langle k^2\rangle}{\langle k\rangle}\left(\frac{\gamma_2+l}{\gamma_2+l+\alpha}-\eta\right)-\frac{\beta_2\alpha\langle k^2\rangle}{\langle k\rangle}\left(\frac{(\gamma_2+l)}{\gamma_2+l+\alpha}-\eta\right)\right]I_k(t)\\
&\quad-\sum_k P(k)k(\gamma_2+\alpha+l)(\gamma_1+l)I_k(t)\\
&=\sum_k P(k)k\left[\left[\beta_1(\gamma_2+l)-\beta_2\alpha\right]\frac{\langle k^2\rangle}{\langle k\rangle}\left(\frac{\gamma_2+l}{\gamma_2+l+\alpha}-\eta\right)-(\gamma_2+l+\alpha)(\gamma_1+l)\right]I_k(t)\\
&>\sum_k kP(k)I_k(t)\ge 0
\end{aligned}$$

Hence $V(t)\to\infty$ as $t\to\infty$, which contradicts to the boundedness of $V(t)$. This completes the proof.

4. Numerical Simulations

In this section, several numerical simulations are presented to illustrate our analysis. We consider the system (1) on a complex social network with $P(k)=k^{-3}$, where the parameter η satisfies $\sum_{k=1}^{n}\eta k^{-3}=1, n=1000$.

In **Figure 2**, the parameters are chosen as $l=0.1,\alpha=0.3,\gamma_1=0.45,\gamma_2=0.3,\beta_1=0.5,\beta_2=0.4,$ then the basic reproductive number $\rho=0.9462<1$. We can see that when $\rho<1$, I_k grows to zero, *i.e.*, the infectious individuals will ultimately disappear.

In **Figure 3**, we choose $l=0.1,\alpha=0.3,\gamma_1=0.1,\gamma_2=0.3,\beta_1=0.5,\beta_2=0.01,$ thus the basic reproductive number $\rho=6.4073>1$. We can see that when $\rho>1$, the rumor is persist and the infected individuals' number will converge to a positive constant respectively.

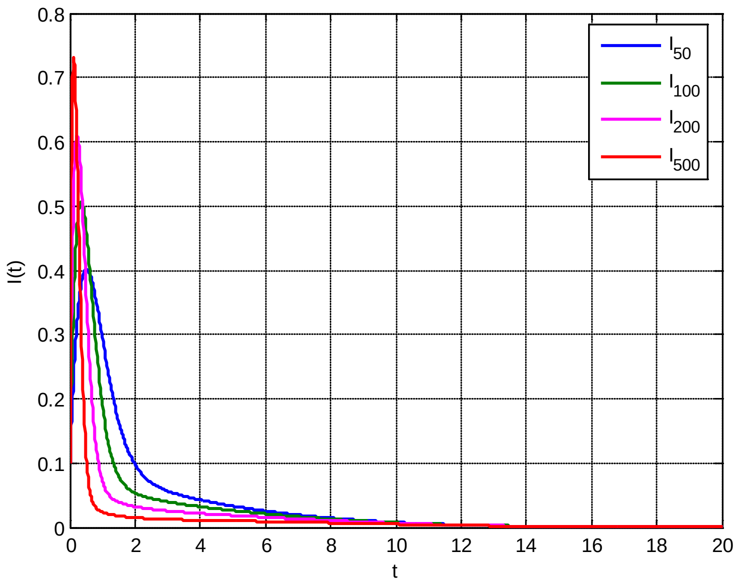

Figure 2. The time series of system (1) with $\rho = 0.9462 < 1$ and initial values $I_k(0) = 0.1$, $k = 50,100,200,500$.

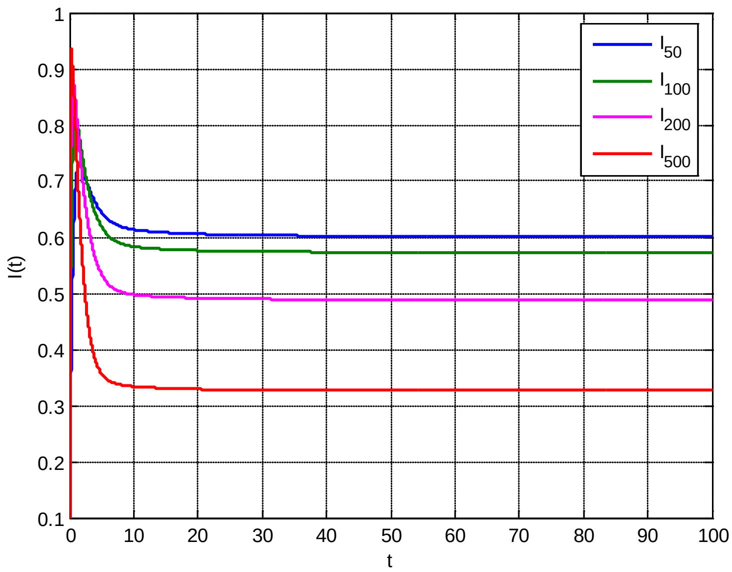

Figure 3. The time series of system (1) with $\rho = 6.4073 > 1$ and initial values $I_k(0) = 0.1$, $k = 50,100,200,500$.

In **Figure 4**, numerical simulations show the spread of *SICS* model on complex social networks with $l = 0.1, \alpha = 0.3, \gamma_1 = 0.45, \gamma_2 = 0.3, \beta_1 = 0.5$ and $\beta_2 = 0.3, \beta_2 = 0.4, \beta_2 = 0.6$. The condition of $\rho < 1$, that different β_2 leading to different states. In addition, it is also found that the larger the β_2 is, the rumor dies out faster.

In **Figure 5**, numerical simulations show the spread of *SICS* model on complex social networks with $l = 0.1, \alpha = 0.3, \gamma_1 = 0.1, \gamma_2 = 0.3, \beta_1 = 0.5$ and $\beta_2 = 0.01, \beta_2 = 0.04, \beta_2 = 0.1$. The condition of $\rho > 1$, that different β_2 leading to different states. In addition, it is also found that the larger the β_2 is, the positive equilibrium will be lower. The simulations indicate that the numerical results are well consistent with the theoretical analysis.

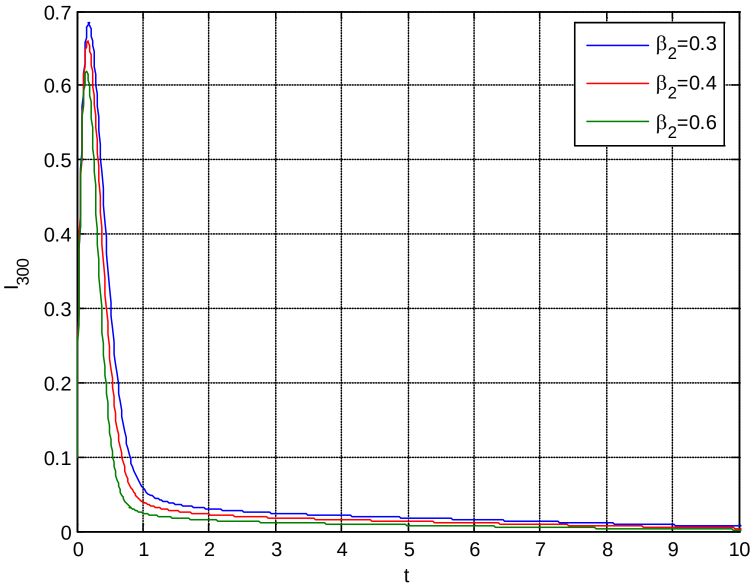

Figure 4. The prevalence I_{300} versus *t* corresponding to different β_2 with identical initial value $I_{300}(0) = 0.1$.

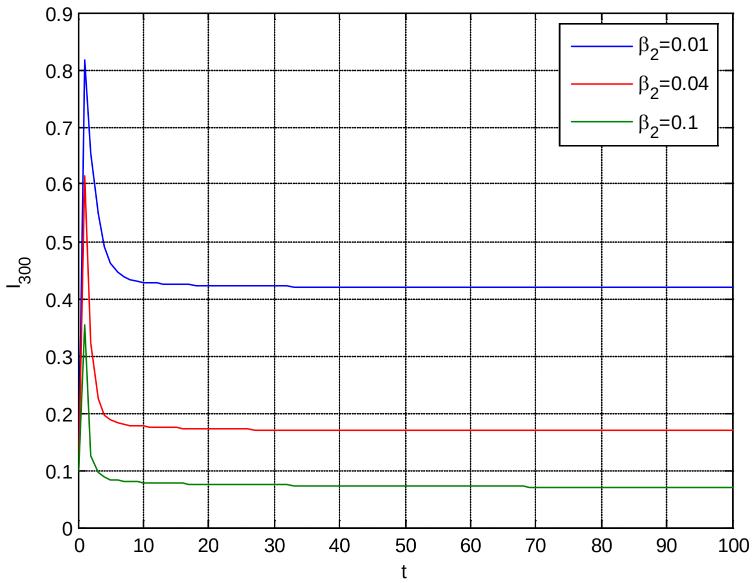

Figure 5. The prevalence I_{300} versus *t* corresponding to different β_2 with identical initial value $I_{300}(0) = 0.1$.

5. Conclusion

In summary, we present a new *SICS* rumor spreading model with counter mechanism on complex social networks. By using the mean-field theory, we obtain the basic reproductive number and equilibriums. Theoretical results indicate that the basic reproductive number is significantly dependent on the topology of the underlying networks and the counter mechanism. The basic reproductive number is in direct proportion to $\langle k^2 \rangle / \langle k \rangle$. So, network heterogeneity makes rumor easy to spread. Moreover, we found that the greater β_2 can decrease the basic reproductive number ρ, *i.e.*, lower average rumor density and shorter rumor prevalent decay time. The global stability of rumor-free equilibrium and the permanence of rumor are proved in detail. Our theoretical and numerical simulation results give a novel explanation for rumor spreading. This study has valuable guiding significance in effectively preventing rumor spreading.

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