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# The Relation between the Pressure Gradient of General Relativity and Its Newtonian Counterpart with Respect to Certain Stellar Models

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## Abstract

**The non-uniqueness of the relation between the pressure gradients of general relativity and its Newtonian counterpart is demonstrated. This may lead to a non-unique relation between the two force laws.**

## Keywords

**Pressure Gradient, General Relativity, Newtonian Gravity**

**Subject Areas:** **Fluid Mechanics, Theoretical Physics**

## 1. Introduction

It is a well-known fact in Newtonian gravitation that the gravitational force and pressure gradient are proportional to each other in a fluid. In general relativity, on the other hand, the concept of "gravitational force" has to be properly defined before we can relate it with the general relativistic pressure gradient. In the present study, we bypass this tricky question of "general relativistic gravitational force" and try to relate the pressure gradient of Newtonian gravity with that of general relativity for different stellar models. To our surprise, we find that the relation between the general relativistic pressure gradient and its Newtonian counterpart is not unique and is very much model dependent. One of the consequences of this is that it can lead to a non-unique relation between the Newtonian force law and its general relativistic counterpart. The units used are those in which the constant of gravitation $G = 1$ and the speed of light $c = 1$.

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## 2. The Expressions for the Pressure Gradients in General Relativity and in Newtonian Gravity for a Class of Stellar Models

Here we restrict ourselves to a narrow class of stellar models [1] for which the density $\rho(r)$ (a function of *r* the radial coordinate) is

$$\rho(r)=\frac{1}{8\pi r^2}\left[\frac{2n-n^2}{2n+1-n^2}+\frac{n^2\left(3+5n-2n^2\right)}{(n+1)\left(2n+1-n^2\right)}\frac{1}{(2n+1)}\left(\frac{r}{a}\right)^{2\left(2n+1-n^2\right)/(n+1)}\right]. \tag{1}$$

The number *n* determines the density of the stellar material for a specified radius *r* = *a* of the star. The mass of the star *m* is related to these two quantities *n* and *a* by

$$\frac{m}{a}=\frac{n}{2n+1}. \tag{2}$$

The pressure $p(r)$ is

$$p(r)=\frac{n^2}{8\pi r^2\left(2n+1-n^2\right)}\left[1-\left(\frac{r}{a}\right)^{2\left(2n+1-n^2\right)/(n+1)}\right] \tag{3}$$

from which after differentiation with respect to *r* we obtain

$$\frac{\mathrm{d}p}{\mathrm{d}r}=\frac{n^2}{8\pi r^3\left(2n+1-n^2\right)}\left[-2-\frac{2\left(n-n^2\right)}{n+1}\left(\frac{r}{a}\right)^{2\left(2n+1-n^2\right)/(n+1)}\right]. \tag{4}$$

This is the general relativistic expression for pressure gradient. The Newtonian pressure gradient is obtained by invoking the concept of equilibrium (hydrostatic) in Newtonian physics [2] (see equation numbers (1) and (4) of chapter 3 of this reference) $M(r)=\int_0^r 4\pi r^2\rho(r)\mathrm{d}r$ and $\frac{\mathrm{d}p}{\mathrm{d}r}=-\frac{M(r)}{r^2}\rho(r)$ from which we obtain

$$\begin{aligned}\frac{\mathrm{d}p}{\mathrm{d}r}=-\frac{1}{16\pi r^3}&\left[\frac{2n-n^2}{2n+1-n^2}+\frac{n^2\left(3+5n-2n^2\right)}{(n+1)\left(2n+1-n^2\right)(2n+1)}\left(\frac{r}{a}\right)^{2\left(2n+1-n^2\right)/(n+1)}\right]\\ &\times\left[\frac{2n-n^2}{2n+1-n^2}+\frac{n^2}{\left(2n+1-n^2\right)(2n+1)}\left(\frac{r}{a}\right)^{2\left(2n+1-n^2\right)/(n+1)}\right].\end{aligned} \tag{5}$$

## 3. Calculation of the Pressure Gradients for Models Having Different *n* but with the Almost Same Value of Mass *m*

The radius *a* of the stellar model is given by Equation (2) for the chosen values of *m* and *n*. We take for the first case $n=2.122\times10^{-6}$ and $a=696\times10^3$ km which roughly corresponds to *m* of the Sun. We keep the Newtonian value of $\frac{\mathrm{d}p}{\mathrm{d}r}$ same for all cases and the values of *r* corresponding to the different *n* values are shown in column 3 of **Table 1**. The general relativistic values of $\frac{\mathrm{d}p}{\mathrm{d}r}$ are shown in the last column of this table.

**Table 1.** The general relativistic pressure gradients for the same value of its Newtonian counterpart.

| *n* | *a* (km) | *r* (km) | $\frac{\mathrm{d}p}{\mathrm{d}r}$ (Newtonian) | $\frac{\mathrm{d}p}{\mathrm{d}r}$ (GR) |
|---|---|---|---|---|
| $2.122\times10^{-6}$ | $696\times10^3$ | $1.1\times10^5$ | $-2.692144\times10^{-28}$ | $-2.692161\times10^{-28}$ |
| $4.244\times10^{-6}$ | $348\times10^3$ | $1.7461361\times10^5$ | -do- | $-2.692176\times10^{-28}$ |
| $1.061\times10^{-6}$ | $1392\times10^3$ | $6.929578\times10^4$ | -do- | $-2.692152\times10^{-28}$ |

Thus without a rigorous proof, one can make an important conjecture: *The deviation of force law in general relativity from its Newtonian counterpart is dependent not only on the strength of the force but also on other parameters which may be dependent on or independent of it but strictly dependent on the model*. This in our opinion will cause serious difficulties in defining the Newtonian limit of general relativity.

## Referencess

[1] Durgapal, M.C. and Gehlot, G.L. (1968) Exact Internal Solutions for Dense Massive Stars. *Physical Review*, **172**, 1308-1309. http://dx.doi.org/10.1103/PhysRev.172.1308

[2] Chandrasekhar, S. (1958) An Introduction to the Study of Stellar Structure. Dover Publications Inc., Mineola.