

Capacitance of a Three-Dimensional Interdigitated (MIM) Capacitor

Frederick J. Young

School of Communications and Arts, University of Pittsburgh at Bradford, Bradford, PA, USA Email: <u>youngfj@youngbros.com</u>

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Abstract

The capacitance of a Metal-Insulator-Metal (MIM) capacitor first described in United States Letters Patent 20070278551 (2007) is analysed in three dimensions. The finite element computer program is given so that others may make similar calculations with their own dimensions and dielectric materials. Plots of potentials and fields are presented.

Keywords

MIMS Capacitor, 3D Finite Element Solution, Voltage Rating, BaTi₁0₃ Dielectric

Subject Areas: Applied Physics, Electric Engineering

1. Capacitance of a Three-Dimensional Interdigitated (MIM) Capacitor

The geometry considered here was taken from a US Letters Patent [1]. The abstract of the invention given in the patent follows:

"Abstract: An interdigitated Metal-Insulator-Metal (MIM) capacitor provides self-shielding and accurate capacitance ratios with small capacitance values. The MIM capacitor includes two terminals that extend to a plurality of interdigitated fingers separated by an insulator. Metal plates occupy layers above and below the fingers and connect to fingers of one terminal. As a result, the MIM capacitor provides self-shielding to one terminal. Additional shielding may be employed by a series of additional shielding layers that are isolated from the capacitor. The self shielding and additional shielding may also be implemented as an array of MIM capacitors."

The geometry of the capacitor considered is shown in the Z = 0 plane in **Figure 1**. Regions 21 and 22 are the capacitor plates with region 21 shielding the top by being held at ground potential. Region 22 is the hot side of the capacitor and regions 24 and 25 are part of the shielding ground. The space between regions 21 and 22 is the dielectric. **Figure 2** shows a simplified view of the capacitor to aid in setting up the geometry for the script. Here the blue lines are the remains of region 1 being an 8 by 34 mil rectangle. Region 2 comprises barium



Figure 1. The geometry given in United States Letters Patent 20070278551.



Figure 2. Right half of MIM with boundary conditions.

titanate (BaTi₁O₃) that covers the path leading from (0.3) to (1.3) to (1.29) to (5.29) to (5.3) to (7.3) to (7.31) to (1.31) to the upper blue line at (1.34). For a distance of 1 mil to the right of this path, it is the layer of barium

titanate. The upper compartment is part of the first region of air dielectric. The two other compartments must be defined as voids so that they will not enter into the calculations. These are given by regions 3 and 4 in the script. To find the capacitance a field calculation is necessary. The potential in the capacitor can be calculated by solving Laplace's equation [1] in the interior of the capacitor [2]. It is given by

$$\nabla^2 \varphi = 0 \tag{1}$$

subject to boundary conditions (1) $\varphi = V_0$ on region 22 and (2) $\varphi = 0$ on region 21. Symmetry $\partial \varphi / \partial n = 0$ has been invoked by default on the edges of region 1. This causes the solution to the right-half of the XY plane to be a mirror image of the left-half plane. The second condition is necessary for the outer boundary and is of little importance when Y = -2. The Dirichlet boundary conditions are applied in region 2 to the barium titanate dielectric as depicted in Figure 2. Other barium titanate boundaries are held at the potentials given also in said figure. This problem is most easily solved by extruding the geometry of Figure 2. In three dimensional problems it is always best to inspect the various means of extrusion in order to find the simplest. Figure 3 shows the outlines of the extrusion used here. The same condition is used on the XY plane at two mills above the capacitor plates. The boundary condition on the XY plane at Z = 0 is $\partial \varphi / \partial Z = 0$ to yield symmetry about that plane. The capacitor half-height in the Z-direction is 24 mils. Thus by symmetry one quarter of the problem is worked. The conducting plates of the capacitor are modeled as voids. The voltage boundary conditions are applied to region 2 which is the high dielectric filling of the capacitor. The script for the solution of this problem is given below. Lines 2 and 3 set up three dimension coordinates X, Y, Z Lines 4 and 5 set the number of contours on the graphs to 4, and cause the coordinate labels to be in mil (thousands of an inch) and the error limit to be $\Box 0.001$. Lines 6 and 7 set the main variable to be V, the electrostatic potential. Lines 8 - 13 give definitions and relate various quantities to each other. Line 9 sets the relative primitivities kdie = 8000, kair = 1, k = kair, the ground potential $V_0 = 0$, the high potential $V_1 = 1$ and the permittivity of air Eps0 = 2.2489e-16 farads/mil. Line 10 establishes energy relationships. Here the stored energy density is given by

$$W_p = \kappa \epsilon_0 \left(\nabla \varphi \right)^2 \tag{2}$$

and the total stored energy is given by



$$W = \iiint_{\text{vol}} W_p dX dY dZ \tag{3}$$

Line 11 defines the capacitance in microfarads as

$$C = 2.0e6 \cdot W / (V_1 - V_0)^2 \tag{4}$$

and the electric field as

$$\mathbf{E} = -\nabla \varphi \tag{5}$$

and the electrostatic induction as

$$\mathbf{D} = \kappa \epsilon_0 \mathbf{E} \tag{6}$$

Line 12 sets geometrical variables and establishes the magnitude of the electric field in all space as

$$E_{\rm mag} = |\mathbf{E}| \tag{7}$$

In line 13 are the instructions for finding the largest magnitude of the electric field and its coordinates. Lines 14 and 15 determine the equation to be solved by finite elements. In line 15

$$\nabla \cdot \mathbf{D} = \mathbf{0} \tag{8}$$

is to be executed subject to Equations (5) and (6). Extrusion of the geometry in the Z-direction begins in line 15. Lines 16 - 24 define the surfaces on various XY planes and set their boundary conditions. In lines 25 - 39 the XY geometry to be extruded is defined and boundary conditions are established. There the air or dielectric in each region is specified. Line 40 allows the solution to be seen at various stages before the error is sufficiently reduced. In line 41 various plots and reports are made and saved when the solution is finished.

2. Computer Program

DigiCap3D.pde

1. TITLE 'Three Dimension Interdigitated Capacitor Analysis'

{ The geometry for this example was taken from Metal-insulator-metal capacitors:United States Patent 20070278551.

Inventor: Anthony, Michael P. (Andover, MA, US) Link to this page:http://www.freepatentsonline.com/20070278551.html } 2. COORDINATES 3. CARTESIAN3 4. SELECT

5. contours = 4 alias(x) = "X(mil)" alias(y) = "Y(mil)" alias(z) = "Z(mil)" errlim = 0.0016. VARIABLES

7. V

8. DEFINITIONS

9. kdie = 8000 kair = 1 k = kair V0 = 0 V1 = 1 Eps0 = 2.2489e-16 { Farads/mil }

10. Wp = $0.5 K^{eps0}$ grad(V)942 { Stored Energy Density } W = vol_integral(Wp) { Total Stored Energy }

11. C = 1.0e6*2*W/(V1-V0)942 { Capacitance in microFarads } E = -grad(V) D = K*Eps0*E

12. Cth = 2 Dth = 1 Xmax = 2.5*Cth+3*Dth Ymax = 34 Z0 = 2 Zth = 24 Emag = magnitude(E)

13. Emax = globalmax(Emag) XX = globalmax_x(Emag) YY = globalmax_y(Emag) ZZ = globalmax_z(Emag)
14. EQUATIONS
15. DIV(D) = 0

16. EXTRUSION

17. SURFACE "Bottom" Z = 0

18. LAYER "Dielectric"

19. SURFACE "Top Metal - Dielectric" Z = Zth

20. LAYER "Air"

21. SURFACE "Top" Z= Z0 + Zth

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22. BOUNDARIES
  23. SURFACE "Bottom" natural(V)=0
  24. SURFACE "Top" natural(V)=0
  25. BOUNDARIES
  26. REGION 1 'Universe'
  27. surface "Bottom"
  28. layer 'Dielectric' k = kair
  29. start(0.Cth) line to (Xmax.Cth) to (Xmax.Ymax) to (0.Ymax) to close
  30. REGION 2 'Dielectric'
  31. SURFACE 'Top Metal - Dielectric'
  32. LAYER "Dielectric" k = kdie
  33. start(0,Cth+Dth) value(V) = V1 line to (Cth/2,Cth+Dth) to (Cth/2,Ymax-2*Cth-Dth) to
(2*Dth+1.5*Cth,Ymax-2*Cth-Dth) to (2*Dth+1.5*Cth,Cth+Dth) to (2*Dth+2.5*Cth,Cth+Dth) to
(2*Dth+2.5*Cth,Ymax-Cth-Dth) to (Cth/2,Ymax-Cth-Dth) to (Cth/2,Ymax) natural(V)=0 line to
(Dth+Cth/2,Ymax) value(V)=V0 line to (Dth+Cth/2,Ymax-Cth) to (3*Dth+2.5*Cth,Ymax-Cth) to
(3*Dth+2.5*Cth,Cth) to (Dth+1.5*Cth,Cth) to (Dth+1.5*Cth,Ymax-2*Cth-2*Dth) to
(Dth+Cth/2,Ymax-2*Cth-2*Dth) to (Dth+Cth/2,Cth) to (0,Cth) natural(V) = 0 line to close
  34. REGION 3
  35. SURFACE 'Top Metal - Dielectric'
  36. layer "Dielectric" void
  37. start(0,Cth+Dth) line to (Cth/2,Cth+Dth) to (Cth/2,Ymax-2*Cth-Dth) to
(2*Dth+1.5*Cth,Ymax-2*Cth-Dth) to (2*Dth+1.5*Cth,Cth+Dth) to (2*Dth+2.5*Cth,Cth+Dth) to
(2*Dth+2.5*Cth,Ymax-Cth-Dth) to (Cth/2,Ymax-Cth-Dth) to (Cth/2,Ymax) to close
  38. start (Dth+Cth/2,Cth) line to (Dth+Cth/2,Ymax-2*Cth-2*Dth) (Dth+1.5*Cth,Ymax-2*Cth-2*Dth) to
(Dth+1.5*Cth,Cth) to close
  39. start(Xmax,Cth) line to (Xmax,Ymax) to (Dth+Cth/2,Ymax) to (Dth+Cth/2,Ymax-Cth) to
(3*Dth+2.5*Cth,Ymax-Cth) to close
  40. MONITORS
  contour(V) on Z = (Z0 + Zth)/2 contour(V) on X = 0 vector(E) on Z = (Z0 + Zth)/2 report C
  41. PLOTS
  grid (x,y,z) grid(x,y) on Z = Z0+Zth grid(y,z) on X = 3 contour(V) on Z = (Z0 + Zth)/2 report(C)
report(Emax)
  vector(E) on Y = 3 as 'Electric intensity' vector(E) on Y = Ymax/2 as 'Electric intensity'
  vector(E) on X = 2^{*}(Dth+Cth) \operatorname{zoom}(0,17,10,10) as 'Electric intensity'
  vector(E) on Z = Z0 zoom(Dth+2.5*Cth,Ymax-Cth-2*Dth,2*Dth,2*Dth) as 'Electric intensity'
  vector(E) on Z = (Z0 + Zth)/2 zoom(Dth+2.5*Cth,Ymax-Cth-2*Dth,2*Dth,2*Dth) as 'Electric intensity'
  vector(E) on Z = Zth/2 zoom(Dth+2.5*Cth,Ymax-Cth-2*Dth,2*Dth,2*Dth) as 'Electric intensity'
  vector(E) on Z = 13.8851 zoom(XX-0.05,YY-0.05,0.1,0.1) report(Emax) surface(Emag) on Z = 13.8851
  vector(E) on z = (Z0 + Zth)/2 zoom(0,26,8,8) as 'Electric intensity' vector(E) on z = (Z0 + Zth)/2
zoom(0,1,8,8) as 'Electric intensity'
  report(C) as 'Total capacitance in ufarads'
  42. SUMMARY
  Report(C) as 'Total Capacitance in ufarads'
  REPORT(C/Zth) as "Capacitance(uF/mil in z dimension)"
  REPORT(W) as "Stored Energy"
  REPORT(Emax) as 'The maximum electric intensity'
  REPORT(XX) as 'The X-coordinate of the maximum electric intensity'
  REPORT(YY) as 'The Y-coordinate of the maximum electric intensity'
  REPORT(ZZ) as 'The Z-coordinate of the maximum electric intensity'
  END
```

3. Results

Note the aforementioned symmetry conditions under boundaries. The total field energy is calculated by

integrating over the entire volume. From that the capacitance is calculated for this quarter of the whole capacitor. The dimensions of various parts of the device are in literal form so that different designs may be done easily. The global max function is applied to the magnitude of the electric intensity vector to determine where the largest value occurs. Figure 3 shows the potential distribution in the dielectric of the capacitor on an internal Z plane. It is important to check this figure to make sure that the correct boundary conditions on potential have been used. The Dirichlet conditions have been met and the symmetry condition required at X = 0 has been met for $2 \le Y \le 3$ or in more general terms $C_{th} \le Y \le C_{th} + D_{th}$. The equipotentials are crowded near some of the corners, indicating larger electric intensities there than elsewhere. In Figure 4 and Figure 5 the vector plots of the upper and lower corners in the $Z = (Z_0 + Z_{th})/2$ plane are shown. In the region not exhibited the electric intensity is almost constant as it would be in a simple one dimensional parallel plate capacitor.

The electric intensity is important because it must not exceed the breakdown electric intensity of the dielectric material. Previous calculations would indicate that the electric intensity may be greatest near sharp edges or corners. Figure 6 shows the electric intensity on the Z = 13.8851 plane where the electric intensity reaches its maxima. It is sometimes difficult to present the E vectors on the correct plane to capture the largest electric intensity vector in a plot.

The script for solving this problem contains many more plots of the mesh and vectors at various places. The summary yields total capacitance of $4.06 \times 10^{-3} \mu$ farads and a maximum electric intensity of 3.97 volts/mil at the coordinates (2.28, 13.8851). An estimate of the capacitance is obtained by expressing the plate area as

$$A_p = l_{\text{mean}} Z_{th} \tag{9}$$

where A_p = the plate area, l_{mean} = the mean length of the dielectric in the X and Y directions and Z_{th} = the height of the plates. This calculation which neglects fringing estimates the total capacitance to be $4.19 \times 10^{-3} \mu$ farads. For the entire capacitor these values of capacitance must be multiplied by four because of the symmetry used in this script. In this geometry the greatest value of a three-dimensional calculation is prediction of where the largest value of electric intensity occurs. To actually see the largest electric intensity vector would require the calculation of more electric intensity plots and is left as a task for the interested reader. For every dielectric there exists a value of electric intensity, called the dielectric breakdown strength, which will







Figure 5. Electric intensity in the vicinity of its maxima.



DiCap3D2g: Grid#5 p2 Nodes=177167 Cells=118465 RMS Err=9.3e-4 Integral= 94.58383

Figure 6. The magnitude of the electric intensity on the Z = 13.89 plane.

result in arcing and dielectric damage. The exact value depends upon temperature, density and material composition. The variation of relative permittivity with material composition is given by V. A. Russel [3]. For a dielectric containing 65% BaTiO₃ and 35% SrTiO₃ the breakdown strength of the dielectric is given in Table 1.

Table 1. Dielectric break down voltage versus density.	
Density (g/cc)	Breakdown (volts/mil)
5.41	122
5.44	142
5.46	190
5.56	291
5.57	295
5.59	350

Table 1 indicates that the breakdown voltage can be as large as 350 volts/mil for the densest dielectric. In that case the capacitor under consideration could operate with an applied voltage less than 350/3.97 = 88.16 volts without being damaged. In Figure 6, the surface of the magnitude of the electric intensity on the Z = 13.8851 plane is exhibited. The sharp peaks in the magnitude of the electric intensity in corners are rather large compared to the value of the electric intensity in the rest of the dielectric.

References

- [1] Anthony, M.P. and Andover, M.A. (2007) Metal-Insulator-Metal Capacitors. US Patent 20070278551. http://www.freepatentsonline.com/20070278551.html
- [2] Panofsky, W.K.H. and Phillips, M. (1955) Classical Electricity and Magnetism. Addison-Wesley Publishing Company, Inc., Cambridge, Mass.
- Russel, V.A. (1962) Ceramic Material of High Dielectric Strength Containing Barium Titanate and Method of Manufacture. US Patent 3049431. <u>http://www.freepatentsonline.com/3049431.pdf</u>